HYDRAULIC MACHINES

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By

Dr. JAGDISH LAL
D Sc. Tech. (ETH Zurich), B.M E. Hons. (Jadavpur)
A M.I.Mech.E. (London), A.M.I.E. (Ind.)
Professor & Head of Mechanical Engineering Department,
Punjab Engineering College, Chandigarh

Metropolitan Book Co. Private Ltd.
BOOKSELLERS & PUBLISHERS
1, FAIZ BAZAR, DELHI.

Published by
Shri B. V. Gupta for Metropolitan Book Co. Private Ltd.,
1, Faiz Bazar, Delhi-6.

THIRD EDITION (Revised & Enlarged)

1961

Printed by
Shri Gopinath Seth, at Navin Press,
6, Faiz Bazar, Delhi-6.

To

My Professors at home & abroad

Prof. Dr. S. C. Bhattacharyya

Ex-Head of Mechanical Engineering
Department
Jadavpur University
Calcutta

Prof. R. Dubs

Ex-Head of Hydraulics &
Hydraulic Machines Section
Swiss Federal Institute of Technology (ETH)
Zurich (Switzerland)

FOREWORD

to

First Edition

Immediately on his return from Zurich, Dr. Jagdish Lal joined the Indian Institute of Technology. He was given the responsibility for organising and equipping the Hydraulic Machines Laboratory. Such a laboratory, as distinct from the Hydraulics Laboratory of the Civil Engineering Department is now considered an essential feature in a modern Technical Institute.

In, a rapidly developing country, it is difficult to decide where the responsibility of the hydraulic engineer ends and that of hydraulic machine engineer begins. It is best that each know as much of the other's job as possible. This book thus covers a field of study which is rapidly gaining in importance.

Dr. Lal has the enthusiasm that we associate with a teacher who remembers the days when he was a brilliant student. This book has grown out of that enthusiasm, and has made available to the students in a handy form, knowledge and information which will be of value not only for the completion of their courses but also for practising their profession.

The book emphasises the need of young engineers acquiring great efficiency in using the tool of mathematics. To them the study of the Chapter on Dimensional Analysis is specially recommended.

The book has the merit that theoretical treatment has been well blended with descriptive and practical details, and should add to the reputation of Dr. Lal as a teacher of the subject.

J. C. Ghosh

(Dr. J.C. Ghosh)
Late Member Planning Commission,
New Delhi.

PREFACE TO THE THIRD EDITION

The second edition which was thoroughly revised and enlarged, exhausted earlier than it was expected. In the present edition the five basic sections, in which the book was divided when it appeared in second edition, remain unaltered ; however their contents have been revised in the light of the decision of Government of India to change over from FPS to metric system.

All formulae, equations and tables have now been given in both the systems. The solutions of some of the problems have been given in metric units together with FPS units. Some problems, solved as well as unsolved, relating to metric units have also been added. Readers are therefore requested to go through the "Introduction to Metric Units" (Page xix). For the use of conversions from one system to the other, Appendices 1 and 2 will be very helpful.

It was felt that the procedure for testing turbines and pumps in the laboratory should be fully explained A few more articles (15.11 to 15.17) have, therefore, been added showing the above details as well as the data to be measured and to be calculated for conducting experiments in the laboratory. The tables (15.1 to 15.5) have been given showing the methods of recording the readings of different brake tests. These will be helpful for the students and research scholars in evaluating the results of the tests performed in the laboratories, as well as for the field engineers conducting experiments on the model and acceptance tests.

For the benefit of those who like to take up the design of a reciprocating pump, an article on "*Design of Valves*" (Art. 11.22) has been added in this new edition. *Deriaz Runner* which is now gaining importance for its use in pumped-storage plants has now been fully explained (See Art 10.18 and 10.19). It is not out of place to include the important items like *Surge Tank* and *Forebay* (Art. 8.15 and 8.16) under the Safety Devices for Penstock (Art. 8.14)

The following articles have thus been added to bring the book to the present requirements—

Safety devices for penstocks : surge tanks and forebay (Art 8.14).

Details of Deriaz runner and its use for pumped-storage plants (10.18 and 10.19).

Tidel Power Projects (Art 10.20). Banki Turbine (Art 6.18).

Boundary layer theory, its separation and prevention (Art 10.24 to 10.26).

Design of valves for reciprocating pumps (Art 11.22).

Layout, accessories and starting of centrifugal pump (Art 12.15).

Procedure of Testing for Turbines and Pumps (Art 15.11 to 15.17).

Hydraulic Crane (Art 16.15), Hydraulic Lift (Art 16 16).

The specifications of turbines given in the last edition of this book, have been checked. The data of a few more turbines, installed in India during the last few years, have been added. Any discrepency in regards to these specifications, pointed out by the readers will be appreciated.

The author is grateful to Shri S.P. Battoo of Indian Standard Institution (ISI), New Delhi for supervising the printing work and in the preparation of new illustrations. Shri A.D. Kapur Asstt. Prof. checked to a large extent answers of unsolved problems. Shri M.M. Sehgal Asstt. Prof. helped in reading the proofs. My thanks are due to both of them. The publishers as usual have spared no pain to maintain the standard of printing and getup.

Many thanks are due to the readers who sent their suggestions from time to time indicating the changes to be made as well as pointing out some errors. Almost all of them have been incorporated in this edition.

The author expects the readers to continue sending their comments and suggestions to remove errors, if any, and to enhance the value of this book.

Chandigarh
April 1961

J. L.

PREFACE TO THE SECOND EDITION

The first edition of this book having been exhausted, the author has utilised this opportunity to make the book a little more comprehensive by completely revising and enlarging it. The book has now been divided into five sections and various chapters grouped under them. It is believed that this will be a more useful way of grouping the various chapters.

Under the section 'Testing of Hydraulic Machines' a chapter on "Testing and Characteristics of Turbines and Pumps" has been added which is useful for laboratory work of the students. This will also be of interest as a reference to the persons engaged in research work.

The old three chapters on 'Water Measurements,' 'Power and Efficiency Measurements, and 'Measurement of Pressure, Level and Speed' have been grouped under one chapter renamed as 'Hydraulic Measurements' which forms the basis of the section 'Testing of Hydraulic Turbines'. The methods of discharge-measurement which were not fully developed at the time of writing the first edition of this book, have been described in detail now. These methods are : *Gibson Inertia-Pressure Method* (Art 14.18) and *Thermometric* and *Thermodynamic Methods* (Art 14.19). The modern methods of discharge measurement *viz* : *Ultrasonic Flowmeter* recently developed has also been described. The *Manometric Piston Gauge* (Art 14.27), meant for measuring precision pressures, has recently been developed by Dr. D. N. Singh, Principal, Bihar Engineering College, Patna. Measurement of speed by electronic revolution-counter developed by NOHAB (Sweden), has been indicated under Art 14.29.

To keep the book up-to-date, a new chapter on 'Recent Trends in the Developments of Water Turbines' has been added. In this chapter, in addition to the discussion on recent developments of individual water turbines, the following topics have been included : *Pump Storage Plants*, *Reversible Turbine Pump*, *Underground Power Station*, *Tubular Turbines*, *Deriaz Runner*, *Aero foil Theory* and its application to axial flow turbo-machinery.

The chapter on *Water Wheels* has been renewed as *Water Wheels and Development of Water Turbines* by adding the different types of water turbines which were developed before modern types of turbines came into existence. Articles 6.17 'The Theory of Turgo-Impulse Turbine' and 1.12 'Jet Propulsion' have also been added.

The chapter on *Governing of Water Turbines* has been rewritten on modern lines.

In chapter on 'Reciprocating Pumps', the theory of working of air vessel and its design has been added which will be helpful in designing Reciprocating Pumps.

The practical data on frictional loss of head in pipe lines and pipe-fittings, which were given in the first edition as Appendices, have been arranged under Art 12.19.

The Article 12.41 on 'Pump Defects and their Remedies' given in tabular form, will be useful to the practical engineers as well as to the students preparing for *viva-voce* on the subject.

The chapter on 'Recapitulation of Elementary Fluid Mechanics' has been deleted from this book and included in the second edition of author's another book named 'Hydraulics'.

Great care has been taken to give up-to-date data about Turbine Installations included in Section II. The author will be glad to know of any discrepancy left therein.

The author is highly thankful to Shri N. Venkata Row, Prof. M. L. Mathur, Dr. S. C. Bhattacharyya and Dr. D. N. Singh for sending their suggestions. Shri S.P. Battoo of the Indian Standards Institutions, New Delhi, has taken great pains in supervising the work of printing and of preparation of figures by Shri M. J. S. Rooprai. The author's thanks are due to them. The publishers have taken great interest to see that the printing and get up is of good standard. Inspite of difficulty in procurement of paper and increase in its cost, the price has been kept moderate.

Any comments and suggestions to remove errors, if any, and to enhance the value of this book will be gratefully received.

Chandigarh
October 1959 J. L.

PREFACE TO THE FIRST EDITION

With the launching of ambitious programmes of multipurpose projects in India, the importance of hydraulic machines is assuming ever-increasing importance. It has been experienced, however, that many of the existing text books in English on fluid mechanics generally deal more elaborately with mainly hydraulics and less with hydraulic machines such as water turbines, pumps, fluid-pressure machines, etc., usually covering each topic in a single chapter. This book, which is the result of a prolonged, specialised study of the subject by the author abroad, is designed to meet the growing need for a comprehensive treatment of the various topics.

The book is written primarily for students preparing for a University degree in engineering or intending to sit for the membership examinations of the Engineering Institutions or for the Public Service Commission Examinations. The practical side, however, has not been kept out of view. The publication should be useful to the engineers working in the field, as a reference book for basic knowledge of Hydraulic Machines.

Whereas the book covers the fundamental principles of Hydraulic Machines as also certain principles of designs, detailed designs are considered beyond its scope. There is a large number of solved examples in the text to illustrate almost every article, and at the end several exercises for each chapter are added for the students to solve. For the purpose of making it more comprehensive, some of the examples are drawn from different examination papers. The appendices at the end include a few useful charts, practical data and some theory as a guide to further study. As the Government of India has decided to change over to the metric system, Appendix 6 has been added to facilitate conversion of the prevailing Foot-Pound (English) units to the metric ones.

The practical values of various non-dimensional factors, ratios and co-efficients, tabulated in Chapter 9 to 15 are based partly on the author's own experience gained in several firms manufacturing hydraulic machinery and partly on data suggested by Prof. R. Dubs in his lectures delivered at the Swiss Federal Institute of Technology at Zurich.

The author is highly thankful to Mr. S. D. Sharma for his valuable help at every stage in the preparation of the book and to Messrs K. L. Aggarwal and S. P. Battoo for their advice and help, specially in the preparation of figures. The author is grateful to the publishers for their cooperation in bringing out this book which is necessarily of a complex nature, at a comparatively moderate cost.

The author will be glad to receive any comments and suggestions for enhancing the value of this book in future editions.

Chandigarh
July 1956

J L.

CONTENTS

Paragraphs headings are given on the first page of each Chapter

SYMBOLS

A		area
B		breadth
C		Venturimeter & nozzle co-efficient
	C_d	co-efficient of discharge
D		diameter
	D_1	mean diameter of Pelton runner or mean bucket circle diameter. In general inlet diameter of turbine runner or pump impeller
E		energy, voltage and Euler number
F		force
	F_x	dynamic force in horizontal or X-direction
	F_y	dynamic force in vertical or Y-direction
	F_r	Froude number
G		weight and modulus of rigidity
H		head
	H_o	designed head
	H_a	accelerating head
	H_{mano}	manometric head
	H_{total}	total head
	H_L	loss of head
I		current
	I_p	polar moment of inertia
	K_s	Non-dimentional factor for specific speed
	K_{v_1}	$\frac{v_1}{\sqrt{2gH}}$ = non-dimentional factor for v_1 or velocity (v_1) co-efficient
	K_{u_1}	$\frac{u}{\sqrt{2gH}}$ = speed ratio or non-dimentional factor for u_1
L		length
M		moment
	M_a	Mach number
	M_1	$\frac{M}{H}$ = moment reduced to unit head
N		speed
	N_1	$\frac{N}{\sqrt{H}}$ = speed reduced to unit head
	N_s	specific speed
P		power in HP or KW
	P_a	available power or water power

	$\dot{P}_H$	power considering the head lost *i.e.* available power *minus* power consumed due to head lost
	P_h	hydraulic power (P_a *minus* hydraulic losses)
	P_{mech}	mechanical losses
	P_t	brake horsepower or net power developed by a turbine
	P_1	$\dfrac{P}{H^{\frac{3}{2}}}$ = power reduced to unit head
	P_{11}	$\dfrac{P}{D^2 . H^{\frac{3}{2}}}$ = power reduced to unit head and unit diameter (runner or jet)
Q		quantity of fluid flowing per unit time or discharge
	Q_1	$\dfrac{Q}{\sqrt{H}}$ = Q reduced to unit head
	Q_{11}	$\dfrac{Q}{D^2 . \sqrt{H}}$ = Q reduced to unit head and unit diameter (runner or jet)
	ΔQ	loss of Q
R		radius
	R_e	Reynolds' number
S		stroke
T		absolute temperature and tare
V		volume
W		work done and wattage
a		area
b		breadth
c		constant
d		diameter
	d_o	nozzle diameter
	d_1	least jet diameter
f		friction co-efficient
	f_s	shear stress
g		gravitational acceleration = 32·2 ft/sec² (or 981 cm/sec²)
h		head or height
k		constant
l		length
m		mass and hydraulic mean depth
p		intensity of pressure
q		partial discharge
r		radius
s		distance, specific gravity and stroke
t		time
u		peripheral or circumferential velocity of water
v		absolute velocity of water

	v_m	velocity of flow
	v_u	velocity of whirl
w		relative velocity of water and specific weight (weight density)
	w_s	specific weight (weight density)
x		horizontal distance
y		vertical distance
z		centrifugal head developed and height above datum
	z_1	number of vanes of a guide wheel
	z_2	number of vanes of a runner
α		angle between absolute and peripheral velocity and angular acceleration
β		angle between peripheral and relative velocity
ρ		mass density
η		efficiency
	η_h	hydraulic efficiency
	η_H	head efficiency
	η_{mano}	manometric efficiency
	η_{mech}	mechanical efficiency
	η_p	pump efficiency
	η_Q	volumetric efficiency
	η_t	overall efficiency
σ		Thoma's cavitation factor
λ		fraction
μ		absolute viscosity
ν		kinemetic viscosity
ϕ		co-efficient
ω		angular velocity
$\alpha, \beta, \theta, \phi, \psi$		angles (in general)
suffix 1		for inlet
suffix 2		for outlet

INTRODUCTION TO METRIC UNITS

(For Conversion See Appendices 1 and 2, pages 589, 590)

The following are the three systems in which the units are generally expressed—

1. FPS or Foot Pound Second System
2. CGS or Centimetre Gram Second System
3. MKS or Metre Kilogram Second System

The units expressed by the first system are generally termed as *Feet Units*. The second and third are the *metric systems*. The units expressed by the second system are smaller in size than that of the third, therefore MKS system is commonly employed to express engineering units.

	Symbol	Metric Units	FPS Units
1. Basic Units—			
a) Time	t	second, minute, hour, day and year	second, minute, hour, day and year
b) Distance	s	millimetre, centimetre, metre, kilometre 10 millimetres (mm) = 1 centimetre 10 centimetre (cm) = 1 decimetre 10 decimetre (dm) = 1 metre 1,000 metres (m) = 1 kilometre 1,000 kilometre (km) = 1 megametre 1,852 metres = 1 nautical mile (international)	inch, feet, yards, mile 12 inches (in.) = 1 foot 3 feet (ft) = 1 yard 220 yards (yd) = furlong 8 furlongs (= 1,760 yds = 5,280 ft) = 1 mile 6,080 ft = 1 nautical mile (British) = 1 knot
c) Force	F	*a*) *Technical*— gram, kilogram, tonne 1,000 gram (g) = 1 kilogram 1,000 kilogram (kg) = 1 tonne	*a*) *Technical*— ounce, pound, hundred-weight, ton 16 ounces (oz) = 1 pound 112 pounds (lb) = 1 hundred-weight 2,240 pounds (lb) = 1 ton [= 20 hundred weight (cwt)]

	Symbol	Metric Units
		b) *Physical*— $1\text{ dyne} = \frac{1}{981 \times 10^3} = \frac{1}{0{\cdot}981 \times 10^6}\text{ kg}$ ($\because$ $g = 981\text{ cm/sec}^2 = 9{\cdot}81\text{ m/sec}^2$)
2. Derived Units—		
a) Capacity or Volume	*V*	litre=Volume occupied by one kg of water $=(\text{dm})^3 = 1{,}000$ cubic centimetres (cc) cubic centimetre (cm^3), cubic decimetre (dm^3), cubic metre (m^3)
b) Density	*w*	one gram per cubic centimetre (1 g/cc) of water $=1{,}000\text{ kg/m}^3$ of water
c) Work	*W*	*a*) *Technical*— kg cm, kg m, tonne m *b*) *Physical*— 1 erg=1 dyne$\times$1 cm $= \frac{1}{0{\cdot}981 \times 10^6 \times 10^2} = \frac{1}{0{\cdot}981 \times 10^8}\text{ kg m}$ 1 joule$=10^7$ ergs $= \frac{10^7}{0{\cdot}981 \times 10^8} = \frac{1}{9{\cdot}81}\text{ kg m}$
d) Power $= \frac{\text{work}}{\text{time}}$	*P*	kg cm/sec, kg m/sec 1 watt = 1 joule/sec 1 kilowatt (kw) $= \frac{1{,}000}{9{\cdot}81} = 102$ kg m/sec 1 metric horsepower=75 kg m/sec =0·736 kw

b) *Physical*—

1 poundal$=\frac{1}{32\cdot2}$ lb ($\because$ $g=32\cdot2$ ft/sec^2)

1 poundal$=453\cdot6\times30\cdot48$ dynes

gallon (imperial)$=$volume occupied by 10 lb of water

$$=\frac{10}{62\cdot4}=\frac{1}{6\cdot24}\ \text{ft}^3$$

cubic inch (in.^{3}), cubic feet (ft^3)

62·4 pounds per cubic foot (62·4 lb/ft^3)

1 g/cc$=$62·4 lb/ft^3

(ΥΥ)

a) *Technical*—

lb ft, ton ft

b) *Physical*—

foot-poundal$=\frac{1}{32\cdot2}$ ft. lb.

a) *Technical*—

lb ft/sec, lb ft/min

1 horsepower (British)$=$550 lb ft/sec

$=$76·2 kg m/sec

$=$0·746 kw

CHAPTER 1

DYNAMIC ACTION OF FLUID

1.1 Definition—The velocity of stream of fluid entering a machine, such as a hydraulic or steam turbine, pump or fan, has more or less a defined direction. A force is always required to act upon the fluid to change its velocity either in direction or in magnitude. According to the Law of Action and Reaction (Newton's Third Law of Motion), an equal and opposite force is exerted by the fluid upon the body that causes this change. This force exerted by virtue of fluid motion is called a *Dynamic Force* and must be distinguished from hydraulic pressure. Whereas hydrostatic pressure implies no motion, dynamic force always involves a change of momentum.

1.2 Impulse-Momentum Equation—This equation is derived from Newton's Second Law of Motion which states that "The rate of change of momentum is proportional to the applied force and takes place in the direction of force."

Momentum of a body is the product of its mass and velocity. Let m be the mass of fluid moving with velocity v and let the change in velocity be dv in time dt,

Then change in momentum $= m \,.\, dv$

and rate of change in momentum $= m \,.\, \dfrac{dv}{dt}$

According to above law,

Dynamic Force = Rate of change in momentum

or $$F = \frac{m \,.\, dv}{dt} \qquad \ldots(1.1)$$

This equation can also be written as

$$F \,.\, dt = m \,.\, dv \qquad \ldots(1.2)$$

i.e., Impulse of dynamic force = change in momentum of body.

This is the *Impulse-Momentum Equation.*

The quantities in above equations are vector quantities and must be added or subtracted vectorially. *i.e.*,

$$(\Sigma F_x) \,.\, t = m \,.\, (\Sigma v_x) \quad \ldots\text{in } X\text{—direction} \qquad \ldots(1.2a)$$

and $$(\Sigma F_y) \,.\, t = m \,.\, (\Sigma v_y) \quad \ldots\text{in } Y\text{—direction} \qquad \ldots(1.2b)$$

1.3 Dynamic Force Exerted by Fluid Jet on Stationary Flat Plate :

a) **Plate Normal to Jet** (See Fig 1.1)—A fluid jet issues from a nozzle and strikes a flat plate with a velocity v. The plate is held stationary and perpendicular to the centre line of the jet.

Let Q=quantity of fluid falling on the plate, in cu ft/sec or m^3/sec

$$=\frac{\text{volume}}{\text{time}}$$

then $w \cdot Q$=weight of fluid in lb/sec or Kg/sec

and $\frac{w \cdot Q}{g}$=mass of fluid per sec

If the fluid is so deflected by the plate that it loses all the velocity v normal to the plate, then

Dynamic force=Change of Momentum/sec

=mass striking the plate/sec×change of velocity normal to the plate

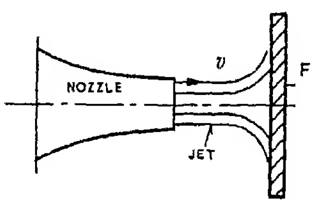


Fig 1.1 Fluid Jet on Stationary Vertical Plate

or $$F=\frac{w \cdot Q}{g} \cdot (v-0)$$

$$F=\frac{w \cdot Q}{g} \cdot v \qquad \ldots(1.3)$$

Further Equation of Continuity is written as

$$Q=a \cdot v$$

where a=area of cross-section of jet

$$F=\cdot\, w \cdot a \cdot v^2 \qquad \ldots(1.3a)$$

b) **Inclined Plate**—Again it can be assumed that all the velocity normal to the plate is lost. Then dynamic force also acts normal to the plate and is given, as before, by

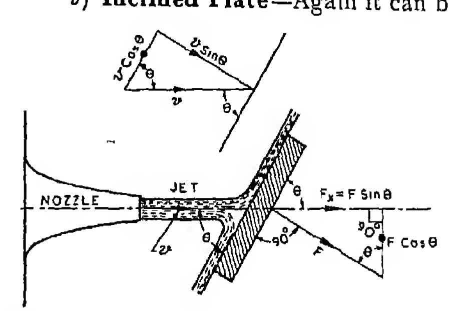


Fig 1.2 Fluid Jet on Stationary Inclined Plate

F=mass striking the plate per sec×change of velocity normal to the plate

Velocity normal to the plate=$v \sin\theta$. This is the normal component of velocity v as shown in Fig 1.2

$$F=\frac{w \cdot Q}{g} \cdot v \cdot \sin\theta$$

$$=\frac{w \cdot a \cdot v^2}{g} \cdot \sin\theta \qquad \ldots(1.4)$$

Component of this force F in the direction of jet (See Fig 1.2) is

$$F_x = F \cdot \sin\theta = \frac{w \cdot Q}{g} \cdot v \cdot \sin^2\theta \qquad \ldots(1.4a)$$

No work is done in the above cases.

Problem 1.1 Find the force exerted by a 2 in. diameter water jet directed against a flat plate held normal to the axis of stream. The velocity of the jet is 115 ft per sec.

Solution

$d=2$ in. $\quad v=115$ ft/sec $\quad w=62{\cdot}4$ lb/cu ft $\quad g=32{\cdot}2$ ft/sec^2

Rate of flow $\quad Q=a \cdot v \quad$ (Equation of Continuity)

$$= \frac{\pi}{4} \times \left(\frac{2}{12}\right)^2 \times 115 \text{ cu ft/sec}$$

Applying Eqn 1.3,

Dynamic force $\quad F = \dfrac{w \cdot Q \cdot v}{g}$

or

$$F = \frac{62{\cdot}4 \times \left\{\frac{\pi}{4} \times \left(\frac{2}{12}\right)^2 \times 115\right\} \times 115}{32{\cdot}2} \text{ lb}$$

$$= \frac{62{\cdot}4 \times \pi \times 4 \times 115 \times 115}{4 \times 144 \times 32{\cdot}2} \text{ lb}$$

Force = **558 lb** *Answer*

Problem 1.2 A 2.5 cm diameter water jet exerts a force of 90 kg in the direction of flow against a flat plate which is held inclined at an angle of 30° with the axis of stream. Find the rate of flow.

Solution

$d=2{\cdot}5$ cm $\quad F_x=90$ kg $\quad \theta=30°, \quad \sin 30°=0{\cdot}5$

Applying Eqn 1.4a, Dynamic force in the direction of jet,

$$F_x = F \cdot \sin\theta = \frac{w \cdot Q}{g} v \cdot \sin^2\theta$$

$$= \frac{w \cdot Q}{g} \cdot \frac{Q}{a} \cdot \sin^2\theta$$

or

$$90 \times 1{,}000 = \frac{Q}{981} \times \frac{Q}{\frac{\pi}{4}(2{\cdot}5)^2} \times 0{\cdot}5^2$$

$$Q = \sqrt{\frac{90 \times 1{,}000 \times 981 \times \pi \times 2{\cdot}5^2}{4 \times 0{\cdot}5^2}} = 41{,}640 \text{cm}^3/\text{sec}$$

or $\quad$ $Q =$ **41·64 litres/sec** *Answer*

If the plate were held normal to the axis of jet instead of being inclined at 30°, the rate of flow Q to exert the same force would obviously be 20·82 litres/sec, as sin θ will then be unity and Q varies inversely with sin θ.

Problem 1.3 A square plate weighing 28 lb, and of uniform thickness and 12 in. edge is hung so that it can swing freely about the upper horizontal edge. A horizontal jet $\frac{3}{4}$ in. diameter and having a

velocity of 50 ft/sec impinges on the plate. The centre line of the jet is 6 in. below the upper edge of the plate, and when the plate is vertical the jet strikes the plate normally and at its centre. Find what force must be applied at the lower edge of the plate in order to keep the plate vertical.

If the plate is allowed to swing freely, find the inclination to the vertical which the plate will assume under the action of the jet.

(*London University*)

Solution

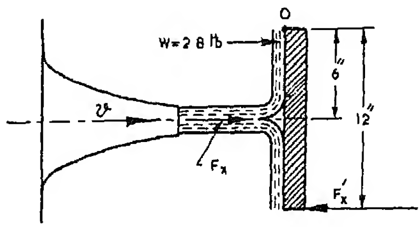


Fig 1.3

$W=28$ lb

Square plate 12 in. × 12 in.

$$d=\frac{3}{4}\text{ in.}=\frac{3}{48}\text{ ft}$$

$v=50$ ft/sec

Dynamic force applied at the centre of the plate in X—direction,

$$F_x=\frac{w\,.\,Q}{g}\,.\,v \quad \text{(See Eqn 1.3)}$$

or

$$F_x=\frac{w\,.\,a\,.\,v^2}{g}=\frac{62{\cdot}4\times\frac{\pi}{4}\times\left(\frac{3}{48}\right)^2\times 50^2}{32{\cdot}2}$$

$$=14{\cdot}8 \text{ lb}$$

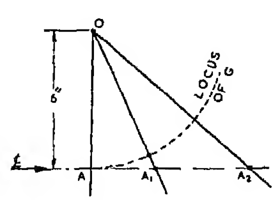


Fig 1.4(a)

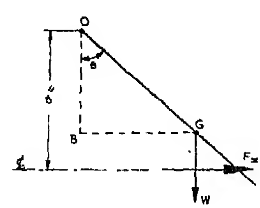


Fig 1.4(b)

Let F'_x be the force applied horizontally at the bottom edge of the plate (See Fig 1.3) to keep the plate in vertical position. Taking moments about O, the upper edge of plate,

$$F'_x\times 12=14{\cdot}8\times 6$$

or $$F'_x=\frac{14{\cdot}8\times 6}{12}=\mathbf{7{\cdot}4\ lb} \quad \textit{Answer}$$

b) If the plate is allowed to swing freely, the vertical distance from the pivot O to the centre line of the jet remains fixed at 6 in. (See Fig 1.4a), however the distance from O to the point where it strikes the plate, varies with the angle of inclination θ. Thus with angle θ_1, the distance is OA_1 and with θ_2 it changes to OA_2.

Let the actual inclination be θ,

then distance $BG=OG\sin\theta=6\sin\theta$ (See Fig 1.4b)

Now there are two forces acting on the plate—

Force F_x acting at 6 in. from O

and force W acting at a distance 6 sin θ from O.

If the plate is to be in equilibrium, the moment of these forces about O must balance each other.

$\therefore \quad F_x \times 6 = W \times 6 \sin\theta$

or $\quad \sin\theta = \dfrac{F_x}{W} = \dfrac{14 \cdot 8}{28} = 0 \cdot 53$

or $\quad \theta = \sin^{-1} 0 \cdot 53 = \mathbf{31^\circ - 21'}$ *Answer*

1.4 Force on Moving Flat Plate—Let the plate in Fig 1.1 move with a velocity u in the same direction as the jet. Now, after the jet has struck the plate, it acquires a velocity u in the same direction and the change in velocity is $(v-u)$. Also the quantity of water striking the plate per second is given by cross-sectional area multiplied by the velocity of jet *relative* to plate.

i.e. $\quad Q = a \,.\, (v-u)$

Again, $\quad$ Force = mass striking/sec × change of velocity

$$F = \frac{w \,.\, Q}{g}(v-u) = \frac{w \,.\, a}{g}(v-u)^2 \qquad \ldots(1.5)$$

Here the distance between plate and nozzle is constantly increasing by u ft/sec. A single moving plate is, therefore, not a practical case. If, however, a series of plates (See Fig 1.5) were so arranged that each plate appeared successively before the jet in the same position and always moving with a velocity u in the direction of jet, weight of water striking the plate would be

$$= \frac{w \,.\, a \,.\, v}{g}$$

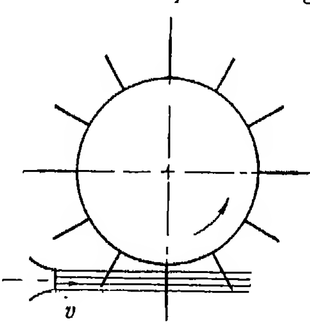


Fig 1.5 Fluid Jet on a Series of Moving Plates

and $F = \dfrac{w \,.\, a \,.\, v}{g}(v-u) \qquad \ldots(1.6)$

Work done on the plates $= F \,.\, u$

$$= \frac{w \,.\, Q}{g}(v-u)u \qquad \ldots(1.7)$$

Kinetic energy of jet $\quad = \dfrac{v^2}{2g}(w \,.\, Q)$

$\therefore$ Efficiency of system, $\eta = \dfrac{\text{Work obtained}}{\text{Energy input}}$

$$= \frac{\dfrac{w \,.\, Q}{g}(v-u)u}{\dfrac{v^2}{2g}(w \,.\, Q)} = \frac{2(v-u)u}{v^2} \qquad \ldots(1.8)$$

For $\quad \eta_{max}, \quad \dfrac{d\eta}{du} = 0$

i.e. $\quad \dfrac{d}{du}(vu - u^2) = 0$

or $\quad v - 2u = 0$

$\therefore \quad u = \dfrac{v}{2}$

Substituting the value of u in Eqn 1.8

$$\eta_{max}=\frac{2\left(v-\frac{v}{2}\right)\cdot\frac{v}{2}}{v^2}$$

$$=\tfrac{1}{2} \text{ or } 0{\cdot}5 \qquad \ldots(1.9)$$

Problem 1.4 A jet of water 3 in. in diameter moving with a velocity of 40 ft per sec strikes a flat vane which is normal to the axis of the stream. (*a*) Find the force exerted by the jet if the vane moves with a velocity of 15 ft per sec. (*b*) Determine the force exerted by the jet if instead of one flat vane, there is a series of vanes so arranged that each vane appears successively before the jet in the same position and always moving with a velocity of 15 ft per second.

Solution

$d=3$ in.
$v=40$ ft/sec
$u=15$ ft/sec

$\therefore$ Cross-sectional area of jet $a=\frac{\pi}{4}\times\left(\frac{3}{12}\right)^2$ sq ft

a) Force exerted by jet on a single vane

$$F=\frac{w\cdot a}{g}(v-u)^2 \qquad \text{(See Eqn 1.5)}$$

$$=\frac{62{\cdot}4\times\frac{\pi}{4}\left(\frac{3}{12}\right)^2\times(40-15)^2}{32{\cdot}2}$$

$=$**59·5 lb** *Answer*

b) Force exerted by jet on a series of vanes

$$F=\frac{w\cdot a\cdot v}{g}(v-u) \qquad \text{(See Eqn 1.6)}$$

$$=\frac{62{\cdot}4\times\frac{\pi}{4}\left(\frac{3}{12}\right)^2\times 40\times(40-15)}{32{\cdot}2}$$

$=$**95·2 lb** *Answer*

1.5 Pressure due to Deviated Flow—Velocity of a stream of fluid moving in a machine changes its direction and the dynamic force resulting from the consequent change of momentum is to be found out.

Let the time taken in negotiating the bend in Fig 1.6 from point (1) to point (2) be t. Mass of fluid flowing in this time,

$$=\frac{w\cdot Q}{g}\cdot t$$

Velocity of each particle of fluid and therefore of the whole mass m, has changed in time t, from v to $v\cdot\cos\alpha$ in X-direction and from zero to $v\sin\alpha$ in Y-direction.

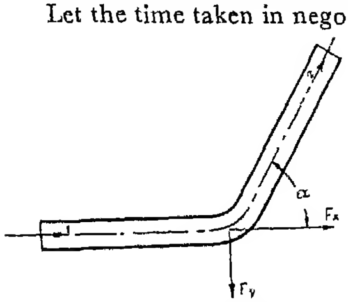


Fig 1.6 Flow in a Bend

Average acceleration (strictly, retardation) in X-direction,

$$f_x = \frac{v - v\cos\alpha}{t} = \frac{v(1-\cos\alpha)}{t}$$

Average acceleration in Y-direction

$$f_x = \frac{v\sin\alpha}{t}$$

Since Force = mass × acceleration,

$$F_x = m \,.\, f_x$$

$$= \frac{w\,.\,Q}{g}\,.\,v(1-\cos\alpha) \qquad ...(1.10)$$

$$F_y = m \,.\, f_y$$

$$= \frac{w\,.\,Q}{g}\,.\,v\sin\alpha \qquad ...(1.11)$$

In Fig 1.6 dynamic forces F_x and F_y are shown in the direction in which they act on the supports keeping the bend in position.

Now,

$$F_x = \frac{w\,.\,Q}{g}\,.\,v\,.\,(1-\cos\alpha)$$

$$= \frac{2\,.\,w\,.\,a\,.\,v^2}{2g}\,(1-\cos\alpha)$$

If $\alpha = 90°$ (See Fig 1.7),

$$F_x = 2\,.\,w\,.\,a\,\frac{v^2}{2g} \qquad ...(1.12)$$

i.e., twice the pressure corresponding to the velocity head considered acting hydrostatically on an area equal to that of the cross-section of the stream.

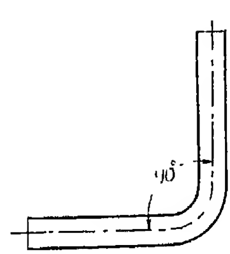


Fig 1.7 Flow in a 90°—Bend

Practical Uses—A pipe line of large diameter, carrying water for Town Water Supply or Water Power Plants is required to be bent in order to lay it on the ground of different elevations. At the pipe bend large forces are brought into play in different directions, for which *Anchor Blocks* of reinforced concrete are built round the bend to withstand these forces by using Eqns 1.10 and 1.11. Such a force will be maximum when the pipe has a 90°—bend.

If $\alpha = 180°$,

$$F_x \text{ is max and} = \frac{2\,.\,w\,.\,Q}{g}\,.\,v = 4\,.\,w\,.\,a\,.\,\frac{v^2}{2g} \qquad ...(1.13)$$

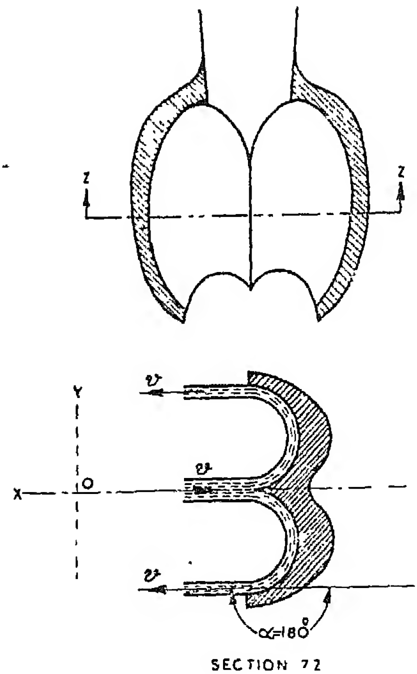


Fig 1.8 Jet on a Moving Vane of Double Hemispherical Type

Practical Use—Fig 1.8 shows a moving vane of double hemispherical type used for Pelton turbines (See Chapter 6). The water stream impinges at the centre of the vane and deflects through an angle of 180°. At the same time the water stream is symmetrically divided at the centre of vane, with which the Y-components of dynamic force (F_y), one of them acting towards the bottom and the other acting towards the top, neutralise each other. Thus the remaining force F_x is twice the ordinary dynamic force shown in Eqn 1.3*a*. This force is utilised to move the wheel of modern water turbine known as "Pelton Turbine" (See Chapter 6).

Problem 1.5 Water under a pressure of 50 lb per sq in. is flowing through a 12 in. pipe at the rate of 9 cusecs, If the pipe is bent by 135°, find the magnitude and direction of the resultant force on the bend.

Solution

$p=50$ lb/sq in.
$d=12$ in.
$Q=9$ cu ft/sec
$\alpha=135°$

Velocity of water flowing in the pipe$=\dfrac{Q}{a}=\dfrac{9}{\dfrac{\pi}{4}\times 1^2}=11{\cdot}45$ ft/sec.

Force in X-direction, applying Eqn 1.10—

$$F_x=\frac{w\,.\,Q}{g}\,.\,v\,.\,(1-\cos\alpha)$$

$$=\frac{62{\cdot}4\times 9}{32{\cdot}2}\times 11{\cdot}45\times(1+\cos 45°)$$

$$=\frac{62{\cdot}4\times 9\times 11{\cdot}45\times 1{\cdot}707}{32{\cdot}2}=\mathbf{341\ lb}$$

Force in Y-direction, applying Eqn 1.11—

$$F_y=\frac{w\,.\,Q}{g}\,.\,v\,.\,\sin\alpha$$

$$=\frac{62{\cdot}4\times 9}{32{\cdot}2}\times 11{\cdot}45\times 0{\cdot}707=\mathbf{141\ lb}$$

Force exerted by water under 50 lb/sq in.

$$=p\,.\,A=50\times\left(\frac{\pi}{4}\times 1^2\times 144\right)=5{,}655\text{ lb}$$

Component of this force in X-direction

$$=5{,}650(1-\cos\alpha)=5{,}655\times 1{\cdot}707=9{,}653\text{ lb}$$

Component of this force in Y-direction $=5{,}655\times\sin\alpha$

$$=5{,}655\times 0{\cdot}707=3{,}998\text{ lb}$$

Total force in X-direction $=341+9{,}653=9{,}994$ lb

Total force in Y-direction $=141+3{,}998=4{,}139$ lb

Hence resultant force $=\sqrt{9{,}994^2+4{,}139^2}=\mathbf{10{,}817\ lb}$

Direction of resultant $=\tan^{-1}\dfrac{4{,}139}{9{,}994}=\mathbf{22^\circ-30'}$

with the original direction of flow.

Answers

Problem 1.6 A 4 ft diameter pipe line carries 65 cusecs of water for the town supply. A 90°-bend has to be fitted in the line by anchoring the end of the pipe with the help of tie rods at right angles to the pipe at the ends of the bend. Find the tension in each tie rod.

Solution

$$d=4\text{ ft}\qquad Q=65\text{ cfs}$$
$$\alpha=90^\circ$$

Velocity of water in the pipe line, $v=\dfrac{Q}{a}$

or
$$v=\frac{65}{\frac{\pi}{4}\times 4^2}=5{\cdot}175\text{ ft/sec}$$

Dynamic Force $F_x=\dfrac{w\,.\,Q}{g}\,v(1-\cos\alpha)$ .. (See Eqn 1.10)

$$=\frac{w\,.\,Q}{g}\,v(1-\cos 90^\circ)$$

$$=\frac{w\,.\,Q}{g}\,v=\frac{2w\,.\,Q}{2g}\,v$$ ...(See Eqn 1.12)

or
$$F_x=\frac{62{\cdot}4\times 65}{32{\cdot}2}\times 5{\cdot}175=\mathbf{652\ lb}$$ *Answer*

As the tie rods are fitted at right angles to the bend, the amount of tension in each rod is 652 lb.

Problem 1.7 A jet of water moving with a velocity of 60 ft/sec (or 18·3 *m*/sec) impinges at the centre of a double hemispherical cup and is deflected through 180°. Find the dynamic force of jet per lb (or 0·4536 kg) of water if

a) the cup is stationary,

b) the cup is moving with a velocity of 25 ft/sec (or 7·625 *m*/sec),

c) instead of one cup, there is a series of cups, so arranged that each cup appears successively before the jet in the same position and always moving with a velocity of 25 ft/sec (or 7·625 *m*/sec).

Solution

$v=60$ ft/sec (or 18·3 *m*/sec)

$u=25$ ft/sec (or 7·625 *m*/sec)

$w \,.\, Q=1$ lb (or 0·4536 kg)

$\alpha=180°$

a) As explained in Fig 1·8, the *Y*-components of dynamic force will neutralise each other, then the force acting will be in *X*-direction only. Applying Impulse-Momentum Equation. (See Eqn 1.2*a*)

$$(\Sigma F_x)\, t=m\, \Sigma (v_x)$$

or
$$F_x \,.\, t=\frac{w \,.\, Q}{g}\, t(v-v \cos \alpha)$$

or
$$F_x=\frac{w \,.\, Q}{g}\, v(1-\cos \alpha)$$

This equation has already been derived under Eqn 1.10.

Now substituting the value of α,

$$F_x=\frac{2w \,.\, Q}{g} \,.\, v \qquad \text{(same as Eqn 1.13)}$$

$$=\frac{2\times 1}{32\cdot 2}\times 60=\mathbf{3\cdot 73\ lb} \quad \textit{Answer}$$

$$\left[\text{or } \frac{2\times 0\cdot 4536}{9\cdot 81}\times 18\cdot 3=\mathbf{1\cdot 695\ kg} \quad \textit{Answer}\right]$$

b) $w \,.\, Q=1$ lb (or 0·4536 kg) $\qquad v=60$ ft/sec (or 18·3 *m*/sec)

$\therefore \quad Q=\frac{1}{62\cdot 4}$ cfs (or 453 6 cm^3/sec or 0 4536 lit/sec)

[$\because$ 1 lit/sec$=1$ $(dm)^3$/sec]

and Cross-sectional area of jet per lb (or 0·4536 kg) of water

$$a=\frac{Q}{v}=\frac{1}{62\cdot 4\times 60}\text{ sq ft}\left(\text{or } \frac{0\ 4536}{18\cdot 3\times 10}=0\cdot 248\times 10^{-4}\text{ sq m.}\right)$$

The dynamic force F_x is given by

$$F_x=\frac{2\, w \,.\, Q}{g} \,.\, v=4\, w \,.\, a\, \frac{v^2}{2g} \qquad \text{...(See Eqn 1.13)}$$

As the cup is moving with a velocity u, then

$$F_x=4\, w \,.\, a \,.\, \frac{(v-u)^2}{2g}$$

$$=4\times 62\cdot 4\times \frac{1}{62\cdot 4\times 60}\times \frac{(60-25)^2}{64\cdot 4}=\mathbf{1\cdot 27\ lb} \quad \textit{Answer}$$

$$\left[\text{or } F_x = 4\times1000\times0{\cdot}248\times10^{-4}\times\frac{(18{\cdot}3-7{\cdot}625)^2}{2\times9{\cdot}81} = \mathbf{0{\cdot}575\ kg}\right]$$

Answer

c) With a series of cups moving, the dynamic force

$$F_x = \frac{2\ w\ .\ Q}{g}(v-u)$$

$$= \frac{2\times1}{32{\cdot}2}\times(60-25) = \mathbf{2{\cdot}175\ lb} \quad \textit{Answer}$$

$$\left[\text{or } F_x = \frac{2\times0{\cdot}4536}{9{\cdot}81}\times(18{\cdot}37-7{\cdot}625) = \mathbf{0{\cdot}985\ kg}\right] \quad \textit{Answer}$$

1.6 Fluid Jet on Curved Plate—

a) **Stationary Plate**—The jet impinges on a curved plate Fig 1.9, at an angle α_1 and is deviated to an angle α_2, both angles being measured with respect to X-direction. Let v_1 and v_2 be the velocities of jet at inlet and at outlet respectively. The velocity of jet at inlet v_1 and the velocity of jet at outlet v_2 will be same as long as there is no friction on the plate.

Velocity of jet at inlet in X-direction $= v_1 \cos \alpha_1$

Velocity of jet at outlet in X-direction $= v_2 \cos \alpha_2$

$\therefore$ Force exerted by the jet on the plate in X-direction can be determined by applying Impulse-Momentum Equation—

$(\Sigma F_x)\ .\ t = m\ (\Sigma\ v_x)$...$X$-direction ...(See Eqn 1.2a)

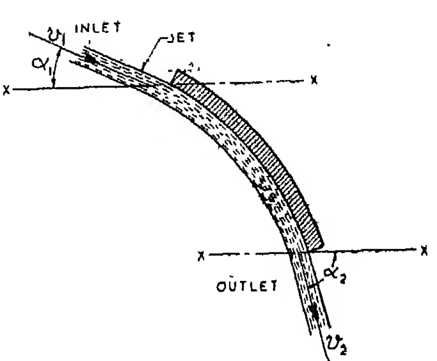


Fig 1.9 Jet Falling on Stationary Curved Plate with Acute Discharge Angle

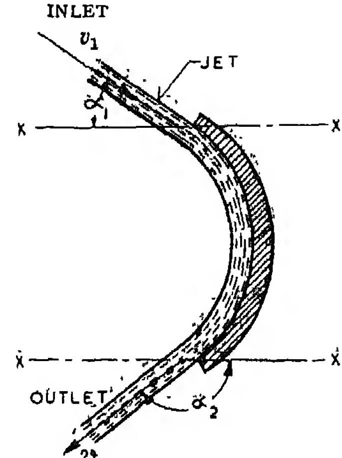


Fig 1.10 Jet Falling on Stationary Curved Plate with Obtuse Discharge Angle

or $\qquad F_x = \frac{m}{t}\times$ change of velocity in X-direction

or $\qquad F_x = \frac{w\ .\ Q}{g}(v_1 \cos \alpha_1 - v_2 \cos \alpha_2) \qquad$ ...(1.14)

where $Q = av_1$ = Quantity of water per second falling on the plate ...(1.15)

$$\therefore \quad F_x = \frac{w \cdot a \cdot v_1}{g} (v_1 \cos \alpha_1 - v_2 \cos \alpha_2) \quad ...(1.16)$$

If the curvature of the plate at outlet is such that outlet angle α_2 is more than 90° (See Fig 1.10), then the second term in the bracket of Eqn 1.16 (*i.e.* $v_2 \cos \alpha_2$) will be negative.

Hence in order to get more force, the curvature of the plate should be such that the outlet angle α_2 is obtuse.

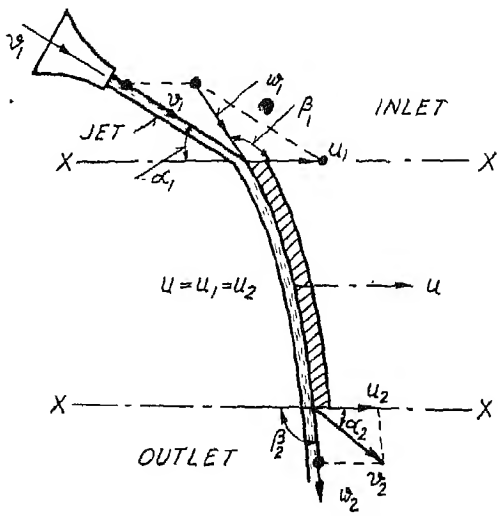


Fig 1.11 Jet Falling on Moving Curved Plate with Acute Discharge Angle α_2

b) **Moving Plate**—The jet having an inlet velocity v_1 impinges on a moving curved plate at an angle α_1 (See Fig 1.11) with respect to X-direction. The curvature of the plate at the point where the jet strikes may or may not make the same angle α_1 with X-direction. Let the angle of curvature of the plate at inlet with the reversed direction of motion *i.e.* $-u$, be β_1 (Fig 1.11). The plate is moving with a velocity u in X-direction. As soon as the jet falls over the plate, the velocity of jet remains no more equal to v_1, but it becomes the velocity of jet relative to the motion of the plate. This velocity is denoted by w_1. Its direction will be tangent to the point of inlet. Its magnitude is determined by subtracting u from v_1 vectorially by Law of Parallelogram of Velocities. The jet glides over the curved surface of the plate with a velocity w_1. Thus when the jet leaves the plate, its relative velocity will remain equal to w_1 provided there is no decrease in velocity due to friction on the surface of flow. Let the velocity of jet relative to the plate motion at outlet be denoted by w_2, then $w_1 = w_2$. Let the plate at outlet be inclined at an angle β_2. This angle β_2 is measured from the reversed direction of motion *i.e.* $-u_2$. Now the absolute velocity of water at outlet v_2, will be the vector sum of the following two velocities—

i) Velocity of water with which the water leaves the plate *i.e.* w_2,

ii) Velocity with which the plate moves in X-direction *i.e.* u.

$$\therefore \quad \vec{v_2} = \vec{w_2} + \vec{u} \quad ...(1.17)$$

The magnitude and direction of absolute velocity v_2 is determined by applying Law of Parallelogram of Forces (See Fig 1.11). The angle which the absolute velocity of water v_2 makes with X-direction or with the direction of motion, is denoted by α_2.

Force exerted by the jet on the plate in X-direction or in the direction of motion, is determined by applying Impulse-Momentum Equation—

$$(\Sigma F_x)\ t = m\ (\Sigma v_x) \ldots\ldots\ldots\text{in } X\text{-direction} \qquad \ldots(\text{See Eqn } 1.2a)$$

or $$F_x = \frac{m}{t} \times \text{change of velocity in } X\text{-direction}$$

or $$F_x = \frac{w \cdot Q}{g} (v_1 \cos \alpha_1 - v_2 \cos \alpha_2) \qquad \ldots(1.18)$$

where $Q = a\ (v_1 - u)$ ...(1.19)

The water issues from the jet at the rate of av_1. However the plate moves away from the jet with velocity u, therefore the velocity of the jet falling upon the plate is reduced by u, or the velocity with which the jet falls upon the plate $= v_1 - u$.

This is the velocity of water relative to the motion of the plate *i.e.* w_1

$$\therefore \quad F_x = \frac{w \cdot a\ (v_1 - u)}{g} (v_1 \cos \alpha_1 - v_2 \cos \alpha_2) \qquad \ldots(1.20)$$

For $\alpha_2 > \frac{\pi}{2}$, $\cos \alpha_2 < 0$ (See Fig 1.12)

Then the second term in the bracket of Eqn 1.20 (*i.e.* $v_2 \cos \alpha_2$) will be negative. Hence in order to get more force, the curvature of plate should be such that α_2 is obtuse.

1.7 Absolute Path of Water—When the jet strikes the moving plate, its position is given by *full* lines (See Fig 1.12). As the plate moves with velocity u, it reaches the position shown by *dotted* lines when the jet leaves it. Now there are two paths traced by water jet, one over the plate surface which is relative to the motion of plate and therefore appears to be moving with the plate; and the other is known as *absolute path* which appears to be stationary with respect to earth.

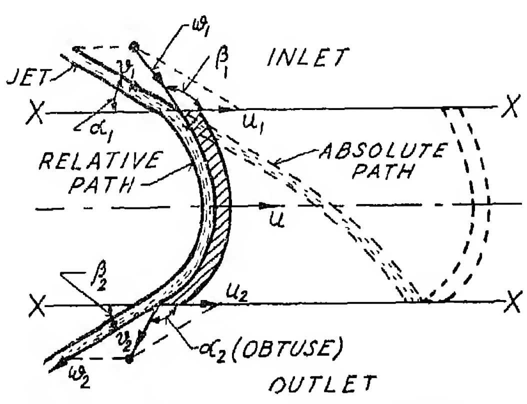


Fig 1.12 Jet Falling on Moving Curved Plate with Obtuse Discharge Angle α_2

To determine the absolute path of water parcticle (See Fig 1.13) take any six points (0 to 5) from the inlet to the outlet of the plate.

Take the distances Sw_{0-1}, Sw_{0-2}, Sw_{0-3} etc along the curved path of the plate from the point of entrance 0 to points 1, 2, 3 etc. These

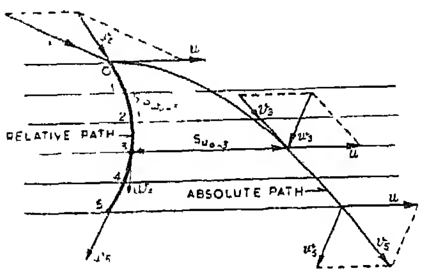


Fig 1.13 Determination of Absolute Path of Water Particle from its Relative Path

are the distances traversed by the water particle with w, the velocity of water relative to the motion of the plate in times t_1, t_2, t_3 etc respectively. Now take the distances Su_{0-1}, Su_{0-2}, Su_{0-3} etc in the horizontal direction from points 1, 2, 3 etc respectively. These are the distances travelled by the plate moving with u, its peripheral velocity in time t_1, t_2, t_3 etc respectively. Join the points Su_{0-1}, Su_{0-2}, Su_{0-3} etc taken in horizontal direction, with a curve which indicates the absolute path of water particle.

The direction of absolute velocity of water at any point will be tangent to the absolute path of water. Similarly the direction of relative velocity of water at any point will be tangent to the relative path of water. The direction of the peripheral velocity of plate is always horizontal. The direction of all the three velocities u, v and w being known, the velocity triangle can be drawn at any point of the path. The velocity triangles have been shown at points 0, 3 and 5 in Fig 1.13.

Problem 1.8 A circular water jet having a cross-sectional area of 4 sq in. moves with a velocity of 110 ft per sec and strikes tangentially a curved plate. The angle of curvature of plate at outlet is 120° with the X-direction. Assuming the plate to be frictionless, find the force exerted by the jet on the plate in X-direction—

a) when the plate is stationary,

b) when the plate is moving in the direction of jet with the velocity of 50 ft/sec.

Solution

Cross-sectional area of jet $a=4$ sq/in.

Velocity of jet at inlet $v_1=110$ ft/sec

Angles $\alpha_1=0°$, $\alpha_2=120°$

Quantity of water falling on the plate per sec

$$Q=av_1=\frac{4}{144}\times 110=3{\cdot}056 \text{ cfs}$$

a) **Jet falling upon stationary plate**—Velocity of jet at inlet in X-direction$=v_1\cos\alpha_1=v_1\cos 0°=v_1=110$ ft/sec

As the plate is stationary, the velocity of water at outlet will be the same as that at the inlet *i.e.* $v_2=v_1$, because there is no friction on the plate (See Fig 1.10). The velocity v_2 at the outlet makes an angle α_2 with X-direction.

$\therefore$ Velocity of jet at outlet in X-direction

$$=v_2\cos\alpha_2=110\cos 120°=-110\cos 60°=-55 \text{ ft/sec}$$

Force exerted by the jet on the plate in X-direction$=$mass of water/sec$\times$change of velocity of water in X-direction

or $$F_x = \frac{w \cdot (a \cdot v_1)}{g} (v_1 \cos \alpha_1 - v_2 \cos \alpha_2) \qquad \text{...(Eqn 1.16)}$$

$$= \frac{62 \cdot 4}{32 \cdot 2} \times \left(\frac{4}{144} \times 110 \right) \left\{ 110 - (-55) \right\}$$

$$= \frac{62 \cdot 4}{32 \cdot 2} \times \frac{4}{144} \times 110 \times 165 = \mathbf{978\ lb} \quad \textit{Answer}$$

b) **Jet falling upon moving plate**—Peripheral velocity of plate $u = 50$ ft/sec.

Velocity of water relative to the motion of plate at inlet $w_1 = v_1 - u$ (Vector Subtraction of v_1 and u as shown in Fig 1.14)

$= 110 - 50 = 60$ ft/sec to the right.

Quantity of water falling on the plate/sec,

$$Q = a\,(v_1 - u) = \frac{4}{144} \times 60 \text{ cfs}$$

Velocity of jet at inlet in X-direction

$= v_1 \cos \alpha_1 = v_1 = 110$ ft/sec.

As the plate is moving with a velocity u, the water will be discharging out with a velocity relative to the motion of the plate *i. e.* $v_1 - u = w_1$. Since there is no friction on the plate, this velocity $v_1 - u = w_1 = w_2$. It makes an angle 120° with X-direction or with the direction of the motion of plate. Hence velocity of water at outlet v_2 will be vector sum of the following two velocities—

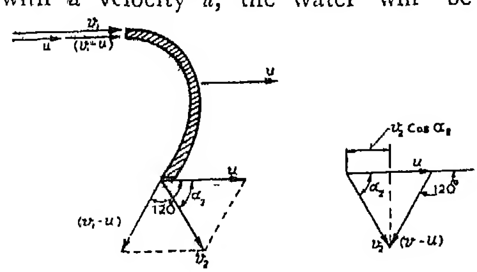


Fig 1.14

i) Velocity of water with which the water is discharging out $w_1 = v_1 - u$

ii) Velocity of the plate $= u$

$$\therefore \quad \vec{v_2} = \overrightarrow{(v_1 - u)} + \vec{u} \qquad \text{...(See Eqn 1.17)}$$

This can be determined by Law of Parallelogram of Forces as shown in Fig 1.14. The angle which the velocity of water v_2, or the resultant, makes with X-direction is α_2.

Velocity of jet at outlet in X-direction

$= v_2 \cos \alpha_2 = u - (v_1 - u) \cos (180° - 120°) = 50 - 60 \times 0{\cdot}5 = 20$ ft/sec

Force exerted by the jet on the plate in X-direction = mass of water/sec × change of velocity of water in X-direction

$$\therefore \quad F_x = \frac{w \cdot a}{g} (v_1 - u)(v_1 \cos \alpha_1 - v_2 \cos \alpha_2) \qquad \text{...(See Eqn 1.20)}$$

$$= \frac{62 \cdot 4}{32 \cdot 2} \times \frac{4}{144} \times 60(110 - 20)$$

$= \mathbf{291\ lb}$ *Answer*

1.8 Fluid Jet on Moving Curved Surface of a Turbine Blade—Fig 1.8 and 1.15 show the sectional plan of a double hemispheri-

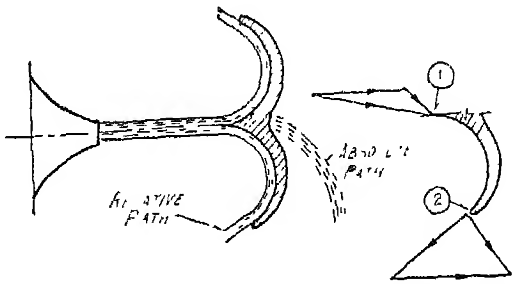


Fig 1.15 Plan of a Double Hemispherical Turbine Bucket

cal blade or bucket as it is sometimes called. Such buckets form the runner of water turbine of Pelton type, shown in Fig 1.16. Each bucket consists of two hemispherical cups separated by a sharp edge

Fig 1.16 Runner of Pelton Water Turbine

at the centre. The water jet impinges at the centre of bucket and is divided by the sharp edge, without shock, into two parts moving

sideways in opposite directions. The jet is thus deflected backward when leaving the bucket (Fig 1.8 and 1.15). The theoretical angle through which the jet deflects is 180° (Fig 1.8), but due to some practical difficulties, the angle of deflection is made equal to about 160° (See Fig 1.15).

The moving curved surface of turbine blade is similar to the moving curved plate described already under Art. 1.6 (*b*). The flow over the curved surface has been shown in Fig 1.12 as the outlet curvature of the plate β_2 is obtuse in case of turbine blade.

The blade moves with a peripheral or circumferential velocity of $u=\omega \cdot r$, where ω is the angular velocity of the wheel to which the blade is fixed and r is the radius of arc of a circle drawn from the *centre of the wheel to the point where the jet impinges.* Now the radius r is same at the point of inlet and at the point where the jet discharges because both the points of inlet and outlet are in the *same* horizontal plane. Hence the circumferential velocity of the blade at inlet and outlet is same

i.e., $$u=u_1=u_2 \qquad \text{...(1.21)}$$

1.9 Velocity Diagrams for Turbine Blades—Fig 1.17 shows the typical velocity diagrams or triangles of turbine blades. They have been taken from Fig 1.12 and are redrawn.

List of Symbols used in Velocity Diagrams or Triangles :

u_1 and u_2 : Circumferential or peripheral velocities of vanes at inlet and outlet respectively.

w_1 and w_2 : Velocities of jet relative to vane at inlet and outlet respectively.

v_1 and v_2 : Absolute velocities (*i.e.*, relative to earth) of jet at inlet and outlet respectively.

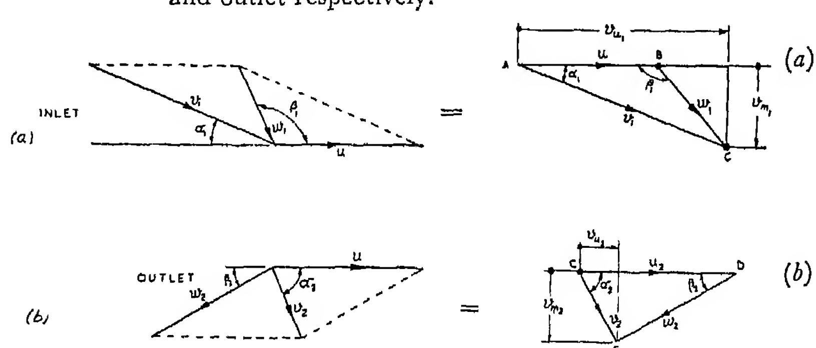


Fig 1.17 Typical Velocity Triangles for the Flow over Turbine Blade
(*a*) Inlet (*b*) Outlet

α_1 and α_2 : Angles between v_1 and u_1, and v_2 and u_2 respectively *i.e.*, angles of jet with direction of motion of vane.

β_1 and β_2 : Angles between w_1 and $-u_1$ and w_2 and $-u_2$ respectively *i.e.*, angles of vane tips. These angles are measured between w and u *reversed.*

The absolute velocities v_1 and v_2 can be resolved in two components :

a) Tangential components $v_1 \cos \alpha_1$ and $v_2 \cos \alpha_2$ are represented by symbols v_{u_1} and v_{u_2} respectively. These components are parallel to the direction of motion of vane *i.e.* u and are therefore responsible for doing the work. Therefore they are termed as *Velocities of Whirl.*

b) Radial or axial components $v_1 \sin \alpha_1$ and $v_2 \sin \alpha_2$ are represented by symbols v_{m_1} and v_{m_2}. These components are perpendicular to the direction of motion of vane and hence they do not do any work on the blades. These components cause the water to flow through the turbine blade and are therefore called the *Velocities of Flow.*

Drawing of Velocity Triangles—The velocity is a vector quantity, therefore the velocity triangle is a vector diagram.

Inlet (See Fig 1.17*a*)—Draw $\overline{AC}=v_1$, the absolute velocity of water at inlet at an angle of α_1 to the wheel tangent. Draw $\overline{AB}=u_1$= the peripheral velocity of wheel in the horizontal direction. Join $\overline{BC}$, which gives w_1, the velocity of water relative to wheel motion at inlet, making angle β_1 with wheel tangent.

$$\vec{w}_1 = \vec{v}_1 - \vec{u}_1$$

Resolve the absolute velocity of water at inlet into two components u_{u_1}, the velocity of whirl at inlet which is the tangential component, and v_{m_1}, the velocity of flow which is the normal or radial component.

Mark the directions of the velocities with arrows as shown in Fig 1.17*a*.

Outlet (See Fig 1.17*b*)—Draw $\overline{CD}=u_2$ the peripheral velocity of wheel at outlet, in the horizontal direction. Draw $\overline{DE}=w_2$, relative velocity of water at outlet, at an angle β_2 to u_2. Join $\overline{CE}$ which gives v_2, the absolute velocity of water at outlet, making an angle α_2 to the wheel motion.

$$\vec{v}_2 = \vec{w}_2 + \vec{u}_2$$

Resolve the absolute velocity of water at outlet into two components v_{u_2}, the velocity of whirl which is the tangential component, and v_{m_2}, the velocity of flow which is the normal or radial component.

Mark the direction of velocities with arrows as shown in Fig 1.17*b*. The velocity of whirl at outlet v_{u_2} may be positive or negative, depending upon the angle α_2 being acute or obtuse respectively.

1.10 Work Done on Tangential Flow Turbine Runner— The tangential flow turbine runner is shown in Fig 1.16. It consists of a number of double hemispherical type blades shown in Fig 1.8. The jet of water issues from a nozzle and impinges on a few blades of runner at a time with an absolute velocity of water v_1. Thus with this arrangement, each blade of the wheel appears successively before the jet in the same position. The wheel revolves with an angular velocity

ω. If r is the radius of this point on the blade where the jet impinges, the peripheral velocity, of this point is $u=\omega \cdot r$. The velocity u is tangent to the wheel motion and in the direction of jet. The velocity of water which is responsible for doing the work on the blades, is the velocity of whirl. This is the component of the absolute velocity of water in the direction of motion.

From Eqn 1.1.

Tangential force on the wheel=mass of impinging water× acceleration.

or Force on vanes in the direction of motion=mass of water per sec×change of velocity in the direction of motion=mass of water per sec×change of velocity of whirl

$$F_u=\frac{w \cdot Q}{g}(v_1 \cos \alpha_1 - v_2 \cos \alpha_2) \qquad ...(1.22)$$

This equation is the same as that already derived under Eqn 1.14 as well as under Eqn 1.18.

In this case $Q=a \cdot v_1$ ...(1.23)

$$\therefore \qquad F_u=\frac{w(a \cdot v_1)}{g}(v_{u_1} - v_{u_2}) \qquad ...(1.24)$$

The factor $(v_{u_1} - v_{u_2})$ is the algebraic difference of the velocities of whirl at inlet and outlet, and can be obtained from velocity triangles (See Fig 1.17). If the water is discharged in the direction of the motion of blade, v_{u_2} will be positive (See Fig 1.11), which means that the change of velocity of whirl will be equal to $(v_{u_1} - v_{u_2})$. However in order to get more force the discharge angle α_2 should be obtuse which makes the direction of discharging water opposite to the blade motion. This makes v_{u_2} negative. Then the change of velocity of whirl will be equal to the arithmetic sum of v_{u_1} and v_{u_2}.

Work done on blade per sec=Force on the blades in the direction of motion×distance moved by the blades per sec.

$$=\frac{w \cdot Q}{g}(v_{u_1} - v_{u_2}) \cdot u \qquad ...(1.25)$$

Horse power developed by the wheel$=\frac{w \cdot Q}{g}(v_{u_1} - v_{u_2}) \cdot \frac{u}{550}$ ...(1.26)

(When units are in FPS system ; Q is in cusecs, velocities in ft/sec, $g=32{\cdot}2$ ft/sec^2 and $w=62{\cdot}4$ lb/ft^3)

or Metric HP developed by the wheel$=\frac{w \cdot Q}{g}(v_{u_1} - v_{u_2}) \cdot \frac{u}{75}$ ...(1.26a)

(Where Q is in m^3/sec, velocities in m/sec, $g=9{\cdot}81$ m/sec^2 and $w=1000$ kg/m^3)

Work done on the blades per lb (or kg) of water$=\frac{(v_{u_1} - v_{u_2}) \cdot u}{g}$ ...(1.27)

Energy supplied to the blades per lb (or kg) of water

$$= \text{Kinetic energy of jet at entrance per lb (or kg) of water}$$

$$= \frac{v_1^2}{2g}$$

$\therefore$ Efficiency of turbine runner $\eta = \dfrac{\text{Output}}{\text{Input}}$

or $\eta = \dfrac{\text{Work done on blades per lb (or kg) of water}}{\text{Energy supplied to the blades per lb (or kg) of water}}$

$$= \frac{\dfrac{(v_{u_1} - v_{u_2}) \cdot u}{g}}{\dfrac{v_1^2}{2g}} \qquad \ldots(1.28)$$

$$= \frac{2u\,(v_{u_1} - v_{u_2})}{v_1^2} \qquad \ldots(1.29)$$

Problem 1.9 A jet of water 2 inches in diameter impinges on a curved vane and is deflected through an angle of 175°. The vane moves in the same direction as that of the jet with a velocity of 110 ft per sec. The rate of flow is 6 cusecs. Determine the component of force on the vane in the direction of motion. How much would be the Horse Power developed by the vane and what would be the water efficiency ? Neglect friction.

Solve the problem if instead of one vane, there is a series of vanes fixed to a wheel.

Solution

$$d = 2 \text{ in.} \qquad u = 110 \text{ ft/sec} \qquad Q = 6 \text{ cfs}$$

$$v_1 = \frac{Q}{a} = \frac{6}{\frac{\pi}{4} \times \left(\frac{2}{12}\right)^2} = 275 \text{ ft/sec.}$$

Assume $\alpha_1 = 0$ and $\beta_1 = 0$; $\beta_2 = 180° - 175°$ (See Fig 1.18*b*)

Velocity of jet in the direction of motion at inlet, $v_1 \cos \alpha_1 = v_{u_1} = v_1$

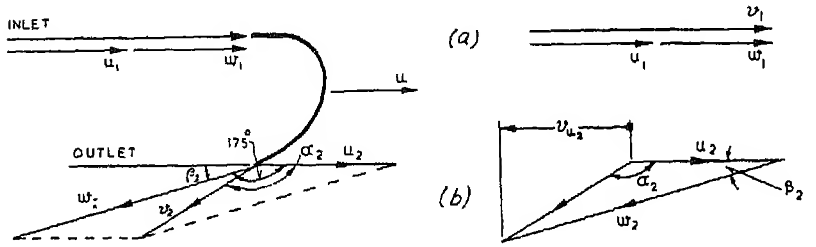


Fig 1.18 Velocity Triangles (*a*) Inlet (*b*) Outlet

and velocity of jet in direction of motion at outlet $= v_2 \cos \alpha_2$

Draw velocity triangles at inlet and outlet of vane (See Fig 1.18)

Considering the inlet velocity triangle first—

$$v_1 = u_1 + w_1$$

$$\therefore \quad w_1 = v_1 - u_1 = 275 - 110 = 165 \text{ ft/sec.}$$

Now $w_1 = w_2$, assuming that no loss occurs along the vane from inlet to outlet. Further the vane is moving with a velocity of 110 ft/sec. The radius of arc of a circle drawn from the centre of the wheel to the point where the jet impinges and to the point where the jet leaves the vane, is the *same*, therefore $u = \omega \cdot r = u_1 = u_2$.

From Outlet Velocity triangle (See Fig 1.18*b*)

$$\begin{aligned} v_2 \cos \alpha_2 = v_{u_2} &= u_2 - w_2 \cos \beta_2 \\ &= 110 - 165 \times \cos 5^\circ \\ &= 110 - 165 \times 0{\cdot}9962 \\ &= 110 - 164{\cdot}5 \\ &= -54{\cdot}5 \text{ ft/sec.} \end{aligned}$$

Force exerted by the jet in the direction of motion, (Eqn 1.20)

$$F_u = \frac{w \cdot a \cdot (v_1 - u_1)}{g} (v_1 \cos \alpha_1 - v_2 \cos \alpha_2)$$

$$= \frac{62{\cdot}4 \times \left\{ \frac{\pi}{4} \left(\frac{2}{12} \right)^2 \times (275 - 110) \right\}}{32{\cdot}2} (275 + 54{\cdot}5)$$

$= \mathbf{2{,}295\ lb}$ *Answer*

HP developed by jet $= \frac{F_u \cdot u}{550} = \frac{2{,}295 \times 110}{550} = \mathbf{460\ HP}$ *Answer*

Water efficiency $= \frac{\text{HP developed} \times 550}{K.E. \text{ of jet } (= \frac{1}{2} m \cdot v_1^2)}$

$$= \frac{460 \times 550}{\frac{1}{2} \times \frac{62{\cdot}4 \times 6}{32{\cdot}2} \times 275^2} = 0\ 575 \text{ or } \mathbf{57{\cdot}5\%} \quad \textit{Answer}$$

b) A series of vanes fixed to a wheel, instead of one vane only—

$$F_u = \frac{w}{g} (a \cdot v_1)(v_1 \cos \alpha_1 - v_2 \cos \alpha_2)$$

$$= \frac{62{\cdot}4}{32{\cdot}2} \times 6(275 + 54{\cdot}5)$$

$= \mathbf{3{,}835\ lb}$ *Answer*

HP developed by jet $= \frac{F_u \cdot u}{550} = \frac{3{,}835 \times 110}{550} = \mathbf{767\ HP}$ *Answer*

Water efficiency $= \frac{\text{HP developed}}{K.E. \text{ of jet}} = \frac{767 \times 550}{\frac{1}{2} \times \frac{62{\cdot}4 \times 6}{32{\cdot}2} \times 275^2}$

$= 0{\cdot}959$ or $\mathbf{95{\cdot}9\%}$ *Answer*

Problem 1.10 A $2\frac{1}{2}$ inches diameter water jet having a velocity of 45 ft per sec impinges on the bucket of a wheel. The axis of the jet

coincides with the axis of the bucket. The bucket is a part of sphere and has a radius of 7 in., the depth being $3\frac{1}{2}$ in. Determine the force exerted by the jet on the bucket when

a) the bucket is fixed,

b) the bucket is moving in the same direction as the jet with a velocity with which the work done per second by the jet on the bucket would be maximum,

c) there is a series of buckets in place of only one, and moving with a velocity such that the efficiency is maximum.

Find the horse power in each case and also give the value of the maximum efficiency.

Solution

$d=2\frac{1}{2}$ in. $\qquad v_1=45$ ft/sec

radius of bucket$=7$ in. $\qquad$ depth$=3\frac{1}{2}$ in.

area of jet $\qquad a=\frac{\pi}{4}=\left(\frac{2\frac{1}{2}}{12}\right)^2=\frac{\pi}{4}\times\frac{25}{4\times 144}=0{\cdot}034$ sq ft

$Q=a\,.\,v_1=0{\cdot}034\times 45=1{\cdot}53$ cu ft/see

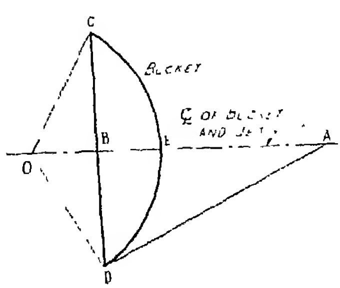


Fig 1.19 Bucket made from a Sphere

From Fig 1.19

$OC=OD=$radius of bucket$=7$ in.

$BE=3{\cdot}5$ in.

$\therefore \quad OB=3{\cdot}5$ in.

and $BD=\sqrt{7^2-3{\cdot}5^2}$

$=\sqrt{49-12{\cdot}25}=\sqrt{36{\cdot}75}$

$=6{\cdot}06$ in.

angle of jet with the axis at outlet—

$\beta_2=\angle BAD=\angle ODB$

$=\sin^{-1}\frac{OB}{OD}=\sin^{-1}\frac{3{\cdot}5}{7}=\sin^{-1}0{\cdot}5$

$\therefore \quad \beta_2=30^\circ$

a) Change of velocity in direction of jet

$=v_1-(-v_1\cos 30^\circ)=v_1+v_1\cos 30^\circ$

Mass of water moving per sec$=\frac{w\,.\,Q}{g}$

$\therefore \qquad$ Force on fixed bucket$=\frac{w\,.\,Q}{g}(v_1+v_1\cos 30^\circ)$

$=\frac{62{\cdot}4\times 1{\cdot}53}{32{\cdot}2}\times 45(1+0{\cdot}866)=$ **250 lb**

Horse Power is **zero**, because the bucket is fixed and no work is done. Hence efficiency is also **zero.**

b) The bucket is moving with a velocity of u in the same direction as that of jet. Drawing the velocity diagrams (See Fig 1.18),

$v_{u_1}=v_1=45$ ft/sec

and $\quad w_1=w_2$......(assuming no loss on the bucket)

$\therefore \quad w_2=w_1=v_1-u$

$=45-u$

From outlet velocity diagram (See Fig 1.18),

$v_{u_2}=u-w_2 \cos 30°$

$=u-(45-u) \,.\, \cos 30°$

$=u-(45-u)\times 0{\cdot}866$

$=1{\cdot}866\,u-39$

Force exerted on the bucket,

$$F_x=\frac{w}{g}\left\{a(v_1-u)\right\}(v_{u_1}-v_{u_2})$$

or $$F_x=\frac{w\,.\,a}{g}(v_1-u)\,(v_{u_1}-v_{u_2})$$

$\therefore$ Work done per sec, $W=\dfrac{w\,.\,a}{g}(v_1-u)\,(v_{u_1}-v_{u_2})\,.\,u$

or $$W=\frac{w\,.\,a}{g}(45-u)(45+39-1{\cdot}866\,u)\,.\,u$$

$$=\frac{w\,.\,a}{g}[45\times 84-\{(45\times 1{\cdot}866)+84\}u+1{\cdot}866\,u^2]\,.\,u$$

For maximum efficiency, $\dfrac{dW}{du}=0$

$$\therefore\quad \frac{w\,.\,a}{g}\,.\,\frac{d}{du}(45\times 84\,u-168\,u^2+1{\cdot}866\,u^3)=0$$

or $\quad 45\times 84-2\times 168\,u-3\times 1{\cdot}866\,u^2=0$

or $$u^2-\frac{2\times 168}{3\times 1{\cdot}866}\,u+\frac{45\times 84}{3\times 1{\cdot}866}=0$$

or $\quad u^2-60\,u+675=0$

$$\therefore\quad u=\frac{60\pm\sqrt{60^2-4\times 675}}{2}=\frac{60\pm\sqrt{900}}{2}$$

$$=\frac{60\pm 30}{2}=45 \text{ or } 15 \text{ ft/sec}$$

If $u=45$ ft/sec, work done$=0$, *i.e.*, minimum, and for maximum work done, $u=15$ ft/sec.

Now $F_x=\dfrac{w\,.\,a}{g}(v_1-u)\,(v_{u_1}-v_{u_2})$

$$=\frac{62{\cdot}4}{32{\cdot}2}\times\frac{\pi}{4}\times\left(\frac{2\frac{1}{2}}{12}\right)^2\times(45-15)(45+39-1{\cdot}866\times 15)$$

$$=\frac{62{\cdot}4}{32{\cdot}2}\times 0{\cdot}034\times 30\times(84-28)$$

$=$**110 lb**

Horse power$=\dfrac{F_x\,.\,u}{550}=\dfrac{110\times15}{550}=$**3 HP**

Efficiency $=\dfrac{\text{Work done/sec}}{\text{Energy supplied}}=\dfrac{110\times15}{\frac{1}{2}\times\frac{62{\cdot}4\times1{\cdot}53}{32{\cdot}2}\times45^2}=\dfrac{1{,}650}{3{,}000}$

$=0{\cdot}55$ or 55%

c) If there is a series of buckets,

Force exerted on the buckets, $F_x=\dfrac{w\,.\,a\,.\,v_1}{g}\,.\,(v_{u_1}-v_{u_2})$

and work done per sec$=\dfrac{w\,.\,a\,.\,v_1}{g}\,(v_{u_1}-v_{u_2})\,.\,u$

as all the water impinges with a velocity of v_1.

$\therefore\ W=\dfrac{62{\cdot}4\times1{\cdot}53}{32{\cdot}2}\times(45+39-1{\cdot}886\ u)\,.\,u$

$=\dfrac{62{\cdot}4\times1{\cdot}53}{32{\cdot}2}\times(84-1{\cdot}866\ u)\,.\,u$

For maximum efficiency, $\dfrac{dW}{du}=0$

i.e. $84-2\times1{\cdot}866\ u=0$

or $u=\dfrac{84}{2\times1{\cdot}866}=22{\cdot}5$ ft/sec

(*i.e.*, for maximum efficiency, $u_1=\frac{1}{2}\,v_1$)

$\therefore$ Force exerted, $F_x=\dfrac{1{\cdot}53\times62{\cdot}4}{32{\cdot}2}\times(84-1{\cdot}866\times22{\cdot}5)$

$=\dfrac{1{\cdot}53\times62{\cdot}4\times42}{32{\cdot}2}=$**124·5 lb**

$\therefore$ Horse power $=\dfrac{F_x\,.\,u}{550}=\dfrac{124{\cdot}5\times22{\cdot}5}{550}=$**5·1 HP**

Efficiency $=\dfrac{\text{Work done/sec}}{\text{Energy supplied}}=\dfrac{124{\cdot}5\times22{\cdot}5}{\frac{1}{2}\times\frac{62{\cdot}4\times1{\cdot}53}{32{\cdot}2}\times45^2}=\dfrac{2{,}805}{3{,}000}$

$=0{\cdot}935$ or **93·5%**

Answers—

Force	Horse power	Efficiency
a) 250 lb	0 HP	0 %
b) 110 lb	3·0 HP	55 %
c) 124·5 lb	5·1 HP	93·5%

1.11 Radial Flow Over Turbine Blade—Radial flow is one in which a particle of water during its flow through the vanes of rotating runner, remains in a plane normal to the axis of rotation, in such a

way that its position changes only with respect to its distance from the axis of rotation. In tangential flow (Art 1.8 to 1.10), the distance of water particle from the axis of rotation remains same *i.e.* r=constant, therefore the action of stream of the vanes is determined by computing the force exerted by the moving fluid by applying Impulse Momentum Equation (See Eqn 1.1). However, in case of radial flow the distance r of the water particle from the axis of rotation varies along its path of flow, which has to be taken into account. Hence instead of finding the force by Impulse-Momentum Equation, the action of stream on the vanes for radial flow is determined by the evaluation of turning moment or torque produced on the vanes. The torque is equal to the Rate of Change of Angular Momentum.

Angular Momentum—If an elementary mass of fluid dM is moving with velocity v in a given direction, perpendicular distance of which is r from a fixed centre,

the angular momentum of weight$=dM \,.\, v \,.\, r$ ...(1.30)

Torque on Vane—Fig 1.20 shows the section through the radial flow turbine. The water from the pipes first reaches the guide vanes which guide the water to the runner vanes of turbine. The guide vanes are stationary and the runner revolves about a fixed centre with angular velocity ω. The flow over the runner vanes is radial.

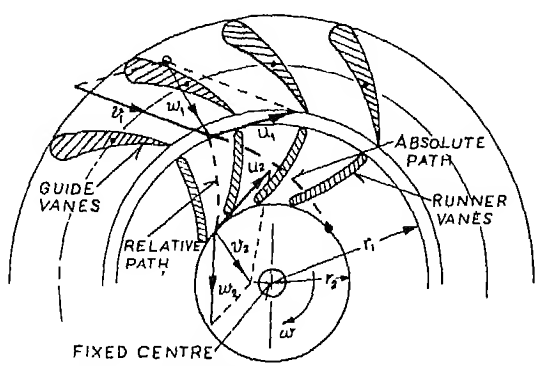


Fig 1.20 Section Through a Radial Flow Turbine

Let an elementary mass of water dM enter the rotating vane of the runner with velocity v_1 at radius r_1 and leave with velocity v_2 at radius r_2 after the time dt. Fig 1.21 shows the velocity triangles at inlet and outlet of turbine vane.

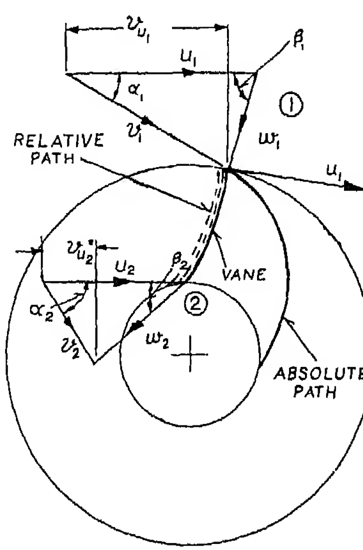


Fig 1.21 Velocity Triangles at Inlet (Pressure Side) and Outlet (Suction Side) of Vane

Peripheral component or component of absolute velocity v_1 in the direction of motion u_1 is

$$v_{u_1}=v_1 \cos \alpha_1$$

Similarly peripheral component of v_2 is

$v_{u_2} = v_2 \cos \alpha_2$

Applying Eqn 1.30,

Angular momentum at inlet $= (dM)(v_1 \cos \alpha_1)\, r_1$

Angular momentum at outlet $= (dM)(v_2 \cos \alpha_2)\, r_2$

$\therefore$ Change of angular momentum in time dt

$$= dM(r_1 \,.\, v_1 \cos \alpha_1 - r_2 \,.\, v_2 \cos \alpha_2)$$

$\therefore$ Rate of change of angular momentum

$$= \frac{dM}{dt}\,(r_1 \,.\, v_1 \cos \alpha_1 - r_2 \,.\, v_2 \cos \alpha_2)$$

but $$dM = \frac{w \,.\, Q}{g}\, dt$$

$\therefore$ Rate of change of angular momentum

$$= \frac{w \,.\, Q}{g}\,(r_1 \,.\, v_1 \cos \alpha_1 - r_2 \,.\, v_2 \cos \alpha_2)$$

which is equal to the torque T produced.

$$\therefore \quad T = \frac{w \,.\, Q}{g}\,(r_1 \,.\, v_1 \cos \alpha_1 - r_2 \,.\, v_2 \cos \alpha_2) \qquad \ldots(1.31)$$

If the torque given by this equation is positive, then this is the torque exerted by the fluid on the runner to revolve it. This is in case of water turbine which gives mechanical energy. If the torque is negative, then it will be the torque exerted on the fluid by the runner which is revolving by the external energy. This is a case of pump, compressor or fan.

Work done/sec or Power=Torque$\times$angular velocity

or $P = T \,.\, \omega$

$$= \frac{w \,.\, Q}{g}\,(r_1 \,.\, v_1 \cos \alpha_1 - r_2 \,.\, v_2 \cos \alpha_2) \,.\, \omega \qquad \ldots(1.32)$$

but $u_1 = \omega\, r_1$ and $u_2 = \omega\, r_2$ ...(1.33)

where u_1 and u_2 are the peripheral velocities of vane at inlet and outlet.

$$\text{Work done/sec or Power } P = \frac{w \,.\, Q}{g}\,(u_1 \,.\, v_1 \cos \alpha_1 - u_2 \,.\, v_2 \cos \alpha_2) \qquad \ldots(1.34)$$

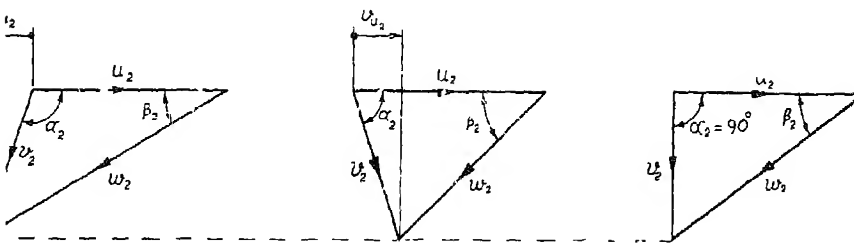
or $$P = \frac{w \,.\, Q}{g}\,(u_1 \,.\, v_{u_1} - u_2 \,.\, v_{u_2}) \qquad \ldots(1.34a)$$

The units of P are ft lb/sec or Kgm/sec.

This is the Fundamental Equation of Fluid Machines. As mentioned above, this is applicable to the turbine as well as to the pump runner. In such a case point 1 will denote the *pressure side* and point 2 the *suction side* of the turbine or pump. (See Fig 1.21). Thus the value of Power P will be positive in case of turbine as it is delivering power and the value is negative in case of pump which is consuming power.

The fluid may leave the vanes of *turbine runner* with an absolute velocity in a direction

a) against the motion of wheel,
b) same as motion of wheel,
or c) radially.



(a) (b) (c)

Fig 1.22 Outlet Velocity Triangles

In case (a), $\alpha_2 > 90°$, v_{u_2} is negative and

substituting this value in Eqn 1.34a,

$$\text{Work done/sec or Power} = \frac{w \cdot Q}{g}(u_1 \cdot v_{v_1} + u_2 \cdot v_{u_2}) \text{ ft lb/sec or } Kgm/\text{sec} \quad ...(1.35)$$

In case (b), $\alpha_2 < 90°$, v_{u_2} is positive,

$$\therefore \text{ Work done/sec or Power} = \frac{w \cdot Q}{g}(u_1 \cdot v_{u_1} - u_2 \cdot v_{u_2}) \text{ ft lb/sec or } Kgm/\text{sec} \quad ...(1.36)$$

In case (c), $\alpha_2 = 90°$, $v_{u_2} = 0$

$$\therefore \text{ Work done/sec or Power} = \frac{w \cdot Q}{g}(u_1 \cdot v_{u_1}) \text{ ft lb/sec or } Kgm/\text{sec} \quad ...(1.37)$$

Problem 1.11 Water enters an inward flow turbine at an angle of 22° to the tangent to the outer rim and leaves the turbine radially. If the speed of the wheel is 300 rpm and the velocity of flow is constant at 10 ft/sec, find the necessary angles of the blades when the inner and outer diameters of the turbine are 12 in. and 24 in. respectively. If width of wheel at inlet is 6 in., calculate HP developed. Thickness of blade may be neglected.

(AMIE—May 1954)

Solution

$\alpha_1 = 22°$
$\alpha_2 = 90°$ (radial discharge)
$N = 300$ rpm
$v_{m_1} = 10$ ft/sec $= v_{m_2}$

($\because$ Velocity of flow remains constant)

$D_1=24$ in. $=2$ ft (inward flow turbine)
$D_2=12$ in. $=1$ ft
$B_1=6$ in.

Peripheral velocity of wheel at inlet, $u_1=\dfrac{\pi D_1 N}{60}$

$$u_1=\frac{\pi\times 2\times 300}{60}=31{\cdot}4 \text{ ft/sec}$$

Peripheral velocity of wheel at outlet, $u_2=\dfrac{\pi D_2 N}{60}$

$$u_2=\frac{\pi\times 1\times 300}{60}=15{\cdot}7 \text{ ft/sec}$$

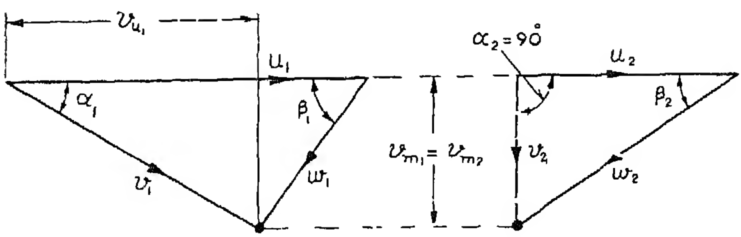


Fig 1.23 Inlet and Outlet Velocity Triangles

Draw the velocity triangle at inlet with u_1, α_1 and v_{m_1}. It is required to find out the blade angle β_1 from the triangle.

$$\tan\beta_1=\frac{v_{m_1}}{u_1-v_{u_1}}=\frac{v_{m_1}}{u_1-v_1\cos\alpha_1} \qquad ..(1)$$

But $\qquad v_{m_1}=v_1\sin\alpha_1$

$$\therefore \qquad v_1=\frac{v_{m_1}}{\sin\alpha_1}=\frac{10}{\sin 22°}=\frac{10}{0{\cdot}3746}=26{\cdot}70 \text{ ft/sec}$$

Substituting v_1, v_{m_1}, α_1 and u_1 in Eqn (1)

$$\tan\beta_1=\frac{10}{31{\cdot}4-26{\cdot}7\times\cos 22°}=\frac{10}{31{\cdot}4-26{\cdot}7\times 0{\cdot}9272}$$
$$=1{\cdot}504$$

$\therefore \qquad$ **$\beta_1=56°-23'$** *Answer*

Similarly to find the blade angle at outlet β_2, draw the velocity triangle at outlet with u_2, α_2 and v_{m_2}

$$\tan\beta_2=\frac{v_{m_2}}{u_2}=\frac{10}{15{\cdot}7}=0{\cdot}637$$

$\therefore \qquad \beta_2=\tan^{-1}0{\cdot}637=$ **$32°-30'$** *Answer*

Work done/sec by the turbine runner

$$=\frac{w\,.\,Q}{g}\,(u_1\,.\,v_{u_1}-u_2\,.\,v_{u_2})$$

...(See Eqn 1.34*a*)

but $\alpha_2=90°$, $\therefore$ $v_{u_2}=0$

$\therefore$ Work done/sec$=\dfrac{w \cdot Q}{g}(u_1 \cdot v_{u_1})$ ...(See Eqn 1·37)

HP of turbine $=\dfrac{w \cdot Q}{550 \cdot g}(u_1 \cdot v_{u_1})$

But $Q=a_1 \cdot v_{m_1}$

and a_1, the area of flow$=\pi D_1 \cdot B_1$

B_1, the width of wheel at inlet$=6$ in.$=0{\cdot}5$ ft

$\therefore$ $a_1=\pi\times2\times0{\cdot}5=3{\cdot}14$ sq ft

$\therefore$ $Q=3{\cdot}14\times10=31{\cdot}4$ cu ft/sec

$v_{u_1}=v_1\cos\alpha_1$

$=26{\cdot}70\times\cos 22°=24{\cdot}75$ ft/sec

Hence HP of turbine$=\dfrac{62{\cdot}4\times31{\cdot}4}{550\times32{\cdot}2}\times24{\cdot}75\times31{\cdot}4$

$=$**86 HP** *Answer*

Problem 1.12 In an inward flow turbine the water comes out of guide vanes and falls with a velocity of 120 ft/sec on the runner consisting of a series of curved blades. The speed of the runner is 300 rpm. The vanes have inlet and outlet diameters of 4 ft and 2 ft respectively. The angle which the guide vanes make with the periphery of the wheel is 30°.

The water after doing work on the runner discharges with an absolute velocity of 10 ft/sec at an angle of 120° to the wheel tangent. Find the water horse power if the rate of flow is 10 cusecs. Determine the best angles of blades.

Solution

$v_1=120$ ft/sec $\quad N=300$ rpm

$D_1=4$ ft $\quad D_2=2$ ft

$\alpha_1=30°$ $\quad \alpha_2=120°$

$v_2=10$ ft/sec $\quad Q=10$ cusecs

$$u_1=\frac{\pi D_1 \cdot N}{60}=\frac{\pi\times4\times300}{60}=62{\cdot}8 \text{ ft/sec}$$

$$v_{u_1}=v_1\cos\alpha_1=120\times\cos 30°=120\times0{\cdot}866=104 \text{ ft/sec}$$

$$v_{m_1}=v_1\sin\alpha_1=120\times\sin 30°=120\times0{\cdot}5=60 \text{ ft/sec}$$

Draw inlet velocity triangle (See Fig 1.24)

$$\tan(180°-\beta_1)=\frac{v_{m_1}}{v_{u_1}-u_1}=\frac{60}{104-62{\cdot}8}$$

$$=\frac{60}{41{\cdot}2}=1{\cdot}452$$

$\therefore$ $(180-\beta_1)=$**55°−16′** or $\beta_1=$**124°−44′** *Answer*

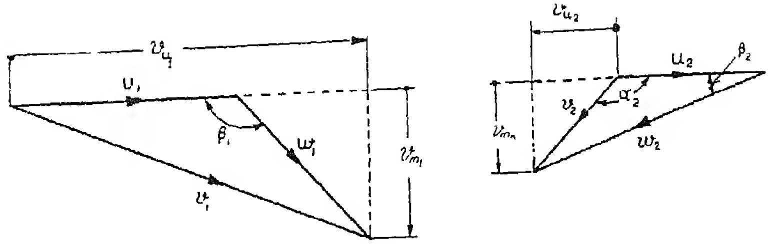


Fig 1.24 Inlet and Outlet Velocity Triangles

$$u_2=\frac{\pi\ D_2\ .\ N}{60}=\frac{\pi\times 2\times 300}{60}=31{\cdot}4 \text{ ft/sec}$$

$$v_{u_2}=v_2\cos\alpha_2=10\times\cos 120^\circ=-5 \text{ ft/sec}$$

$$v_{m_2}=v_2\sin\alpha_2=10\times 0{\cdot}866=8{\cdot}66 \text{ ft/sec}$$

Draw outlet velocity triangle (Fig 1.24)

$$\tan\beta_2=\frac{v_{m_2}}{u_2-v_{u_2}}=\frac{8{\cdot}66}{31{\cdot}4+5}$$

$$=\frac{8{\cdot}66}{36{\cdot}4}=0{\cdot}238$$

$\therefore\ \ \beta_2=\mathbf{13^\circ-24'}$ *Answer*

Work done by water per sec $=\dfrac{w\ .\ Q}{g}\ (u_1\ .\ v_{u_1}-u_2\ .\ v_{u_2})$

...(See Eqn 1.34*a*)

$\therefore$ Hydraulic or Water Horse Power $=\dfrac{w\ .\ Q}{g\times 550}\ (u_1\ .\ v_{u_1}-u_2\ .\ v_{u_2})$

$$=\frac{62{\cdot}4\times 10}{32{\cdot}2\times 550}\times\left\{(62{\cdot}4\times 104)-(31{\cdot}4\times -5)\right\}$$

$$=\frac{62{\cdot}4\times 10\times 6{,}647}{32{\cdot}2\times 550}=\mathbf{234\ HP}$$ *Answer*

1.12 Jet Propulsion—

Principle—An orifice of cross-sectional area a is fixed to tank full of water (Fig 1.25). The jet issues out from the orifice with a large velocity v. Let the quantity of water coming out of orifice per sec be Q. Applying impulse-momentum equation (See Eqn 1.1), the force exerted upon the fluid to change its velocity from 0 to v is

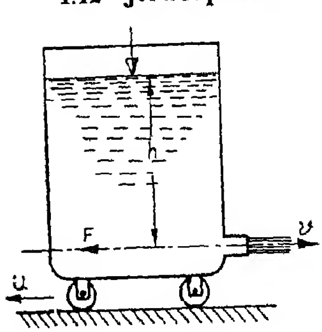


Fig 1.25 Principle of Jet Propulsion

$$F=\frac{w\ .\ Q}{g}\ .\ v=\frac{w\ .\ a\ .\ v^2}{g} \qquad ...(1.38)$$

where $v=K_v\ .\ \sqrt{2gh}$

K'_v=co-efficient of velocity

h=head over the centre line of orifice.

According to Newton's Law of Action and Reaction, an equal force F is exerted upon the tank in a direction opposite to v, shown by the arrow in Fig 1.25. If the tank is provided with wheels, it will start moving towards the left with a velocity u (say).

Propulsion of Ships—The principle explained above may be applied to drive the ship through the water. The ship carries pumps which take water from that surrounding the vessel and discharge by forcing through the orifice at the back of the ship.

Let u=velocity of ship

v=velocity of issuing jet

w_1=velocity of water issuing from the jet relative to the motion of ship

Then $w_1=u+v$ ($\because$ u and v act in opposite directions)

Head of water supplied by the pumps or energy supplied per lb of water= $\dfrac{w_1^2}{2g}$.

Momentum of jet per sec$=\dfrac{w \cdot Q}{g} \cdot v$

Momentum of water entering the ship per sec=0

($\because$ it is at rest).

$\therefore$ Rate of change of momentum$=\dfrac{w \cdot Q}{g} \cdot v$ ...(1.39)

This is equal to the *Dynamic Force* responsible for propelling the ship.

Work done on the ship by the jet per sec$=\dfrac{w \cdot Q}{g} v \cdot u$ ...(1.40)

but $w_1=u+v$ or $v=w_1-u$

$\therefore$ Work done on the ship by the jet per sec$=\dfrac{w \cdot Q}{g}(w_1-u) \cdot u$

...(1.40a)

Energy supplied per sec$=w \cdot Q\times$(energy supplied per lb of water)

$$=w \cdot Q \cdot \frac{w_1^2}{2g}$$

$\therefore$ Efficiency of system $\eta=\dfrac{\text{Work done on the ship by the jet per sec}}{\text{Energy supplied per sec}}$

$$=\frac{\dfrac{w \cdot Q}{g}(w_1-u) \cdot u}{w \cdot Q \cdot \dfrac{w_1^2}{2g}}=\frac{2(w_1-u) \cdot u}{w_1^2} \quad (1.41)$$

For maximum efficiency $\frac{d\eta}{du}=0$

$$\therefore \quad \frac{d}{du}(w_1 \cdot u-u^2)=0$$

or $$w_1-2u=0$$

or $$u=\frac{w_1}{2} \qquad \ldots(1\cdot42)$$

Substituting the value of u in Eqn 1.41,

$$\eta_{max}=\frac{2\left(w_1-\frac{w_1}{2}\right)\cdot\frac{w_1}{2}}{w_1^2}=\frac{1}{2} \text{ or } 50\% \qquad \ldots(1.43)$$

In some cases the water enters the ship through a pipe having an open end at the front of ship. As the water is at rest and the ship moves with a velocity u, the water will enter the pipe with velocity u relative to the motion of the ship.

$\therefore$ Energy supplied by the water per sec$=w \cdot Q\left(\frac{w_1^2}{2g}-\frac{u^2}{2g}\right)$

instead of $w \cdot Q \cdot \frac{w_1^2}{2g}$ as in the previous case. Thus now the energy supplied is less than in the previous case.

Now the work done on the ship by the jet per sec

$$=\frac{w \cdot Q}{g} \cdot (w_1-u)\, u \qquad \ldots(\text{See Eqn } 1.40a)$$

i.e. same as in the previous case.

$$\therefore \text{ Efficiency } \eta=\frac{\frac{w \cdot Q}{g}(w_1 \cdot u) \cdot u}{w \cdot Q\left(\frac{w_1^2}{2g}-\frac{u^2}{2g}\right)}=\frac{2(w_1-u) \cdot u}{(w_1+u)(w_1-u)}$$

$$=\frac{2u}{(w_1+u)} \qquad \ldots(1.44)$$

The principle of jet propulsion is similar to that of rocket, therefore the jet propulsion of ships can be known as water rocket. In practice screw propulsion is employed to move the ships and jet propulsion is no longer used, except in the case of life boats to a limited extent only.

UNSOLVED PROBLEMS

1.1 Define Dynamic Force ? How is it distinguished from hydrostatic pressure ?

1.2 Derive Impulse-Momentum Equation.

1.3 Differentiate between tangential and radial flow.

1.4 What is the difference between the force of jet when it impinges on a single moving flat plate and the force of jet when it strikes on a series of moving plates ?

1.5 How is the force on the bend of a pipe-line running full of water, determined? What precautions are taken if this force is very large?

1.6 Explain the flow over the double hemispherical vane of Pelton turbine.

1.7 How is the absolute path of jet falling on a moving vane, determined?

1.8 How are the velocity diagrams for the flow over the moving vanes drawn?

1.9 What are 'Velocity of Whirl' and 'Velocity of Flow' and why are they so named?

1.10 Show that when a jet of water impinges on a series of curved vanes, maximum efficiency is obtained when the vane is semi-circular in section and the velocity of the vane is half that of the jet.
(*Mysore University—1957*)

1.11 Define angular momentum and explain how is it used to determine the torque and work done by a vane in case of radial flow.

1.12 What is Fundamental Equation of Fluid Machines? How is it applied to both turbines and pumps?

1.13 How is the Fundamental Equation of Fluid Machines affected if angle α_2 is actute, obtuse or complimentary?

1.14 What is jet propulsion? How do the ships move with this principle?

1.15 A jet of water 2 inches in diameter exerts a force of 600 lb on a flat plate held normal to the jet's path, what is the rate of discharge?
(2·6 cfs)

1.16 A jet of water 2 inches in diameter is moving at 50 ft per second. It strikes a flat plate which is inclined at 30° to the jet. Find the force on the plate in the direction of jet, when

a) the plate is stationary,

b) the plate is moving at 10 ft per second in the direction of jet.
(53 lb ; 33·9 lb) (*Roorkee University—1959*)

1.17 A jet of water 2 inches in diameter strikes a curved vane at rest with a velocity of 90 ft/sec and is deflected through 135° from its original direction. Neglecting friction, compute the resultant force on the blade in magnitude and direction.
(630 lb ; 22·5°) (*AICTE—1958 Suppl*)

1.18 A free jet whose sectional area is 20 sq cm and whose velocity is 25 metres/sec, impinges tangentially on a smooth vane which diverts its direction through 120°. What is the magnitude and direction of the resultant force on the vane? (216·5 Kg)

1.19 A rectangular plate weighing 12 lb is suspended vertically by a hinge on the top horizontal edge. The centre of gravity of the plate is 4 in. from the hinge. A horizontal jet of water of 1 in. diameter whose axis is 6 in. below the hinge impinges normally on the plate with a velocity of 18·5 ft per sec. Find the horizontal force applied at the centre of gravity to maintain the plate in its vertical position.

Find the change of velocity of the jet if the plate is deflected through an angle of 30° and the same horizontal force continues to act at the centre of gravity of the plate.

(5·45 lb ; 9·4 ft/sec) (*Annamalai University—1958*)

1.20 A water jet issues from a nozzle 2 in. diameter, strikes a plate with a velocity of 100 ft/sec. The plate makes an angle of 60° to the horizontal. If frictional loss is such as to reduce the velocity of the stream leaving the body to 80 ft/sec, find

a) component of force in the direction of jet,

b) component of force normal to jet,

c) magnitude and direction of resultant of force exerted by water.

[*a*) 254 lb, *b*) 293 lb, *c*) 388 lb at 49°—8′ with direction of jet.]

1.21 A horizontal pipe of 4 in. diameter conveys water at a pressure of 40 lb/sq in. with a velocity of 6 ft/sec. If the diameter of pipe decreases to 2 in., find the axial force on this length of pipe. Assume no loss of energy.

If a bend is connected to the horizontal pipe which turns through 30° and tapers uniformly from 4 in. to 2 in., find the forces exerted on the end other than due to gravity.

(370 lb in the direction of flow, F_x=388·6 lb, F_y=69·29 lb)

1.22 A 60° reducing bend having an inlet diameter 12 in. and outlet diameter 6 in., carries 12 cusecs of water. The pressure of water at inlet is 40 lb/sq in. Find the components of force necessary to hold the bend in position. Neglect friction.

(F_x=3,933 lb, F_y=1,635 lb)

1.23 Water flows through a 100° bend in a 6 ft diameter pipe which carries a discharge of 300 cusecs. The ends of the bend are anchored by tie rods at right angles to the bend. Find the tension on each tie rod. (6060 lb) (*Poona University—1958*)

1.24 A pipe of 60 cm diameter is deflected through 90°. The ends of the pipe are anchored by the rods at right angles to the pipe at the ends of the bend. If the pipe is delivering 1·5 m^3/sec, find the tension in each tie rod. (795 Kg)

1.25 A 90° reducing bend rising upward carries 12·6 cfs of oil, sp gr 0·85, at a pressure of 20·5 lb/sq in. entering the bend at one end. The diameter of bend through which the oil enters is 16 in. and it is reduced to 12 in. on the other end. The volume between two ends is 3·75 cu ft. Neglecting friction, find the force on the bend.

(5,180 lb, −76·1°)

1.26 A jet of water issues from a nozzle 0·1875 in. diameter and impinges axially on an approximately hemispherical cup which is observed to turn it through a total angle of 160°. Quantity of water coming out of nozzle is 24 lb in 42 seconds and the force exerted on the vane is 1·2 lb. Determine the ratio of the velocity of water as it leaves the vane to the velocity of approach. (0·4427)

1.27 A jet of water 2 in. diameter having a velocity of 60 ft/sec, glides without shock into a series of smooth curved vanes moving in a direction parallel to the jet and away from it with a velocity of

26 ft/sec. The water is turned round through an angle of 150° as it passes over the vane. Find the pressure exerted on the vanes in the direction of rotation, and HP developed.

If for any reason the blade is held stationary, what will be the pressure on it? (161 lb, 7·62 HP, 284 lb) (*Bombay University—1948*)

1.28 A series of curved vanes (entrance angle 30° and exit angle 15°) deflects a jet of water 1·5 sq in. in area, moving at 160 ft/sec and inclined at 15° to the line of motion of the vane. Find—

a) the velocity of the vane to avoid shock at entry,

b) the magnitude and direction of the resultant force on the vane and the force in the direction of motion,

c) the magnitude and direction of velocity at exit.

a) 82 5 ft/sec ; *b*) 493 lb at 7·5° and 490 lb in the direction of motion *c*) 21·5 ft/sec at 82·5°. (*Bombay University – 1951*)

1.29 A jet of water having a velocity of 110 ft/sec impinges on a series of vanes moving with a velocity of 60 ft/sec. The jet makes an angle of 30° to the direction of motion of vanes when entering and leaves at an angle of 120°. Draw the triangles of velocities at inlet and outlet and find

a) angles of vane tips so that water enters and leaves without shock,

b) the work done per lb of water entering the turbine,

c) efficiency of the turbine. (132°—42′; 7·5°; 186·4 ft lb/sec; 93·3%) (*AMIE—Nov 1956 and Madras University*)

1.30 In an inward flow turbine, the peripheral velocity of the wheel is 70 ft/sec. The velocity of the whirl at-inlet is 55 ft/sec, and radial velocity of flow is 7 ft/sec. If the flow is 24 cfs and hydraulic efficiency 80%, find the head available and the horse-power of the turbine. Assume radial discharge.

(149·5 ft ; 326 HP) (*Roorkee University – 1959*)

1.31 An inward flow reaction turbine develops 260 HP at an overall efficiency of 78% under a head of 230 ft. The peripheral speed of vanes at inlet is 80 ft/sec. Width of wheel at inlet is $\frac{1}{8}$th the corresponding diameter. Velocity of flow remains constant at 15 ft/sec. Outlet diameter of vanes is $\frac{3}{4}$ inlet diameter. If inlet angle of runner vane is 90° to the tangent, determine the guide blade discharge angle and runner vane blade outlet angle. Velocity of whirl at exit is zero. (10° – 36′, 14°) (*Delhi University – 1956*)

1.32 Guide vanes of a Francis turbine make an angle of 15° with tangent to the wheel, the peripheral speed of which is 40 ft/sec. The absolute velocity of water at inlet is 45 ft/sec. Determine the proper values of vane angles at inlet and outlet, the inner dia of the wheel being half the outer. The water leaves the wheel in outer direction and velocity of flow is constant throughout.

(73·3° ; 30·15°) (*AMIE—April 1948*)

1.33 An inward flow turbine works under a total head of 120 ft. The velocity of the wheel periphery at inlet is 60 ft/sec. The outlet pipe of the turbine is 1 ft in dia, and the turbine is supplied with 8 cusecs. The radial velocity of flow through the wheel is the

same as velocity in the outlet pipe. Neglecting friction, determine—

a) vane angle at inlet,

b) guide blade angle and

c) HP of the turbine. (71·1° 9·1°; 107 HP) (*AMIE—May 1953*)

1.34 A motor-boat with jet propulsion draws 10 cfs through orifices amid ships and discharges it astern through orifices having an effective area of 0·5 sq ft. If the boat travels at 10 mph, find the propelling force. (103 lb) [*AMI Mech E*)

1.35 A locomotive going at 40 mph scoops up water from a trough. The tank is 8 ft above the mouth of the scoop, and the delivery pipe has an area of 50 sq in. If half the available head is wasted at entrance, find the velocity at which the water is delivered into the tank, and the number of tons lifted from a trench 500 yards long. What, under these conditions, is the increased resistance; and what is the minimum speed of train at which the tank can be filled? (34·8 ft/sec; 8·7 tons, 21·8 mph) (*London University*)

1.36 In a jet propelled boat, the resistance to the motion is 400 kg. The boat is provided with two jets each having an area 0·02 sq metres. The water is drawn through orifices amid ship. If the boat travels at a speed of 6 metres/sec, determine the quantity of water to be pumped per second and the efficiency of jet propulsion in this case. (0·534 cu metres; 49·5%)

CHAPTER 2

WHIRLING FLUID

2.1 Definitions—Vortex, Forced Vortex and Free Vortex 2.2 Mathematical Analysis of Forced Vortex—Cylindrical Vortex—Open Vessel and Closed Vessel 2.3 Principle of a Centrifugal Pump 2.4 Spiral Vortex 2.5 Flow along Curved Path 2.6 Free Vortex—Circulatory Flow 2.7 Radial Flow 2.8 Spiral Flow.

2.1 Definitions

Vortex—A mass of fluid in rotation is called a vortex. Vortices are of two types *viz*, forced and free.

Forced Vortex—When a fluid is made to rotate by some external agency, it is known as a *Forced Vortex.* The external agency is generally the mechanical power with which either the fluid is stirred or the vessel containing the fluid is set in motion.

Examples :

a) Rotation of liquid inside the impeller of a centrifugal pump,

and *b*) Rotation of liquid inside the runner of a turbine.

Free Vortex is one which requires no external impressed contact forces to cause rotation. Energy is not expended by any outside source. Here, the rotational motion is usually due to fluid pressure itself and gravity.

Examples :

a) Flow of water in the spiral casing of a turbine before it enters the guide vanes,

and *b*) Flow of water in a centrifugal pump casing after it has left the impeller.

2.2 Mathematical Analysis of Forced Vortex—

Cylindrical Vortex—Liquid is rotated in a cylindrical container by supplying energy from an external source. Let there be a vertical section through the axis of such a vessel (See Fig 2.1).

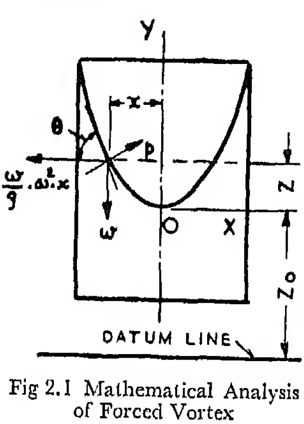


Fig 2.1 Mathematical Analysis of Forced Vortex

Let the section of liquid surface be any curve $y=f(x)$ and let P be any point on it given by (x, y) measured from an origin at the lowest point on the curve. It follows from symmetry that the latter point lies on the axis.

Consider the forces on a particle at P;

i) Weight w acting vertically downwards,

ii) Centrifugal force $\frac{w}{g}\,\omega^2 \,.\, x$ acting horizontally,

and *iii*) Fluid pressure P acting always normal to the surface.

Let the tangent at P make an $\angle\theta$ with x-axis.

For equilibrium, equating components of Forces, in the direction of tangent at P

$$\frac{w}{g}\,.\,\omega^2\,.\,x\cos\theta = w\,.\,\sin\theta$$

or $$\tan\theta = \frac{\omega^2\,.\,x}{g}$$

From elementary knowledge of calculus, $\tan\theta$, the slope of the tangent to curve $y=f(x)$ at point (x, y) is $\frac{dy}{dx}$.

$$\therefore \quad \frac{dy}{dx} = \frac{\omega^2\,.\,x}{g}$$

Integrating, $$y = \frac{\omega^2\,.\,x^2}{2g}$$

or $$x^2 = \frac{2g}{\omega^2}\,.\,y \qquad \ldots(2.1)$$

This evidently is the equation to a parabola of the second order, touching the origin and symmetrical about y-axis.

The surface of liquid in a forced vortex is, therefore, the surface generated by the revolution of a parabola *i.e.* the *Paraboloid of Revolution.*

In engineering practice the mathematical symbols x and y are often replaced by r and z respectively, then

$$r^2 = \frac{2g}{\omega^2}\,.\,z \quad \text{(where } z=\text{head of particle)}$$

or $$z = \frac{\omega^2\,.\,r^2}{2g} = \frac{u^2}{2g} \qquad \ldots(2.1a)$$

(where $u=\omega\,.\,r=$ circumferential velocity of particle)

If z_0 is the height of zero point measured from some datum line, it will become a more general relation,

$$z = \frac{\omega^2\,.\,r^2}{2g} + z_0 = \frac{u^2}{2g} + z_0$$

Difference of head between two points 1 and 2,

$$z_2 - z_1 = \frac{u_2^2 - u_1^2}{2g} \qquad \ldots(2.1b)$$

This is the head developed by the centrifugal action and is known as the *Centrifugal Head.*

A more general relation is

$$\left(\frac{p_2}{w} - \frac{p_1}{w}\right) + (z_2 - z_1) = \frac{\omega^2}{2g}\,(r_2^2 - r_1^2) = \frac{u_2^2 - u_1^2}{2g} \qquad \ldots(2.2)$$

a) **Open Vessel**—Liquid is free to form a paraboloid. Let z_i be the depression of level at centre due to formation of forced vortex (See Fig 2.2).

Since the total volume remains the same,

Volume of liquid in vessel before rotation = volume of liquid in vessel after rotation.

$V_o + \pi r^2 . z_i = V_o + \frac{1}{2} \pi r^2 . z$

(Where V_o = initial volume up-to centre of de-pression)

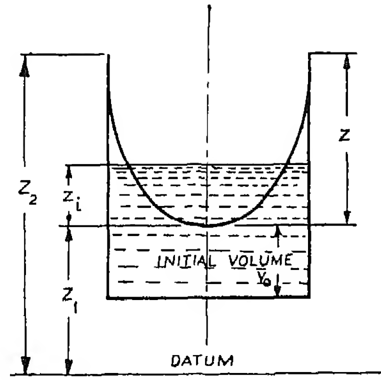


Fig 2.2 Forced Vortex in Open Vessel

$$\therefore \quad z_i = \frac{z}{2} = \frac{z_2 - z_1}{2}$$

$$\text{Since } r^2 = \frac{2g}{\omega^2} . z = \frac{2g}{\omega^2} . 2 . z_i$$

$$\therefore \quad z_i = \frac{r^2 . \omega^2}{4g} = k . \omega^2 \qquad \ldots(2.3)$$

(where k is a constant for the vessel)

Thus the depression of vortex below the original level of still water is proportional to square of angular velocity of rotation, $z_i \propto \omega^2$.

Problem 2.1 An open cylindrical vessel 12 inches in internal diameter and 3 ft long, stands vertically. It is filled with water upto a height of 2 ft from the bottom. Find the speed of rotation about its vertical axis, when the water is just going to spill out of vessel.

Solution

$d = 12$ in. $\qquad l = 3$ ft

$z_{empty} = 3 - 2 = 1$ ft

$$\text{Centrifugal head, } z = \frac{u^2}{2g} = \frac{\omega^2 . r^2}{2g} = \frac{\left(\frac{2\pi N}{60}\right)^2 . r^2}{2g} \qquad \text{(See Eqn 2.1}a)$$

At the point of spilling, the volume of empty space in the vessel before rotation = volume of the paraboloid,

i.e. $\pi r^2 . z_{empty} = \frac{1}{2} \pi r^2 . z$

$\therefore \quad z_{empty} = \frac{1}{2} z$

or $\quad z = 2z_{empty} = 2 \times 1 = 2$ ft

$$\text{But} \quad z = \frac{\left(\frac{2\pi N}{60}\right)^2 . r^2}{2g} = \frac{\left(\frac{2\pi \times N}{60}\right)^2 \times 0{\cdot}5^2}{64{\cdot}4}$$

$\therefore$ Substituting for z,

$$2 = \left(\frac{2\pi \times N}{60}\right)^2 \times \frac{0{\cdot}25}{64{\cdot}4}$$

or $\quad N=\sqrt{\dfrac{2\times 3{,}600\times 64{\cdot}4}{2\times 2\times\pi\times\pi\times 0{\cdot}25}}$

$=\sqrt{47{,}000}$

$=$**216·8 rpm** *Answer*

Problem 2.2 A cylinder 6 in. in dia and 15 in. high containing water is rotated about its vertical axis at a speed of 320 rpm, so that a portion of water spills out.

a) If the cylinder is now brought to rest, what would be the depth of water in it. *(AMIE—Nov 1953)*

b) If now the speed is increased to 600 rpm, how much water will be left in the vessel.

Solution

$d=6$ in. $=0{\cdot}5$ ft $\qquad h=15$ in. $=1{\cdot}25$ ft

$N=320$ rpm, $\qquad r=\dfrac{0{\cdot}5}{2}=0{\cdot}25$ ft

Angular velocity of vessel, $\omega=\dfrac{2\pi N}{60}=\dfrac{2\pi\times 320}{60}=33{\cdot}5$ radians/sec

Peripheral velocity of vessel, $u=\omega\,.\,r=33{\cdot}5\times 0{\cdot}25=8{\cdot}375$ ft/sec

$\therefore$ Centrifugal head generated by rotation, $z=\dfrac{u^2}{2g}=\dfrac{(8{\cdot}375)^2}{64{\cdot}4}$

$=1{\cdot}09$ ft

Volume of water which spills out, will be the volume of the paraboloid of revolution.

Volume of paraboloid of revolution $=\frac{1}{2}\pi r^2\,.\,z$

$=\frac{1}{2}\times\pi\times(0{\cdot}25)^2\times 1{\cdot}09=0{\cdot}107$ cu ft

Volume of vessel when full with water

$=\dfrac{\pi}{4}d^2\,.\,h=\dfrac{\pi}{4}\times(0{\cdot}5)^2\times 1{\cdot}25=0{\cdot}245$ cu ft

$\therefore$ Remaining water $=0{\cdot}245-0{\cdot}107=0{\cdot}138$ cu ft

$\therefore$ *a*) Depth of water after the vessel has come to rest

$=\dfrac{\text{Volume}}{\frac{\pi}{4}d^2}=\dfrac{0{\cdot}138}{\frac{\pi}{4}\times 0{\cdot}25}=0{\cdot}702$ ft

$=$**8·42 inches** *Answer*

b) Centrifugal head generated when the speed is 600 rpm,

$$z=\frac{\left(\dfrac{2\pi\,r\,N}{60}\right)^2}{2g}$$

$$=\frac{(2\pi\times 0{\cdot}25\times 600)^2}{3{,}600\times 64{\cdot}4}=3{\cdot}83\text{ ft}$$

The shape of the free surface will be as shown in Fig 2.3.

The volume of water spilling out will now be equal to the hatched volume in Fig 2.3. This can be determined as follows :—

Let x be the radius of parabola at the bottom of vessel, then

Applying Eqn 2.1 and replacing y by h

$$x=\frac{\sqrt{2gh}}{\omega}=\frac{\sqrt{64\cdot4\times2\cdot58}}{\frac{2\pi\times600}{60}}=0\cdot205 \text{ ft}$$

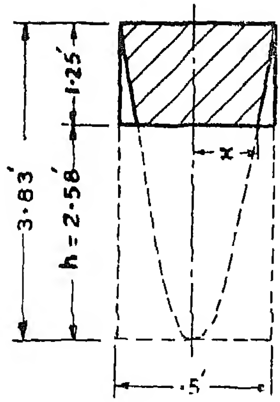


Fig 2.3 Paraboloid of Revolution with a Speed of 600 rpm

$\therefore$ Volume of water spilled

=(Total volume of paraboloid)—(volume of paraboloid under the bottom of vessel)

$$=\{\tfrac{1}{2}\times\pi\times(0\cdot25)^2\times3\cdot83\}-\{\tfrac{1}{2}\times\pi\times(0\cdot205)^2\times2\cdot58\}$$

$$=0\cdot376-0\cdot17=0\cdot206 \text{ cu ft}$$

$$\therefore \text{ Remaining water}=\left(\frac{\pi}{4}\times0\cdot5^2\times1\cdot25\right)-0\cdot206$$

$$=0\cdot245-0\cdot206=\mathbf{0\cdot039 \text{ cu ft}}$$ *Answer*

b) **Closed Vessel**—Let the fluid touch the lid when the angular velocity is ω, (See Fig 2.4). If the speed is increased even further say to ω',

$$(r')^2=\frac{2g}{(\omega')^2}\cdot z'$$

and $$r^2=\frac{2g}{\omega^2}\cdot z$$

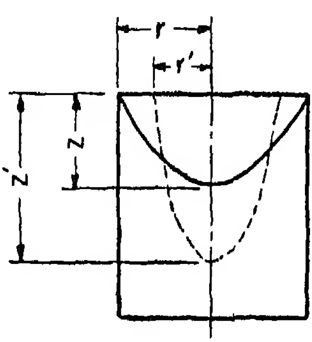


Fig 2.4 Forced Vortex in Closed Vessel

Since the volume of liquid remains the same, the volume of air enclosed also remains the same.

$$\therefore \quad \tfrac{1}{2}\cdot\pi\cdot r^2\cdot z=\tfrac{1}{2}\cdot\pi\cdot(r')^2\,(z')$$

$$\therefore \quad r^2\cdot z=r'^2\cdot z'=\frac{2g}{(\omega')^2}(z')\cdot(z')$$

Hence, $$(z')^2=\frac{r^2z}{2g}\cdot(\omega')^2 \qquad \ldots(2.4)$$

$$=m^2\cdot(\omega')^2$$

or $$z'=m\cdot\omega' \quad \text{(where } m \text{ is a constant)} \qquad \ldots(2.4a)$$

Pressure on the lid is equal to the head to which liquid would have risen in the absence of the lid.

Problem 2.3 A closed cylindrical vessel 10 in. (or 25·4 cm) in internal diameter is completely filled with water and is rotated about

its vertical axis at 1,450 rpm. Determine the difference of pressure between its circumference and the centre in ft (or metres) of water having a specific weight of 62·4 lb per cu ft (or 1,000 kg/m³). If the vessel now contains air of specific weight = 0·075 lb/cu ft (or 1·2 kg/m^3), what would be the pressure difference in ft (or metres) of fluid. Give the values in lb/sq in. (or kg/cm²) in both the cases.

Solution

$d = 10$ in. (or 25·4 cm) $\therefore\ r = \frac{5}{12}$ ft (or 0·127m)

$N = 1{,}450$ rpm $\therefore\ \omega = \frac{2\pi N}{60} = \frac{2\pi \times 1{,}450}{60} = 152$ radians/sec

Difference of pressure or centrifugal head

$$= \frac{p}{w_{water}} = \frac{u^2}{2g} = \frac{\omega^2 \cdot r^2}{2g} = \frac{(152 \times \frac{5}{12})^2}{64 \cdot 4} \text{ ft} \left[\text{or } \frac{(152 \times 0 \cdot 127)^2}{19 \cdot 6}\ m\right]$$

$= \mathbf{62\ ft}$ (or **18·9 m**) **of water** *Answer*

Difference of pressure if the fluid is air

$$= \frac{p}{w_{air}} = \frac{u^2}{2g} = \frac{\omega^2 \cdot r^2}{2g} = \frac{(152 \times \frac{5}{12})^2}{64 \cdot 4} \text{ ft} \left[\text{or } \frac{(152 \times 0 \cdot 127)^2}{19 \cdot 6}\ m\right]$$

$= \mathbf{62\ ft}$ (or **18·9 m**) **of air** *Answer*

Hence the difference of pressure is *same* if it is expressed in ft (or metres) of fluid handled.

Difference of pressure in lb/sq in. (or kg/cm²), if the vessel contains water

$$= \left(\frac{p}{w_{water}}\right) \cdot w_{water} = \frac{62 \times 62 \cdot 4}{144} \text{ lb/in}^2 \left(\text{or } \frac{18 \cdot 9 \times 10^3}{100^2} \text{ kg/cm}^2\right)$$

$=$ **26·8 lb/sq in.** (or **1·89 kg/cm²**) *Answer*

Difference of pressure in lb/sq in. (or kg/cm²), if the vessel contains air

$$= \left(\frac{p}{w_{air}}\right) \cdot w_{air} = 62 \times \frac{0 \cdot 075}{144} \text{ lb/in.}^2 \left(\text{or } \frac{18 \cdot 9 \times 1 \cdot 2}{100^2} \text{ kg/cm}^2\right)$$

$=$ **0·0323 lb/sq in.** (or **0·00227 kg/cm²**) *Answer*

Problem 2.4 A closed cylindrical vessel 6 in. in internal diameter and 3 ft long, stands vertically. It is filled with water upto a height of 2 ft from the bottom, and is rotated about its vertical axis. Find the revolving speed of the vessel when :—

i) the top of the cup formed by water just touches the top lid of the vessel,

and *ii*) the bottom of the cup formed by water just touches the bottom lid of the vessel.

Solution

$d = 6$ in. $l = 3$ ft $z_1 = 3 - 2 = 1$ ft

i) *When top of cup just touches the lid* (See Fig 2.5) :

Volume of hollow cup = Volume of empty space in the cylinder before rotation.

i.e. $\frac{1}{2} \cdot \pi \cdot r^2 \cdot z = \pi \cdot r^2 \cdot z_i$

or $\frac{1}{2} \times \pi \times (0.25)^2 \times z = \pi \times (0.25)^2 \times z_i$

$\therefore\ z = 2z_i = 2 \times 1 = 2$ ft

Centrifugal head developed by rotation,

$$z = \frac{\omega^2 \cdot r^2}{2g} = \frac{\left(\frac{2\pi N}{60}\right)^2 \times r^2}{2g}$$

or $$z = \left(\frac{2\pi N}{60}\right)^2 \times \frac{(0.25)^2}{64.4}$$

$$\therefore\quad N = \sqrt{\frac{2 \times 3{,}600 \times 64.4}{2 \times 2 \times \pi \times \pi \times (0.25)^2}}$$

$= \sqrt{1{,}88{,}000}$

$=$ **434 rpm** *Answer*

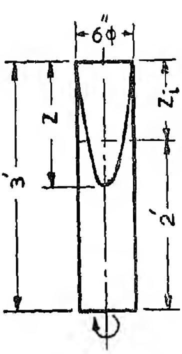


Fig 2.5 Top of Cup Touching the Lid of Closed Vessel

ii) *When the bottom of cup touches the bottom of vessel* (See Fig 2.6) : volume of the paraboloid = volume of the empty space before rotation.

i.e. $\frac{1}{2} \cdot \pi \cdot (r')^2 \cdot (z') = \pi \cdot r^2 \cdot z_i$

In this case the speed of the vessel is increased to ω'

$$\therefore\quad (r')^2 = \frac{2g}{(\omega')^2} \cdot (z')$$

Substituting the value of r' in the above equation,

$$\frac{1}{2} \cdot \pi \left\{ \frac{2g}{(\omega')^2} \cdot z' \right\} z' = \pi \cdot r^2 \cdot z_i$$

$$\therefore\quad (\omega')^2 = \frac{g \cdot (z')^2}{r^2 \cdot z_i}$$

or $$\left(\frac{2\pi N'}{60}\right)^2 = \frac{g \cdot (z')^2}{r^2 \cdot z_i}$$

$$\therefore\quad N' = \sqrt{\frac{g \cdot (z')^2}{r^2 \cdot z_i} \cdot \left(\frac{60}{2\pi}\right)^2}$$

$$= \sqrt{\frac{32.2 \times 3^2}{(0.25)^2 \times 1} \times \frac{3600}{4\pi^2}}$$

$= \sqrt{4{,}22{,}000}$

$=$ **650 rpm** *Answer*

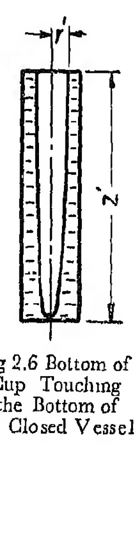


Fig 2.6 Bottom of Cup Touching the Bottom of Closed Vessel

The results obtained from the above mathematical example are very interesting. The speed in the first case is two-third the speed in the second case when the vessel is originally two-third full with water. Similarly, it can be shown that if the vessel is originally three-fourth full, the speed in the first case would be half the speed in the second case. Also if the vessel is half full, the speed in both cases would be the same.

Forming an equation :

$$\text{Since } \frac{N'}{N} = \frac{\frac{60}{2\pi r} \cdot z' \cdot \sqrt{\frac{g}{z_i}}}{\frac{60}{2\pi r} \times \sqrt{2z \cdot g}} = \frac{z'}{\sqrt{2z \cdot z_i}} = \frac{z'}{\sqrt{4z_i^2}} = \frac{z_i}{2z_i}$$

$$\left(\text{provided } \frac{z_i}{z} \lesseqgtr \frac{1}{2}\right)$$

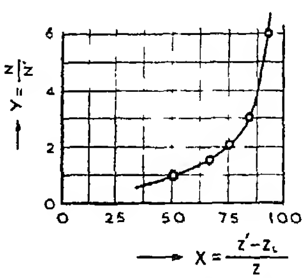


Fig 2.7 Variation of Centrifugal Head with Change of Container Speed

$$\text{or} \quad \frac{z_i}{z'} = \frac{N}{2N'}$$

$$\text{or} \quad 1 - \frac{z'-z_i}{z'} = \frac{1}{2}\left(\frac{N}{N'}\right)$$

Putting, $\frac{z'-z_i}{z'} = \frac{x}{100}$

which is percent full of water

and $\frac{N}{N'} = y$, the ratio of speeds

then $\mathbf{\frac{x}{100} = 1 - \frac{1}{2y}}$,

Tabulating and drawing in Fig 2.7

x	50	66·7	75	83·3	91·7	100
y	1	1·5	2	3	6	∞

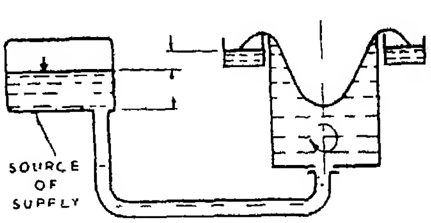


Fig 2.8 Principle of a Centrifugal Pump

2.3 Principle of a Centrifugal Pump—If the liquid is rotated with a sufficiently high velocity so as to enable it to rise beyond the walls of the container and if more liquid is constantly supplied at the centre by some suitable means, the tendency of the liquid would be to flow out as illustrated in Fig 2.8 Such a system in principle is a *Centrifugal Pump*.

If on the other hand, liquid flowed into the tank over the rim from an outside source at a higher elevation and were drawn out at the centre, the flow being inward, the system will constitute a *Francis Water Turbine* in principle.

2.4 Spiral Vortex—In runners of turbines and pumps the flow of fluid is usually a combination of

i) Circulatory flow *i.e.*, flow in concentric circles and *ii*) Radial flow *i.e.*, flow involving a change of distance from the axis of rotation.

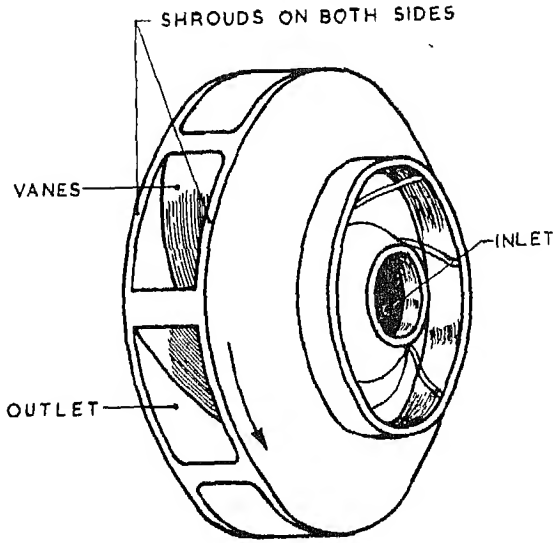


Fig 2.9 Impeller of a Centrifugal Pump

The path resulting from the superimposition of these two motions is of the form of spiral.

The impeller of a centrifugal pump consists of two plates called shrouds between which are fixed a number of vanes (See Fig 2.9). Water enters through an opening provided at the centre and leaves at the periphery. Vanes are made of spiral shape to enable water to have both circulatory and radial flows inside the impeller. As the radius increases from inlet to outlet, the area across flow must also increase (See Fig 2.10) and consequently the relative velocity decreases so that

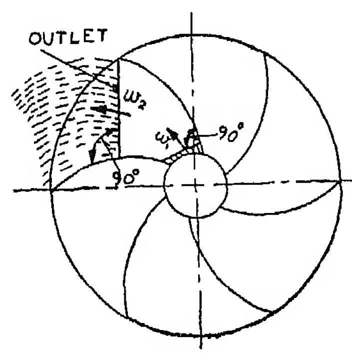


Fig 2.10 Line Diagram of Impeller Blades

$$w_1 = \frac{q}{a_1} \text{ and } w_2 = \frac{q}{a_2}$$

where, q = quantity of water flowing per sec and a_1 and a_2 are areas across flow at inlet and outlet respectively, between two blades.

$\therefore$ Pressure difference due to change of kinetic energy

$$= \frac{w_1^2 - w_2^2}{2g} \text{ per lb (or per kg) of fluid}$$

and pressure difference due to the centrifugal head

$$=\frac{\omega^2}{2g}(r_2^2-r_1^2) \qquad \text{...(See Eqn 2.16)}$$

This pressure difference is due to cylindrical vortex only.

Hence, the total pressure difference due to two flows *i.e.* cylindrical and spiral vortices will be

$$\frac{p_2-p_1}{w_d}=\frac{\omega^2}{2g}(r_2^2-r_1^2)+\frac{w_1^2-w_2^2}{2g}$$

(where w_d=specific weight of fluid)

$$=\frac{u_2^2-u_1^2}{2g}+\frac{w_1^2-w_2^2}{2g} \qquad \text{...(2.5)}$$

Problem 2.5 A centrifugal pump impeller has an inner diameter of 20 in. Its outer diameter is twice the inner diameter. Find the speed of the impeller in rpm at which the lifting will commence against a head of 50 ft.

Solution

R_1 the inner radius of impeller = 10 in. = $\frac{5}{6}$ ft

R_2, the outer radius of impeller = 20 in. = $\frac{5}{3}$ ft

Centrifugal head, $z=\frac{\omega^2}{2g}(R_2^2-R_1^2)$

or $$50=\frac{\left(\frac{2\pi N}{60}\right)^2}{64{\cdot}4}\left\{\left(\frac{5}{3}\right)^2-\left(\frac{5}{6}\right)^2\right\}$$

or $$N=\sqrt{\frac{50\times 64{\cdot}4\times 36\times 60\times 60}{4\times\pi\times\pi\times 75}}$$

$$=\sqrt{1{,}41{,}000}$$

$=$**376 rpm** *Answer*

Problem 2.6 Find the head developed by the impeller in Problem 2·5, if the relative velocities of water at inlet and outlet are 25 ft/sec and 18 ft/sec respectively.

Solution

$R_1=10$ in. $\qquad R_2=20$ in.

Head developed by centrifugal action = 50 ft

$w_1=25$ ft/sec $\qquad w_2=18$ ft/sec

Head developed by impeller $=\frac{u_2^2-u_1^2}{2g}+\frac{w_1^2-w_2^2}{2g}$

$$=50+\frac{25^2-18^2}{64{\cdot}4}$$

$$=50+4{\cdot}67$$

$=$**54·67 ft** *Answer*

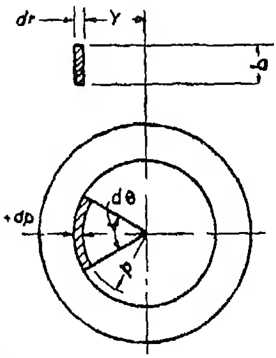


Fig 2.11 Flow along a Curved Path

2.5 Flow along Curved Path—Consider an annular ring of fluid of breadth b and thickness dr moving with an angular velocity ω (See Fig 2.11).

Forces, acting on a small element of the ring, which subtends an angle $d\theta$ at the centre of revolution, can be easily found out.

They are *i*) Centrifugal force acting radially outwards,

ii) Resultant force due to difference of liquid pressure on either side acting radially inwards,

and *iii*) Weight of the fluid, acting vertically downwards

Mass of this element$=\frac{w}{g} \cdot r \cdot d\theta \cdot dr \cdot b$

$\therefore$ Centrifugal force$=\left(\frac{w}{g} \cdot r \cdot d\theta \cdot dr \cdot b\right) \cdot w^2 \cdot r$

and Resultant force due to liquid pressure

$$=dp(r \cdot d\theta \cdot b)$$

(neglecting small quantities of second order)

For considering the equilibrium of forces in a horizontal plane, the vertical force w, weight of liquid, need not be taken into account and then

$$\frac{w}{g} r \cdot d\theta \cdot dr \cdot b \cdot w^2 \cdot r = dp \cdot r \cdot d\theta \cdot b$$

or $$dp=\frac{w \cdot dr}{g} \cdot \omega^2 \cdot r$$

If u be the tangential velocity corresponding to ω ; $u=\omega r$

and $$dp=\frac{w \cdot dr}{g} \cdot \frac{u^2}{r} \qquad \ldots(2.6)$$

This is the *Equation of Flow* in a curved path and it can be seen from here that $dp=0$ when $r=\infty$, which means that for rectilinear flow, pressure remains constant in directions transverse to the direction of motion.

2.6 Free Vortex—As already stated in Art 2.1, the free vortex is the mass of liquid in rotation, caused without applying any external force. A common example for such a flow is emptying a wash-basin which is a shallow vessel having a hole in the bottom through which the liquid discharges out. The motion of fluid in the basin becomes rotary and thus a free vortex is formed.

In free vortex the total head H in Bernoulli's Equation is constant for all stream lines, because no energy is imparted to the liquid in rotation.

Circulatory Flow—Consider a mass m of fluid moving in a circular path with a tangential velocity v_u at a distance r from the centre.

Force of rotating mass = mass × acceleration

$$= m \cdot \frac{v_u}{t}$$

and Torque = moment of resultant force

$$= m \cdot \frac{v_u}{t} \cdot r$$

In free vortex, the torque remains same as there is no addition of energy from external force. It follows from the Law of Conservation of Energy—

$$m \cdot \frac{v_u}{t} \cdot r = \text{constant}$$

but

$$\frac{m}{t} = \frac{w \cdot Q}{g} = \text{constant}$$

$$\therefore \quad v_u \cdot r = \text{constant} \qquad \ldots(2.7)$$

$$\text{or} \quad v_u \cdot r = c \qquad \ldots(2.7a)$$

$$\text{or} \quad v_u = \frac{c}{r} \qquad \ldots(2.8)$$

$$\text{or} \quad v_u \propto \frac{1}{r} \qquad \ldots(2.8a)$$

Hence the tangential velocity v_u is inversely proportional to distance r.

If this value of velocity is substituted in the general equation for curvilinear flow (See Eqn 2.6), then

$$dp = \frac{w \cdot dr}{g} \cdot \frac{v_u^2}{r} = \frac{w \cdot dr}{g} \cdot \frac{c^2}{r^2} \qquad .(2.9)$$

Integrating,

$$\frac{p_2}{w} - \frac{p_1}{w} = \frac{c^2}{2g}\left(\frac{1}{r_1^2} - \frac{1}{r_2^2}\right)$$

$$= \frac{c^2}{2g} \cdot \frac{1}{r_1^2}\left\{1 - \left(\frac{r_1}{r_2}\right)^2\right\}$$

$$= \frac{v_{u_1}^2}{2g}\left\{1 - \left(\frac{r_1}{r_2}\right)^2\right\} \qquad \left(\because \frac{c^2}{r_1^2} = v_{u_1}^2\right)$$

$$\text{i.e.} \quad \frac{p_2 - p_1}{w} = \frac{v_{u_1}^2}{2g}\left\{1 - \left(\frac{r_1}{r_2}\right)^2\right\} \qquad (2.10)$$

This equation may be used for calculating differences of pressure between any two points in a free vortex. This difference of pressure results from circulatory motions. Any calculation of actual pressure difference must also take into account the changes in static head, hydrostatic pressure and radial velocity. The pressure difference due to radial velocity is explained in the next article.

2.7 Radial Flow—Radial flow can be produced by constraining a stream of fluid to flow between two parallel circular plates the openings being at the centre and the periphery.

If b be the distance between two plates (See Fig 2.12) and v_m the radial velocity at a point distant R from the centre,

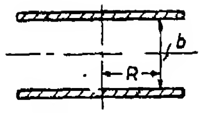


Fig 2.12 Radial Flow

area across flow $=2\pi \cdot R \cdot b$

If the quantity flowing per sec be q,

$$q=(2\pi \cdot R \cdot b) \cdot v_m \qquad \ldots(2.11)$$

where $\quad q=$constant

$\therefore \quad R \cdot b \cdot v_m=$constant

In radial flow distance b is constant, then

$$R \cdot v_m=\text{conssant} \qquad \ldots(2.12)$$

or $$v_m \propto \frac{1}{R} \qquad \ldots(2.12a)$$

It is interesting to note that pressure variation caused due to change of radial velocity is inversely proportional to R just as in circulatory motion.

2.8 Spiral Flow—Similar to that of forced vortex, the spiral flow in the case of free vortex will be the super-imposition of radial flow over circulatory flow. For such a flow the distance between two plates b (See Fig 2.12 and 2.14) will vary, thus

$$R \cdot b \cdot v_m=\text{constant}$$

or $$v_m=\frac{\text{constant}}{R \cdot b} \qquad \ldots(2.13)$$

This shows that the pressure variation caused due to the change in velocity will not be the same as in the previous case.

One way to find out the actual pressure difference is to calculate separately for each kind of flow by applying Eqn 2.8a and 2.12a. This is employed when the factors influencing the two flows are different.

Direct computation of pressure difference from the change in absolute velocity v is also possible, when the total head for either type of flow is the same.

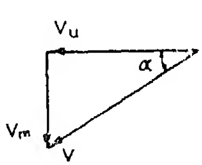


Fig 2.13 Velocity Triangle for Spiral Flow

Since both v_u and v_m have been shown to vary inversely as R, other factors remaining the same, angle α must remain constant (See Fig 2.13). Then the resultant path is an equiangular or *logarithmic spiral.*

Angle α is determined as $\tan \alpha=\dfrac{v_m}{v_u} \qquad \ldots(2.14)$

If a structurally essential obstacle to the flow of a stream of fluid is so designed that the natural path of a fluid is such a spiral, resistance to its motion would be least. A practical example of such a motion is in the volute of a centrifugal pump.

Problem 2.7 The impeller of a centrifugal pump is surrounded by a casing which is made up of two parts—vortex chamber and volute. The outer diameter of the impeller is 20 inches and its width at outlet is 2 inches. The outer diameter of the vortex chamber is 28 inches. The

impeller discharges water into the vortex chamber with a velocity of 50 ft sec at an angle of 20° with the tangent of rotation. The width of volute is 4 inches when the diameter is 34 inches. Determine,

a) the magnitude and the direction of velocity of water when it leaves the vortex chamber and enters the volute,

b) the rise in pressure in the vortex chamber,

c) the magnitude and direction of the velocity of water at a point 17 inches from the centre,

and d) the rise in pressure from the outlet of the vortex chamber to a point in the volute 17 inches from the centre.

Solution

Let point 2 denote the position of impeller outlet as well as the vortex chamber inlet, point 3 denote the position of vortex chamber outlet as well as the volute inlet, and point 4 denote the position of volute outlet. (See Fig 2.14).

$D_2=20$ in.	$D_3=28$ in.	$D_4=34$ in.
$b_2=2$ in.	$b_3=2$ in.	$b_4=4$ in.
$v_2=50$ ft/sec		

a) The water is constrained to flow between two parallel plates in vortex chamber between points 2 and 3. It is a case of free vortex. Applying Eqn 2.8a for circulatory flow.

$$v_u \propto \frac{1}{r}$$

and applying Eqn 2.12a for radial flow,

$$v_m \propto \frac{1}{r}$$

$\therefore$ α = constant (See Fig 2.13), thus $v \propto \dfrac{1}{r}$

$$\therefore \quad v_2 \,.\, R_2 = v_3 \,.\, R_3$$

or $$v_3=\frac{v_2 \,.\, R_2}{R_3}=\frac{50\times 10}{14}=\mathbf{35{\cdot}7\ ft/sec}$$

and $$\alpha_3=\alpha_2=\mathbf{20^\circ}$$

Fig 2.14 Section through Centrifugal Pump Impeller with Vortex Chamber and Volute Casing

b) Rise in pressure in the vortex chamber between points (2) and (3)

$$=\frac{p_3-p_2}{w}=\frac{v_2^2-v_3^2}{2g}=\frac{50^2-(35{\cdot}7)^2}{64{\cdot}4}=\mathbf{19\ ft}$$

c) Direction of motion changes as the water flows through the volute between points (3) and (4), thus angle α does not remain constant.

Because it is free vortex, $v_u \propto \dfrac{1}{r}$ ...(See Eqn 2.8a)

$$\therefore \quad v_{u_4} = \frac{v_{u_3} \cdot R_3}{R_4} = \frac{v_3 \cdot \cos\alpha_3 \cdot R_3}{R_4}$$

$$= \frac{35{\cdot}7 \times \cos 20^\circ \times 14}{17} = 27{\cdot}6 \text{ ft/sec}$$

Since Q remains constant,

$2\pi \cdot R_3 \cdot b_3 \cdot v_{m_3} = 2\pi \cdot R_4 \cdot b_4 \cdot v_{m_4}$ ...(See Eqn 2.11)

$$\therefore \quad v_{m_4} = \frac{R_3 \cdot b_3 \cdot v_{m_3}}{R_4 \cdot b_4} = \frac{R_3 \cdot b_3 \cdot v_3 \cdot \sin\alpha_3}{R_4 \cdot b_4}$$

$$= \frac{14 \times 2 \times 35{\cdot}7 \times \sin 20^\circ}{17 \times 4} = 5{\cdot}02 \text{ ft/sec}$$

$$\therefore \quad v_4 = \sqrt{v_{u_4}^2 + v_{m_4}^2}$$ ...(See Fig 2.13)

$$= \sqrt{(27{\cdot}6)^2 + (5{\cdot}02)^2} = \mathbf{28 \text{ ft/sec}}$$

$$\tan\alpha_4 = \frac{v_{m_4}}{v_{u_4}} = \frac{5{\cdot}02}{27{\cdot}6} = 0{\cdot}182$$

$$\therefore \quad \alpha_4 = \tan^{-1}(0{\cdot}182) = \mathbf{10^\circ - 19'}$$

d) Rise in pressure from point (3) to point (4)

$$\frac{p_4 - p_3}{w} = \frac{v_3^2 - v_4^2}{2g} = \frac{(35{\cdot}7)^2 - 28^2}{64{\cdot}4} = \mathbf{7{\cdot}63 \text{ ft}}$$

Answers

a) Magnitude of velocity of water when it leaves the vortex chamber and enters the volute, *i.e.*, at point (3)

$= \mathbf{35{\cdot}7 \text{ ft/sec}}$

Direction of velocity of water at point (3)

$= \mathbf{20^\circ}$ with the tangent of rotation

b) Rise in pressure in vortex chamber between points (2) and (3)

$= \mathbf{19 \text{ ft}}$

c) Magnitude of velocity of water at point (4)

$= \mathbf{28 \text{ ft/sec}}$

Direction of velocity of water at point (4)

$= \mathbf{10^\circ - 19'}$ with the tangent of rotation

d) Rise in pressure between points (3) and (4)

$= \mathbf{7{\cdot}63 \text{ ft}}$

UNSOLVED PROBLEMS

2·1 Distinguish between Free Vortex and Forced Vortex. Cite Examples. (*AMIE—Nov. 1953*)

2·2 What is centrifugal head ? Derive an equation of the same.

2.3 Prove that in an open cylinder the depression of vortex below the original level of still water, is proportional to the square of angular velocity of rotation.

2.4 Explain with the help of a sketch the working principle of a centrifugal pump.

2.5 What is spiral vortex ? Give an example to illustrate such a flow.

2.6 Derive an 'Equation of Flow' in a curved path, and prove that for rectilinear flow the pressure remains constant transverse to the direction of motion.

2.7 Liquid in a deep vessel is stirred. Prove that it forms two types of curves, the forced vortex in the centre and the free vortex on the outside.

2.8 Prove that in free vortex, the tangential velocity as well as the radial velocity is inversely proportional to r, the distance from the centre.

2.9 Explain the nature of flow in a vortex chamber and volute of a centrifugal pump.

2.10 A glass tube 2 in. in diameter, open at the top, containing a liquid, rotates about its axis, which is vertical, at 700 rpm. What is the depression of the lowest point of the surface below the surface of liquid when at rest ? (3·48 in.) (*London University*)

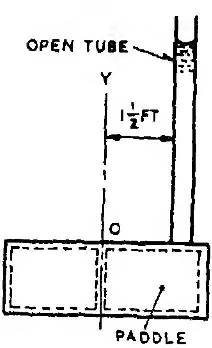


Fig 2.15

2.11 The closed cylindrical container shown in Fig 2.15 is filled with oil having a specific gravity of 0.85. By means of paddles, the oil is rotated so that each particle has an angular velocity of 100 rpm above OY. To what height above O will the oil rise in the open tube ? (3·82 ft)

2.12 An open cylindrical tank 4 ft in diameter and 6 ft deep is filled with water and rotated about its axis at 60 rpm. How much liquid is spilled and how deep in the water at the axis ?

At what speed should the tank be rotated in order that the centre of the bottom of the tank has zero depth of water ?

(15·3 ft^3 ; 3·55 ft ; 9·83 rad/sec or N=93·7 rpm)

2.13 A closed cylinder, 2 ft in diameter and 4 ft high is mounted on a vertical rotating shaft. The cylinder is filled with water upto three-fourth of its height. Show that the ratio of speed to y, where y is the distance from the lowest point of the paraboloid to the top of the cylinder, is constant between certain limits of speed. Find out these limiting speeds.

(108, 216 rpm) (*Gujrat University—1958*)

2.14 A closed cylindrical vessel with 0·4 metre diameter, is rotated about its axis which is vertical. The vessel is 2m long and is filled with

liquid upto a height of 1·5m. Find the maximum speed in rpm at which the vessel can be rotated so that the bottom of the vessel is wholly covered with a thin layer of the liquid. (422·8 rpm)

2.15 A closed cylinder 12 in. diameter and 0·1 in. deep is completely filled with water. It is rotated about its axis which is vertical, at 240 revolutions per minute. Calculate the total pressure of water on each end. (60·9 and 61·308 lb) (*Roorkee University—1955*)

2.16 A centrifugal pump has an inner radius of 10 in. and outer radius of 20 in. It is not fitted with vortex chamber or guide vanes. Determine the speed in rpm at which the lifting will commence against a head of 25 ft. (265·3 rpm) (*AMIE—May 1953*)

2.17 Calculate the least inner and outer diameters of the impeller of the centrifugal pump to just start delivering water to a height of 100 ft. The inside diameter of impeller is half that of the outside diameter and a manometric efficiency is 0·8. Pump runs at 1,000 rpm. (1 ft ; 2 ft)

2.18 The external and internal diameters of a centrifugal pump impeller are 12 in. and $6\frac{1}{2}$ in. respectively. It discharges 2 cusecs of water when running at 1,200 rpm. The net areas at inlet and at outlet are 0·39 sq ft and 0·25 sq ft respectively. The angle of discharge $\beta_2=18^\circ$ and angle of entrance $\alpha_1=90^\circ$. Determine the head generated in the water passing through the impeller.

If the discharge is reduced to 1·5 cusecs and speed is maintained, determine the head created by the impeller. (74·5 ft and 87 ft)

2.19 In a free cylindrical vortex of water the tangential velocity at a radius of 4 in. from the axis of rotation is found to be 24 ft/sec and the intensity of pressure is 40 lb per sq in. Find the intensity of pressure at a radius of 8 inches from the axis. (43 lb/sq in.) (*AMIE—May 1958*)

2.20 A pump impeller is 15 in. diameter, and it discharges water with component velocities of 7 and 42 ft/sec in the radial and tangential directions respectively. The impeller is surrounded by a concentric cylindrical chamber with parallel sides, the outer diameter being 18 in. If the flow in this chamber is in a free spiral vortex, find the component velocities of the water on leaving, and increase in pressure if there is no loss. (5·833 ft/sec, 35 ft/sec, 8·602 ft)

2.21 Water enters a centrifugal pump impeller with the resultant or absolute velocity in an axial direction. The impeller rotates at 1,500 rpm and has an outer diameter of 18 in. The relative velocity at exit is radial. The pump efficiency is 85% and the capacity is 3 cfs. Neglet losses in the pump shaft bearings. What is the pump head and the useful power input to the fluid? (215 ft, 86·2 HP)

2.22 A pump impeller discharges water at a speed of 25 m/sec, the angle of discharge being 10° with the tangent to the periphery. The diameter of the impeller is 0·6 m and width at periphery 10 cm. The impeller is surrounded by a chamber with outer diameter of 0·75 m and width at outer side equal to 12 cm. If the flow in the chamber is free spiral, find the radial and tangential velocities of water when it leaves the chamber, and also find pressure increase due to flow in the chamber. (2·89 m/sec, 16·4 m/sec, 17·7 m)

CHAPTER 3

DIMENSIONAL ANALYSIS, SIMILAR FLOWS & UNIT QUANTITIES

3.1 Introduction—In modern hydraulic engineering, such complex flow phenomena have to be dealt with that often a purely theoretical investigation fails to yield a practical and complex solution. It is then necessary to rely on experimental results. Principles of dimensional analysis and dynamic similarity have been extremely useful in the use and interpretation of experimental data.

Experiments cannot obviously be carried out on full-size dams, rivers, channels or hydraulic machines such as turbines and large pumps, because it is very costly. For the sake of economy and convenience it is essential that small-scale models be made for test purposes. Certain laws of similarity must be observed in order to ensure that the model test-data can be applied to the prototype. In order to apply these laws of similarity, it is the modern trend to express experimental results in terms of non-dimensional factors. Dimensional analysis and dynamic similarity are helpful in finding such non-dimensional factors.

In modern practice the experiments may be performed on the models with a fluid convenient to work with. The results of such experiments are applied to a fluid with which the prototype has to work. For example the blading of hydraulic and steam turbines are tested in a wind tunnel where the fluid used for experiments is air. The results are then converted to work the machines with water or steam as the case may be. This is possible by applying certain laws of similarity with the help of dimensional analysis.

3.2 Dimensional Analysis is a branch of mathematics which deals with the dimensions of quantities. Each physical phenomenon can be expressed by an equation giving relationship between different quantities. Such quantities are dimensional and non-dimensional (absolute numerals). In general any variable present in physical phenomenon will be a dimensional quantity. Dimensional analysis helps us in determining a convenient arrangement of variables in a physical relationship. This is accomplished by forming a number of non-dimensional

groups out of a given number of dimensional quantities, so that variables can be reduced and thus they can be conveniently represented on suitable co-ordinates. Thus dimensional analysis, on the whole, facilitates planning of scientific and reliable experiments.

3.3 Fundamental and Derived Quantities—All physical quantities can be expressed in terms of *fundamental quantities* which are only three in number *i e.*, mass, length and time; generally denoted by the symbols M, L and T. All other quantities such as area, volume, velocity, acceleration, force, energy etc. are termed as *derived quantities*, because they can be expressed in terms of the above fundamental quantities.

Some engineers prefer to use force instead of mass as fundamental quantity, because the former is easy to measure. The system will then be represented by symbols FLT instead of MLT. Some of the derived quantities are given in Table 3.1 below in terms of fundamental units of both systems. However for the solutions of further problems MLT—system has been used in this book.

Table 3.1 Derived Quantities

No.	Quantity	Dimensions: *FPS* System	Dimensions: Metric System	Symbol	*MLT* System	*FLT* System
1	Area	ft²	m^2	A	L^2	L^2
2	Volume	ft³	m^3	V	L^3	L^3
3	Velocity	ft/sec	m/sec	v	LT^{-1}	LT^{-1}
4	Angular velocity	rad/sec	rad/sec	ω	T^{-1}	T^{-1}
5	Acceleration*	ft/sec²	m/sec²	f, g	LT^{-2}	LT^{-2}
6	Mass	lb sec²/ft	kg sec²/m	M	M	FT^2L^{-1}
7	Force	lb	kg	F	MLT^{-2}	F
8	Weight	lb	kg	W or G	MLT^{-2}	F
9	Pressure	lb/ft² or lb/in.²	kg/m^2 or kg/cm^2	p	$ML^{-1}T^{-2}$	FL^{-2}
10	Impulse or momentum	lb-sec	kg-sec	I	MLT^{-1}	FT
11	Discharge	ft³/sec	m^3/sec	Q	L^3T^{-1}	L^3T^{-1}
12	Work or energy	ft lb	$kg\ m$	E	ML^2T^{-2}	FL
13	Torque	ft lb	$kg\ m$	T	ML^2T^{-2}	FL
14	Power	ft lb/sec	$kg\ m$/sec	P	ML^2T^{-3}	FLT^{-1}
15	Specific weight	lb/ft³	kg/m^3	w	$ML^{-2}T^{-2}$	FL^{-3}
16	Density	lb sec²/ft⁴	kg sec²/m^4	ρ	ML^{-3}	FT^2L^{-4}
17	Absolute viscosity	lb sec/ft²	kg sec/m^2	μ	$ML^{-1}T^{-1}$	FTL^{-2}
18	Kinematic viscosity	ft²/sec	m^2/sec	ν	L^2T^{-1}	L^2T^{-1}
19	Shearing stress	lb/ft²	kg/m^2	τ	$ML^{-1}T^{-2}$	FL^{-2}
20	Surface tension	lb/ft	kg/m	σ	MT^{-2}	FL^{-1}
21	Modulus of elasticity	lb/ft²	kg/m^2	E	$ML^{-1}T^{-2}$	FL^{-2}

*The acceleration due to gravity equals 32·1740 ft (or 980·665 cm) per sec per sec at sea level, latitude 45° N.

3.4 Similar Flows—In order that the results of model-tests can be successfully applied to the prototype hydraulic project or machine, it is essential that the two flows should be *mechanically* similar. Mechanical similarity implies two kinds of similarity *viz*, geometric and dynamical.

Geometric similarity, as the name indicates, implies that the geometry of the two systems should be identical *i.e*, every geometric dimension of the model should bear the same ratio to the corresponding dimension of the prototype. This is necessary but not a sufficient condition of similarity of flow.

The other condition is dynamical similarity. In order that the stream line patterns of the two systems may be identical it is essential that the direction of velocity at any point in one system must be the same as the direction of velocity at the corresponding point in the other system. Direction of velocity is solely dependent upon the ratio of forces acting on a fluid particle at that point Therefore dynamical similarity can be realised if the ratios of forces acting on fluid particles at corresponding points in the two flows are equal.

In the problems that shall be met with at the present stage, the following forces predominate :

i) Inertia Forces,

ii) Friction or Viscous Forces,

iii) Gravity Forces,

iv) Pressure Forces,

v) Elastic Forces,

vi) Surface Tension.

To obtain conditions of dynamical similarity, the ratio of the first force and any one of the remaining of the above forces can be considered. Every ratio will obviously be a non-dimensional factor. Some of the most important of such ratios, described as numbers, shall be discussed in the following articles.

3.5 Reynolds' Number—There is a class of flows in which only the inertia and viscous (friction) forces play an important role. Then the condition for mechanical similarity is, that the ratio of inertia forces to viscous forces should be constant.

Now,

$$\begin{aligned}\text{Inertia force} &= \text{mass} \times \text{acceleration}\\ &= \text{volume} \times \text{mass density} \times \frac{\text{velocity}}{\text{time}}\\ &= \frac{\text{volume}}{\text{time}} \times \text{mass density} \times \text{velocity}\\ &= (\text{cross-sectional area} \times \text{velocity}) \times\\ &\qquad\qquad \text{mass density} \times \text{velocity}\end{aligned}$$

Symbolically,

$$F_i = a \cdot v \cdot \rho \cdot v \;\ldots\ldots\ldots\ldots \left(\rho = \frac{w}{g}\right)$$

But $a \propto L^2$... ..(where L=linear dimension)

$$\therefore \qquad F_i = L^2 \,.\, v^2 \,.\, \rho$$

Similarly viscous or frictional force=shear stress due to action of viscosity $\times$ cross-sectional area of flow,

or $\qquad F_f = \mu \,.\, \dfrac{dv}{dy} \,.\, a$

Now $\qquad a \propto L^2$

and $\qquad \dfrac{dv}{dy} \propto \dfrac{v}{L}$

$\therefore \qquad F_f \propto \mu \,.\, L \,.\, v$

and Ratio $\qquad \dfrac{F_i}{F_f} = \dfrac{L^2 \,.\, v^2 \,.\, \rho}{\mu \,.\, L \,.\, v} = \dfrac{v \,.\, L}{\nu}$ ...(3.1)

where $\qquad \nu = \dfrac{\mu}{\rho}$ is the kinematic viscosity of fluid.

This non-dimensional ratio $\dfrac{v \,.\, L}{\nu}$ is named as *Reynolds' Number* (R_e) in honour of Osborne Reynolds (1842-1912), a British Scientist. For the types of flow considered here dynamical similarity is realised if R_e is constant.

$\therefore$ For similar flow in model and prototype,

$$R_{e_m} = R_{e_a} \qquad \text{...(3.2)}$$

where m indicates model and a the actual body.

or $\qquad \dfrac{v_m \,.\, L_m}{\nu_m} = \dfrac{v_a \,.\, L_a}{\nu_a}$ ...(3.2a)

Here L_a may be any linear dimension, since geometric similarity is assumed.

Reynolds' number will, thus, be a measure of relative magnitude of the inertia to the viscous forces occurring in the flow. The higher the Reynolds' number, the greater will be relative contribution of inertia effects. The smaller the Reynolds' number the greater will be the relative magnitude of viscous stresses.

Reynolds' number can be a sufficient criterion for similarity of flow only when other forces *viz.*, elastic, pressure, gravitational and surface forces are not of much consequence. Conditions, therefore, are in order :

i) fluid should be incompressible ;

and *ii*) there should be no free surface and either the flow should be in a closed conduit or around objects completely immersed in fluids.

Such conditions are approximately realised in the following cases :

i) Flow of incompressible fluid in closed pipe ;

ii) Flow of incompressible fluid in open channel so long as there is no surface activity such as waves or hydraulic jump ;

iii) Motion of submarine completely under water ;

iv) Motion of airship ;

and *v*) Flow around structures under moving fluids.

Turbines and pumps also fall in this category. But often R_e is so high that it ceases to be material.

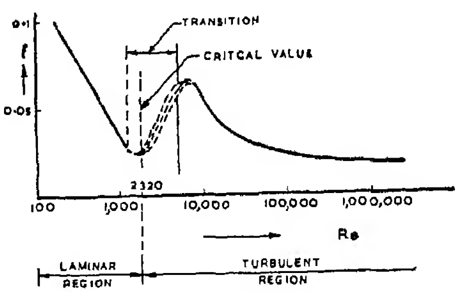


Fig 3.1 $f=f\,(R_e)$

This is shown in Fig 3.1 in which frictional factor f is plotted against R_e. Now the value of R_e for turbines and pumps is generally more than 1,000,000 where the curve becomes nearly a horizontal line showing that f is independent of R_e. However there is a certain value of f which is constant throughout. This is the *relative roughness* of surface over which the flow takes place. Thus for turbulent region—

$$f = a + \frac{b}{R_e^{\,c}} \qquad \ldots(3.3)$$

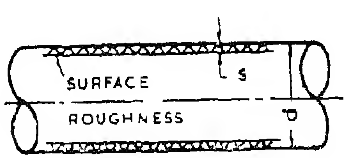


Fig 3.2 Roughness on the Inner Surface of Pipe

where a, b and c are constants and constant a depends upon relative roughness, denoted by δ

$$\therefore \qquad f = f\,(R_e, \delta) \qquad \ldots(3.4)$$

$$\text{where} \qquad \delta = \frac{2s}{d} \qquad \ldots(3.5)$$

s= thickness of roughness of surface of flow

d= diameter of pipe (See Fig 3.2)

$\therefore$ For model and prototype of turbines and pumps,

$$\delta_m = \delta_a \qquad \text{or} \qquad \frac{2\,.\,s_m}{d_m} = \frac{2\,.\,s_a}{d_a} \qquad \ldots(3.6)$$

Problem 3.1 Water flows at 70°*F* through a Venturimeter having a mean velocity of 5 ft per sec in its throat. The main and throat diameters of the meter are 40 in. and 20 in. respectively. Another Venturimeter having a dynamical similar flow was built having a throat diameter of 3 in., and the water has to flow at 50°*F*. Assume kinematic viscosity ν at 70°*F* and 50°*F* as $1{\cdot}08 \times 10^{-5}$ and $1{\cdot}41 \times 10^{-5}$ respectively. Find the velocity in the throat of the latter.

Solution

v_a = 5 ft/sec

d_a = 20 in. $\qquad\qquad d_m$ = 3 in.

$\nu_a = 1{\cdot}08 \times 10^{-5}$ $\nu_m = 1{\cdot}41 \times 10^{-5}$

For dynamically similar flow

$$(R_e)_a = (R_e)_m \qquad \text{.. (See Eqn 3.2)}$$

$$\text{or} \left(\frac{v \,.\, d}{\nu}\right)_a = \left(\frac{v \,.\, d}{\nu}\right)_m$$

$$\therefore \qquad v_m = \frac{v_a \,.\, d_a}{\nu_a} \,.\, \frac{\nu_m}{d_m}$$

$$= \frac{5 \times \frac{20}{12}}{1{\cdot}08 \times 10^{-5}} \times \frac{1{\cdot}41 \times 10^{-5}}{\frac{3}{12}}$$

$= 43\ 5$ **ft/sec** *Answer*

Problem 3 2 A model of an airship was tested in deep water. The length of the model was 10 ft and it had a speed of 25 ft per sec which was measured in water. Determine the speed of the actual sized ship in air when its length is 350 ft. Assume the kinematic viscosity of air as 13 times that of water. The flows are dynamically similar.

What would be the resistance of the actual ship in air, when its model gave a reading of 280 lb in water ?

Solution

$L_m = 10$ ft $L_a = 350$ ft

$v_m = 25$ ft/sec $\nu_a = 13\ \nu_m$

For dynamically similar flow Reynolds' Numbers are equal :

$$\therefore \qquad \frac{v_a \,.\, L_a}{\nu_a} = \frac{v_m \,.\, L_m}{\nu_m} \qquad \text{...(See Eqn 3.2}a\text{)}$$

$$\therefore \quad v_a = v_m \,.\, \frac{L_m}{L_a} \,.\, \frac{\nu_a}{\nu_m} = 25 \times \frac{10}{350} \times 13 = \mathbf{9{\cdot}28\ ft/sec} \qquad \textit{Answer}$$

b) For the resistance of ship—

the ratio of inertia forces to frictional forces must be same.

$$\textit{i.e.} \qquad \frac{F_{i_a}}{F_{f_a}} = \frac{F_{i_m}}{F_{f_m}} \qquad \text{...(See Eqn 3.1)}$$

$$\text{or} \qquad \frac{w_a \,.\, Q_a \,.\, \overline{v_a}}{F_{f_a} \,.\, g_a} = \frac{w_m \,.\, Q_m \,.\, \overline{v_m}}{F_{f_m} \,.\, g_m}$$

assume $g_a = g_m$ ...(being at the same place)

and $Q = a \,.\, \overline{v}$ where $\overline{v}$ = mean velocity

a = cross-sectional area

and $a = k \,.\, d^2 = k_1 \,.\, l^2$

$$\therefore \quad \frac{w_a \cdot k_1 \cdot l_a^2 \, (\overline{v_a})^2}{F_{f_a}} = \frac{w_m \cdot k_1 \cdot l_m^2 \, (\overline{v_m})^2}{F_{f_m}}$$

$$\therefore \quad \frac{F_{f_a}}{F_{f_m}} = \frac{w_a \cdot k_1 \cdot l_a^2 \cdot v_a^2}{w_m \cdot k_1 \cdot l_m^2 \cdot v_m^2} \quad \left\{ \begin{array}{l} k_1 \text{ cancels out,} \\ w_a = 0{\cdot}077 \text{ lb/ft}^3 \\ w_m = 62{\cdot}4 \text{ lb/ft}^3 \end{array} \right.$$

$$\therefore \quad F_{f_a} = \frac{0{\cdot}077 \times 350^2 \times 9{\cdot}28^2}{62{\cdot}4 \times 10^4 \times 25^2} \times 280$$

$$= \mathbf{58{\cdot}4 \text{ lb}} \qquad \textit{Answer}$$

3.6 Froude's Number—There is another class of flows in which gravitational forces are of prime importance. It has been proved that

Inertia Force, $F_i \propto L^2 \cdot v^2 \cdot \rho$

Gravitational Force, $F_g \propto$ mass $\times$ acceleration due to gravity
$\propto$ volume $\times$ mass density $\times g$.

or $F_g \propto L^3 \cdot \rho \cdot g$.

$\therefore$ Non-dimensional ratio :

$$\frac{F_i}{F_g} \propto \frac{L^2 \cdot v^2 \cdot \rho}{L^3 \cdot \rho \cdot g}$$

$$\propto \frac{v^2}{L \cdot g} \qquad \ldots(3.7)$$

The square root of this ratio, $\dfrac{v}{\sqrt{L \cdot g}}$ is known as *Froude's Number* (F_r) after William Froude (1810—79), a British Scientist who first applied it to the practical problems of the resistance of ship or floating body.

$$\text{Thus } \frac{v_m}{\sqrt{L_m \cdot g_m}} = \frac{v_a}{\sqrt{L_a \cdot g_a}} \qquad \ldots(3.8)$$

The equality of Froude's Numbers for two kinds of flow can ensure dynamical similarity only if gravitational forces are of prime importance. This is the case when the free surface of the fluid plays a major role as in :

i) Wave motion set up on the surface of water by a ship,

ii) Flow in an open channel (waves considered),

iii) Flow of jet from an orifice,

and *iv*) Flow over the spillway of a dam.

A little consideration will suggest that both conditions for dynamical similarity cannot be satisfied at the same time *i.e.* the R_e and F_r cannot be simultaneously the same for two flows, because the variation of force co-efficient with Reynolds' number represents the effect of skin friction, while the variation with Froude's number represents the contribution of wave-making resistance.

The towing of model of ship gives the total resistance which is the sum of the skin friction and the wave-making resistance. In order

to determine the wave-making resistance, the computed skin friction must be subtracted from the total resistance. For the same Froude's number, the wave-making resistance of full-size ship may be determined from this result. A computed skin friction for the ship is then added to this value to give the total ship resistance.

Problem 3.3 In an open channel water is flowing with a depth of 2 ft (or 0·61 m). It suddenly forms a jump at a certain point and the depth increases from 2 ft (or 0·61 m) to 3·33 ft (or 1·015 m). The velocity of water in 2 ft (or 0·61 m) depth is 10 ft per sec (or 3·048 m/sec). Another channel was built in which a similar jump was formed. The depth of water in the new channel in which the flow is dynamically similar is 8 ft (or 2·44 m). Calculate the velocity of water in it. Find also the height of the jump in the second case.

Solution

$$v_a = 10 \text{ ft/sec} \quad (\text{or } 3{\cdot}048 \text{ m/sec})$$

$$d_a = 2 \text{ ft} \quad (\text{or } 0{\cdot}61 \text{ m})$$

$$d_b = 8 \text{ ft} \quad (\text{or } 2{\cdot}44 \text{ m})$$

The frictional resistance plays a minor part in this case and the two forces acting here are due to inertia and gravity. Therefore, Froude's Number for the two channels must be same as the flow is dynamically similar.

$$\therefore \quad \frac{v_a}{\sqrt{g_a \cdot d_a}} = \frac{v_b}{\sqrt{g_b \cdot d_b}} \qquad \text{...(See Eqn 3.8)}$$

but $\quad g_a = g_b$ ....($\because$ Both channels are at the same place)

$$\therefore \quad \frac{v_a}{\sqrt{d_a}} = \frac{v_b}{\sqrt{d_b}}$$

$$\therefore \quad v_b = v_a \cdot \sqrt{\frac{d_b}{d_a}} = 10 \times \sqrt{\frac{8}{2}} \text{ ft/sec}$$

$$\left(\text{or } 3{\cdot}048 \times \sqrt{\frac{2{\cdot}44}{0{\cdot}61}} \text{ m/sec}\right)$$

$$= \mathbf{20 \text{ ft/sec} \quad (\text{or } 6{\cdot}1 \text{ m/sec})} \qquad \textit{Answer}$$

Height of the jump in the second case

$$= 3{\cdot}33 \times \frac{8}{2} \text{ ft} \quad \left(\text{or } 1{\cdot}015 \times \frac{2{\cdot}44}{0{\cdot}61} \text{ m}\right)$$

$$= \mathbf{13{\cdot}32 \text{ ft} \quad (\text{or } 4{\cdot}06 \text{ m})} \qquad \textit{Answer}$$

3.7 Euler's and Newton's Numbers—In some cases pressure forces are predominant and then the important non-dimensional ratio is

$$= \frac{\text{Inertia Force}}{\text{Pressure Force}}$$

or $\quad \dfrac{F_i}{F_p} \propto \dfrac{L^2 \cdot v^2 \cdot \rho}{p \cdot L^2}$

(where p is the stress or pressure per unit area and area $\propto L^2$)

$$\propto \frac{\rho \cdot v^2}{p} \qquad \text{...(3.9)}$$

This number is usually denoted by E_u and is known as *Euler's number.*

For similarity between model and prototype

$$(E_u)_m = (E_u)_a \qquad \ldots(3.10)$$

or

$$\frac{\rho_m \cdot v_m^2}{p_m} = \frac{\rho_a \cdot v_a^2}{p_a} \qquad \ldots(3.10a)$$

The reciprocal $\frac{1}{E_u}$ is sometimes called *Newton's number.*

The practical application of the Euler's number is water hammer in penstocks of hydro power plants.

3 8 Mach's Number – In dealing with compressible fluids, elastic forces become important. Then, for dynamical similarity the ratio inertia force/elastic force should be constant.

$$F_i \propto \rho \cdot L^2 \cdot v^2$$

$$F_e \propto E \cdot L^2 \qquad \text{(where } E=\text{volume modulus of elasticity)}$$

∴ Dimensionless ratio :

$$\frac{F_i}{F_e} \propto \frac{\rho \cdot L^2 \cdot v^2}{E \cdot L^2}$$

$$\propto \frac{\rho \cdot v^2}{E} \qquad \ldots(3.11)$$

But $\frac{E}{\rho} = v_s^2$, where v_s is the velocity of sound in that fluid. For a formal proof of this the student should consult some elementary book on sound.

$$\therefore \qquad \frac{F_i}{F_e} \propto \frac{v^2}{v_s^2} \qquad \ldots(3.12)$$

The accoustic velocity ratio $\frac{v}{v_s}$ equal to the square root of this number, is commonly called the *Mach's Number* (M_a) in honour of *E.* Mach, an Austrian Scientist. A critical value is $M_a = 1$. It becomes very significant when

$$v > v_s \quad i.e. \; M_a > 1$$

This number finds its application when an aeroplane is flying with a velocity more than the velocity of sound.

3.9 Weber's Number—When the forces due to surface tension are the most important, the dimensionless force ratio

$$\propto \frac{\text{Inertia Force}}{\text{Surface Tension}}$$

$$\propto \frac{\rho \cdot v^2 \cdot L^2}{\tau \cdot L} \qquad \text{(where } \tau=\text{surface tension per unit length)}$$

$$\propto \frac{\rho \cdot v^2 \cdot L}{\tau}$$

This ratio is called *Weber's Number*. For similarity between model and prototype

$$\frac{\rho_m \cdot v_m^2 \cdot L_m}{\tau_m} = \frac{\rho_a \cdot v_a^2 \cdot L_a}{\tau_a} \qquad \ldots(3.13)$$

Practical applications :

a) Capillary tube action in general,

b) A very thin sheet of liquid flowing over a surface,

c) Flow of liquid from an open-ended tube or nozzle leading to the formation of a spray of liquid drops.

3.10 Buckingham's Theorem*—The first and foremost point which should be borne in mind is that in any equation giving relationship between numbers of different physical quantities, each term in the equation must have the same dimensions. This is known as *dimensional homogeneity* which is expressed mathematically with the help of Buckingham's Theorem. The theorem states,

"If n quantities $(Q_1, Q_2, \ldots Q_n)$ which are independent of each other, completely determine a physical phenomenon and which involve m fundamental units, then the relation between one of the variables say Q_n in terms of a number of other independent variables $Q_1, Q_2 \ldots$ etc.,

i.e., $$Q_n = f(Q_1, Q_2 \ldots Q_{n-1}) \qquad \ldots(3.14)$$

[On the assumption that whatever the form of the function each term can be expressed binomically,

$$Q_n = Q_1^{p_1} \cdot Q_2^{p_2} \cdot Q_3^{p_3} \ldots Q_{n-1}^{p_{n-1}} \qquad \ldots(3.15)]$$

can be expressed in a similar form involving $(n-m)$ non-dimensional factors

$$\pi_1 = \phi(\pi_2, \pi_3, \ldots \pi_{n-m}) \qquad \ldots(3.16)$$

these factors involving quantities $Q_1, Q_2 \ldots$ etc."

Method—Each of the quantities in Eqn 3.15 can be expressed in terms of m fundamental units. As the fundamental units of each side of equation must balance, m equations can be obtained by equating the indices (exponents) of each of these units on the left side of equation *i.e.*, in Q_n to all the indices of such units on the right of the equation. Thus m quantities can be eliminated, reducing the relation (Eqn 3.14) to the form

$$\pi_1 = \phi(\pi_2, \pi_3 \ldots \pi_{n-m})$$

Illustration—To find non-dimensional factors for studying the thrust of a screw propeller completely immersed in a fluid, quantities those may be involved are,

F the thrust,

D the propeller diameter,

*For a formal proof, not given here, see Physical Review, Vol. 4, Oct. 1914, page 345.

and $\dfrac{F}{\rho \cdot v^2 \cdot m^2} = \phi\left(\dfrac{v \cdot m}{\nu}, \dfrac{t}{m}\right) = k$ (some constant) ...(3.18)

This constant in each case$=\dfrac{f}{2}=\dfrac{g}{C^2}$ where C is Chezy's constant

or Chezy's Constant $C=\sqrt{\dfrac{2g}{f}}$

where f=frictional factor.

iii) **Aeroplanes, Air Ships and Screw Propellers**—If it is assumed that inertia and viscous forces are predominant, it may be directly inferred that for dynamical similarity, their ratio *i.e.*, Reynolds' number should be constant.

It can also be derived *ab initio* from dimensional analysis.

The quantities involved are :

R the air-resistance,
v the velocity,
l some characteristic length,
μ the viscosity of air,
and ρ the mass density of air.

Now,

$$R=\phi\ (v \cdot l \cdot \rho \cdot \mu) \qquad ...(1)$$

or
$$R=v^a \cdot l^b \cdot \rho^c \cdot \mu^d \qquad ...(2)$$

Dimensionally,

$$MLT^{-2}=(LT^{-1})^a \cdot (L)^b \cdot (ML^{-3})^c \cdot (ML^{-1}\ T^{-1})^d \qquad ...(3)$$

Equating indices of M, L and T on either side,

$$1=c+d$$
$$1=a+b-3c-d$$
$$-2=-a-d$$

Whence, $a=2-d$, $b=2-d$, $c=1-d$

$\therefore$ $R \propto v^{2-d} \cdot l^{2-d} \cdot \rho^{1-d} \cdot \mu^d$ [By eliminating a, b and c from Eqn (2)]

or $R \propto (v^2 \cdot l^2 \cdot \rho)\left(\dfrac{\mu}{v \cdot l \cdot \rho}\right)^d$

$$\therefore \frac{R}{\rho \cdot v^2 \cdot l^2}=\phi\left(\frac{v \cdot l \cdot \rho}{\mu}\right)=\phi\ (R_e) \qquad ...(3.19)$$

Dynamical similarity is realised if R_e is same for two flows.

iv) **Ships**—A moving ship is only partly submerged and therefore she has to overcome frictional and wave-making resistance. A little consideration will suggest that so far as surface-wave-formation is concerned, gravitational forces are the most important.

$\therefore$ For dynamical similarity, Froude's Number should be same for two flows.

The same can also be derived from dimensional analysis. The quantities involved are:

R the resistance,
l the length of ship,
g the acceleration due to gravity,
ρ the mass density of fluid,
μ the viscosity of water,
and v the speed of ship.

Now $\quad R = \phi\,(v\,.\,l\,.\,\rho\,.\,\mu\,.\,g)$ ...(1)

or $\quad R \propto v^a\,.\,l^b\,.\,\rho^c\,.\,\mu^d\,.\,g^e$ ...(2)

Dimensionally,

$$MLT^{-2}=(LT^{-1})^a\,.\,(L)^b\,.\,(ML^{-3})^c\,.\,(ML^{-1}\,T^{-1})^d\,.\,(LT^{-2})^e \qquad \text{...(3)}$$

Equating the indices of M, L and T on two sides,

$$1=c+d$$
$$1=a+b-3c-d+e$$
$$-2=-a-d-2e$$

Solving,

From first, $\quad c=1-d$

From last, $\quad a=2-d-2e$

From second, $\quad b=1-a+3c+d-e$

$$=1-(2-d-2e)+3(1-d)+d-e$$

(Substituting for a and c)

$$=2-d+e$$

Eliminating a, b and c from eqn (2)

$$R \propto v^{2-d-2e}\,.\,l^{2-d+e}\,.\,\rho^{1-d}\,.\,\mu^d\,.\,g^e$$

$$\propto v^2\,.\,l^2\,.\,\rho\,.\left(\frac{\mu}{v\,.\,l\,.\,\rho}\right)^d.\left(\frac{g\,.\,l}{v^2}\right)^e$$

or $$\frac{R}{\rho\,.\,v^2\,.\,l^2}=\phi\left(\frac{v\,.\,l}{\nu},\ \frac{v^2}{l\,.\,g}\right)=\phi\,(R_e,\,F_r) \qquad \text{...(3.20)}$$

$\therefore$ For similar flow both R_e and F_r should be same. This is obviously impossible. Experience shows that for wave-making resistance, similar flow is possible if F_r for model and actual sized ship is same.

$\therefore$ Condition is—

$$\frac{v_m}{\sqrt{l_m\,.\,g_m}}=\frac{v_a}{\sqrt{l_a\,.\,g_a}} \qquad \text{.. (See Eqn 3.8)}$$

or $$\frac{v_m}{\sqrt{l_m}}=\frac{v_a}{\sqrt{l_a}} \qquad \text{...}(\because\ g_m=g_a)$$

If the moving body is totally submerged in water to a great depth, the resistance to motion will be mainly due to viscous forces. This is the case of submarine which behaves just like an aeroplane which has been discussed under the foregoing Art 3.11 (iii).

For the moving ship which is always partly submerged, the true dynamical similarity is obtained only if both the numbers R_e and F_r are same for the model and the actual-sized body, thus

a) $$\frac{v_m \cdot l_m}{\nu_m} = \frac{v_a \cdot l_a}{\nu_a} \text{ (for same } R_e\text{)}$$

Using the same fluid at the same temperature, for floating the model and the actual-sized ship,

$$\nu_m = \nu_a$$

$$\therefore \quad v_m \cdot l_m = v_a \cdot l_a$$

or $$v_m = v_a \frac{l_a}{l_m} \qquad \ldots(3.21)$$

b) $$\frac{v_m^2}{l_m \cdot g_m} = \frac{v_a^2}{l_a \cdot g_a} \text{ (for same } F_r\text{)}$$

Testing the model and actual-sized ship at the same place,

$$g_m = g_a$$

$$\therefore \quad \frac{v_m^2}{l_m} = \frac{v_a^2}{l_a}$$

or $$v_m = v_a \cdot \sqrt{\frac{l_m}{l_a}} \qquad \ldots(3.22)$$

From the above two Equations 3.21 and 3.22 it is seen that the corresponding speed v_m of model is different in each case. It is not possible, therefore, to test the model for the determination of total resistance. Froude suggested the following method :

The total resistance R is divided into two portions—

a) the frictional resistance or drag R_f

and b) the wave-making resistance R_w

$\therefore$ for actual-sized ship

$$R = R_f + R_w$$

Let r = total resistance of the model.

Dividing r into two portions r_f and r_w,

$$r = r_f + r_w$$

Froude suggested the following formula for computing the frictional drag of the ship or its model :

$$R_f = F \cdot A\, v^n \qquad \ldots(3.23)$$

where F and n = constants

A = surface area

v = speed of ship or model.

The values of the constants F and n are determined by towing long thin boards through water. The surface area of towing boards being large, the frictional drag is much more than the wave making resistance and thus the latter can be neglected. By substituting the values of the constants in Eqn 3.23, the frictional resistance R_f can

be calculated. Similarly the frictional resistance r_f of model can also be determined from Eqn 3.23.

Now tow the model through water at a speed v_m given by the Eqn 3.22 and find the *total* resistance r of the model.

The total resistance r and the frictional resistance r_f of the model being known, the wave resistance of model will be

$$r_w = r - r_f \qquad \ldots(3.24)$$

For true dynamically similar conditions—

$$\frac{R}{\rho \cdot v^2 \cdot l^2} = \phi\,(F_r) \qquad \ldots(\text{See Eqn } 3.20)$$

$$= \text{constant}$$

$$\therefore \qquad \frac{R_w}{\rho_a \cdot v_a^2 \cdot l_a^2} = \frac{r_w}{\rho_m \cdot v_m^2 \cdot l_m^2}$$

or
$$R_w = r_w \cdot \frac{\rho_a}{\rho_m} \cdot \left(\frac{v_a}{v_m}\right)^2 \cdot \left(\frac{l_a}{l_m}\right)^2$$

From Eqn 3.22,

$$\frac{v_a}{v_m} = \sqrt{\frac{l_a}{l_m}}$$

$$\therefore \qquad R_w = r_w \cdot \frac{\rho_a}{\rho_m} \cdot \left(\frac{l_a}{l_m}\right)^3 \qquad \ldots(3.25)$$

By substituting r_w from Eqn 3.24, R_w is calculated. The total resistance R of the ship is found by adding R_f from Eqn 3.23 and R_w from Eqn 3.25,

or
$$R = R_w + R_f \qquad \ldots(3.26)$$

v) **Turbines and Pumps**—

The various quantities involved are :

Q the rate of flow,

N the speed in rpm,

D the diameter of runner or impeller,

μ the viscosity of fluid,

ρ the mass density of fluid,

H the total head,

and E the energy per unit mass $= g \cdot H$

Now $\quad Q = \phi\,(N \cdot D \cdot \mu \cdot \rho \cdot H \cdot E) \qquad \ldots(1)$

or $\quad Q \propto N^a \cdot D^b \cdot \mu^c \cdot \rho^d \cdot H^e \cdot E^f \qquad \ldots(2)$

Balancing dimensionally,

$$L^3T^{-1} = (T^{-1})^a \cdot (L)^b \cdot (ML^{-1}\,T^{-1})^c \cdot (ML^{-3})^d \cdot (L)^e \cdot (L^2\,T^{-2})^f \qquad \ldots(3)$$

Equating the indices of M, L and T on either side,

$$0 = c + d$$
$$3 = b - c - 3d + e + 2f$$
$$-1 = -a - c - 2f$$

Whence $c=-d$

$$f=\frac{1}{2}-\frac{a}{2}+\frac{d}{2}$$

and $b=3+c+3d-e-2f$

$$=3-d+3d-e-(1-a+d) \quad \text{...(Substituting for } c \text{ and } f)$$

$$=2+a+d-e$$

Eliminating b, c and f from Eqn (2)

$$Q \propto N^a \,.\, D^{2+a+d-e} \,.\, \mu^{-d} \,.\, \rho^d \,.\, H^e \,.\, E^{\frac{1}{2}-\frac{a}{2}+\frac{d}{2}}$$

or $$Q \propto D^2 \,.\, E^{\frac{1}{2}} \,.\, \left(\frac{N \,.\, D}{E^{\frac{1}{2}}}\right)^a \,.\, \left(\frac{D \,.\, \rho \,.\, E^{\frac{1}{2}}}{\mu}\right)^d \,.\, \left(\frac{H}{D}\right)^e$$

or $$\frac{Q}{E^{\frac{1}{2}} \,.\, D^2}=\phi\left(\frac{N \,.\, D}{E^{\frac{1}{2}}},\ \frac{D \,.\, E^{\frac{1}{2}}}{\nu},\ \frac{H}{D}\right)$$

or $$\frac{Q}{\sqrt{gH} \,.\, D^2}=\phi\left(\frac{N \,.\, D}{\sqrt{g \,.\, H}},\ \frac{D \,.\, \sqrt{g \,.\, H}}{\nu},\ \frac{H}{D}\right) \quad \text{...(3.27)}$$

Three conditions cannot be satisfied simultaneously.

Experiments show that considerable deviations can be made from geometrical similarity *i.e.*, $\frac{H}{D}$ need not be constant within wide limits without influencing similarity of flow provided relative roughness is same.

Also for turbines and pumps, Reynolds' Number is so large that small variations become immaterial. The term $\frac{D \,.\, \sqrt{g \,.\, H}}{\nu}$ is therefore unimportant.

$\frac{Q}{H^{\frac{1}{2}} \,.\, D^2}$ is then constant if the number $\frac{N \,.\, D}{\sqrt{g \,.\, H}}$ is constant.

$\frac{Q}{H^{\frac{1}{2}} \,.\, D^2}$ is known as *Specific Flow* (See Eqn 3.35).

For dynamically similar turbines

$$\frac{N \,.\, D}{\sqrt{g \,.\, H}}=\text{constant}$$

or $$\frac{\frac{\pi \,.\, N \,.\, D}{60}}{\sqrt{2g \,.\, H}}=\text{constant}$$

or $$\frac{u}{\sqrt{2g \,.\, H}}=\text{constant} \quad \text{...(3.28)}$$

(Where u=peripheral velocity)

or K_u=constant ...(3.28*a*)

Where $K_u = \frac{u}{\sqrt{2gH}}$ is known as **Speed Ratio.**

It can also be shown that velocity ratios are also equal for similar pumps and turbines.

For model and prototype

$$\frac{N_m \, . \, D_m}{\sqrt{g \, . \, H_m}} = \frac{N_a \, . \, D_a}{\sqrt{g \, . \, H_a}} \qquad \ldots (3.29)$$

i.e. $$(K_u)_m = (K_u)_a \qquad (3.29a)$$

Problem 3.4 *a*) Assuming that the thrust T of a screw propeller is dependent upon diameter d, speed of advance v, fluid density ρ, revolutions per second n, and co-efficient of viscosity η; show, using the principle of dimensional homogeneity, that it can be represented by,

$$T = \rho \, . \, d^2 \, . \, v^2 \, . \, \phi \left\{ \frac{\eta}{\rho \, . \, d \, . \, v}, \; \frac{d \, . \, n}{v} \right\}$$

and hence explain the condition of dynamical similarity usually assumed for propellers.

b) The characteristics of a propeller of 12 ft diameter and rotational speed 100 rpm are examined by means of a geometrically similar model of 18 in. diameter. When the model is rotated at 360 rpm by a torque of 16 lb ft the thrust developed is 52 lb and the speed of advance is 4·8 knots. Determine the torque, thrust, speed of advance and efficiency of the full scale propeller. (1 knot = 6,080 ft/hr)

(Punjab University—1959A)

Solution

a) $$T = \phi \, (d, v, \rho, n, \eta)$$

By Buckingham's Theorem—

$$T \propto d^a \, . \, v^b \, . \, \rho^c \, . \, n^d \, . \, \eta^e$$

Expressing the different quantities in terms of fundamental units MLT—

Thrust	T = mass × acceleration	$= MLT^{-2}$
Diameter	d = Length	$= L$
Speed of advance	v = ft/sec	$= LT^{-1}$
Fluid density	ρ = lbsec²/ft⁴	$= ML^{-3}$
Revolutions per sec	n = rad/sec	$= T^{-1}$
Co-efficient of viscosity	η = lb ft^{-2} sec	$= ML^{-1}T^{-1}$

$$\therefore \quad MLT^{-2} \propto L^a \, . \, (LT^{-1})^b \, . \, (ML^{-3})^c \, . \, (T^{-1})^d \, . \, (ML^{-1}T^{-1})^e$$

Equating the indices of each fundamental quantity MLT on either side

$$1 = c + e$$
$$1 = a + b - 3c - e$$
$$-2 = -b - d - e$$

or $\qquad 2=b+d+e$

Whence $\qquad c=1-e$

$$b=2-d-e$$

$$\begin{aligned} a&=1-b+3c+e\\ &=1-(2-d-e)+3(1-e)+e\\ &=1-2+d+e+3-3e+e\\ &=2+d-e \end{aligned}$$

$$\therefore \quad T \propto (d)^{2+d-e} \cdot (v)^{2-d-e} \cdot (\rho)^{1-e} \cdot n^d \cdot \eta^e$$

or $\quad T \propto (d^2 \cdot v^2 \cdot \rho) \cdot (d \cdot v^{-1} \cdot n)^d \cdot (d^{-1} \cdot v^{-1} \cdot \rho^{-1} \cdot \eta)^e$

$$\propto (d^2 \cdot v^2 \cdot \rho) \left(\frac{d \cdot n}{v}\right)^d \cdot \left(\frac{\eta}{\rho \cdot d \cdot v}\right)^e$$

Hence $\qquad T = \rho \cdot d^2 \cdot v^2 \cdot \phi\left\{\frac{\eta}{\rho \cdot d \cdot v}, \frac{d \cdot n}{v}\right\}$

Now $\quad \frac{\eta}{\rho \cdot d \cdot v} = \frac{\nu}{d \cdot v} = \frac{1}{R_e}$

where $\qquad R_e$=Reynolds' number.

$\therefore$ Propeller should be governed by Reynolds' number and factor $\frac{d \cdot n}{v}$. However, it is impossible to satisfy both conditions at the same time, therefore it is usual to neglect the effect of the viscosity and make the factor $\frac{n \cdot d}{v}$ same for the model and the prototype.

b) $\quad d_m=18$ in.$=1{\cdot}5$ ft $\qquad d_a=12$ ft

$n_m=360$ rpm $\qquad n_a=100$ rpm

$(Torque)_m=16$ lb ft

$T_m=52$ lb

$v_m=4{\cdot}8$ knots

i) For the speed of advance the following condition of geometrical similarity will apply—

$$\frac{d_m \cdot n_m}{v_m} = \frac{d_a \cdot n_a}{v_a}$$

or $\qquad v_a=\frac{d_a}{d_m} \cdot \frac{n_a}{n_m} \cdot v_m$

$$=\frac{12}{1{\cdot}5} \times \frac{100}{360} \times 4{\cdot}8 = \mathbf{10{\cdot}67\ knots} \quad \textit{Answer}$$

or $\quad 10{\cdot}67 \times 6{,}080 = 65{,}000$ ft/hr

ii) For Thrust, $\frac{T}{\rho \cdot d^2 \cdot v^2}$ = constant

or $\qquad \frac{T_m}{\rho_m \cdot d_m^2 \cdot v_m^2} = \frac{T_a}{\rho_a \cdot d_a^2 \cdot v_a^2}$

$\rho_m = \rho_a$ —media being same,

$$\therefore \quad T_a = T_m \cdot \left(\frac{d_a}{d_m}\right)^2 \cdot \left(\frac{v_a}{v_m}\right)^2$$

$$= 52 \times \left(\frac{12}{1 \cdot 5}\right)^2 \times \left(\frac{10 \cdot 67}{4 \cdot 8}\right)^2 = 52 \times 64 \times 4 \cdot 94$$

$= \mathbf{16{,}440 \ lb}$ *Answer*

iii) Efficiency of model $= \dfrac{\text{Output}}{\text{Input}}$

$$= \frac{\text{Thrust} \times \text{speed of advance}}{\text{torque} \times \text{angular speed}}$$

$$= \frac{52 \times \dfrac{(4 \cdot 8 \times 6{,}080)}{60}}{16 \times 2\pi \times 360} \times 100 \ \%$$

$= \mathbf{70\%}$ *Answer*

iv) Toique developed by prototype

$$= \frac{\text{Thrust} \times \text{speed of advance}}{\text{angular speed} \times \text{efficiency}}$$

Taking efficiency of prototype and model to be the same, the former will however be slightly higher than the latter—

$$\text{Torque of prototype} = \frac{16{,}440 \times (10 \cdot 67 \times 6{,}080)}{(2\pi \times 100) \times 60 \times 0 \cdot 7}$$

$= \mathbf{40{,}500 \ lb \ ft}$ *Answer*

3.12 Unit and Specific Quantities—The rate of flow, speed, power etc., of hydraulic machines are all fuuctions of the working head which is one of the most fundamental of all quantities that go to determine the flow phenomena associated with machines such as turbines and pumps. To facilitate correlation, comparison and use of experimental data, these quantities are usually reduced to unit heads. Each is expressed as a function of head and its value corresponding to a unit value of head is determined. These reduced quantities are known as unit quantities *e.g.* unit flow,* unit speed, unit force, unit power and unit torque etc.

For similar reasons it is also convenient to use some specific quantities. A specific quantity is obtained by reducing any quantity to a value corresponding to unit head and some unit size. The latter dimension is the inlet diameter of runner in case of reaction turbines and least jet diameter in Pelton turbines.

Thus specific flow is the rate of flow corresponding to unit head and unit diameter. Similarly specific power is the power corresponding to a unit head and unit diameter.

The term '*specific*' is however used in a slightly different sense in connection with speed. The specific speed of a turbine is defined as the

*This should not be confused with numerically unit quantities. Thus one cusec is not unit flow in this sense of the term nor is one rpm a unit speed. Following pages will make it clear.

speed of a geometrically similar turbine working under a unit head and developing unit power. The specific speed of a pump is the speed of a geometrically similar pump working against a unit head and raising unit quantity of water.

The concept of specific speed is very important in the study of turbines and pumps. It is the modern basis of a scientific classification of turbines and pumps. (See Chapters 5 and 12 on Hydro-Electric Plants and Centrifugal Pumps). It does not follow that a given machine can actually operate with unit and specific quantities.

For instance it would be more precise to say that unit flow is the flow in a geometrically similar machine working with unit head, and the same applies to other unit and specific quantities.

The condition of similarity is essential to justify the assumption that the following co-efficients k_1, k_2, k_3 .....etc are all constants.

Expressions† will now be derived for unit and specific quantities.

3.13 Unit Quantities—

i) Unit Rate of Flow—

Rate of Flow = Cross-sectional area × velocity of flow

symbolically, $Q \propto v_{m_o}$

But $v_{m_o} = Kv_{m_o} \cdot \sqrt{2g \cdot H}$

Where H is the head and Kv_{m_o} some velocity co-efficient

$\therefore \quad Q \propto \sqrt{H}$

or $\quad Q = k_1 \cdot \sqrt{H}$

Now by definition unit rate of flow Q_1 is the value of Q when $H = 1$

$\therefore \quad Q_1 = k_1 \cdot \sqrt{1} = k_1$

i.e., Q_1 is numerically equal to k_1

$\therefore$ Numerically, unit rate of flow

$$Q_1 = \frac{Q}{\sqrt{H}} \qquad \ldots(3.30)$$

ii) Unit Speed—

Let N rpm be the speed of the turbine then linear or peripheral velocity of runner at inlet,

$$u_1 = \frac{\pi \cdot D_1 \cdot N}{60}$$

Also $\quad u_1 = K_{u_1} \cdot \sqrt{2g \cdot H}$

$\therefore \quad N \propto u_1 \propto \sqrt{H}$

†The following expressions have been derived for turbines but those for flow, speed and power hold good for pumps also.

or $N = k_2 \cdot \sqrt{H}$

Where k_2 is some co-effieient.

Now, by definition, unit speed

$$N_1 = k_2 \cdot \sqrt{1} = k_2$$

$\therefore$ Unit speed is numerically equal to k_2,

or numerically,

$$N_1 = k_2 = \frac{N}{\sqrt{H}} \qquad ...(3.31)$$

iii) **Unit Power—**

The available horse-power of a turbine

$$P_a = \frac{w \cdot Q \cdot H}{550} \quad \left[\text{or } P_a = \frac{w \cdot Q \cdot H}{75} \text{ metric HP} \right]$$

and brake horse-power

$$P_t = \eta_t \cdot \frac{w \cdot Q \cdot H}{550} \quad ...(\text{where } \eta_t = \text{turbine overall efficiency})$$

or in general, horse-power

$$P \propto Q \cdot H$$

But $Q \propto \sqrt{H}$

$\therefore$ $P \propto H \cdot \sqrt{H}$

or $P = k_3 \cdot H^{\frac{3}{2}}$

where k_3 is some co-efficient.

Now, by definition, unit power

$$P_1 = k_3 \cdot (1)^{\frac{3}{2}} = k_3$$

or numerically,

$$P_1 = k_3 = \frac{P}{H^{\frac{3}{2}}} \qquad ...(3.32)$$

iv) **Unit Force—**

The force exerted by jet on Pelton runner at its periphery is given by

$$F = \frac{w \cdot Q}{g} (v_{u_1} - v_{u_2})$$

i.e., $F \propto Q \cdot v_u$

But $Q \propto \sqrt{H}$

and $v_u \propto \sqrt{H}$

$\therefore$ $F \propto H$

or $F = k_4 \cdot H$

Now, by definition, unit force on periphery of runner,

$$F_1 = k_4 \cdot 1 = k_4$$

$\therefore$ Numerically,

$$F_1 = k_4 = \frac{F}{H} \qquad \ldots(3.33)$$

v) **Unit Torque—**

Torque or turning moment on runner=Force at Periphery × Radius.

Symbolically,

$$T = F \, . \, R \qquad \text{or} \qquad T \propto F$$

But $F \propto H$

$\therefore$ $T \propto H$

or $T = k_5 \, . \, H$

Now by definition, unit torque

$$T_1 = k_5 \, . \, 1 = k_5$$

$\therefore$ Numerically,

$$T_1 = k_5 = \frac{T}{H}. \qquad \ldots(3.34)$$

3.14 Specific Quantities—

i) **Specific Rate of Flow or Specific Flow—**

For a reaction turbine,

$$Q = (\pi \, . \, D_o \, . \, B_o) \, . \, v_{m_o}$$

The dimensions B_o and D_o generally have linear relations with D_1, the runner diameter at inlet, and, therefore, since

$$v_{m_o} \propto \sqrt{H}$$

$$Q \propto D_1^2 \, . \, \sqrt{H}$$

or $Q = k_6 \, . \, D_1^2 \, . \, \sqrt{H}$

Now, by definition, specific rate of flow

$$Q_{11} = k_6 \, . \, 1^2 \times \sqrt{1}$$

Numerically,

$$Q_{11} = k_6 = \frac{Q}{D_1^2 \, . \, \sqrt{H}} \qquad \ldots(3.35)$$

For a Pelton turbine,

$$Q = \frac{\pi}{4} \, . \, d_1^2 \, . \, v_1 \qquad \textit{i.e.,} \quad Q \propto d_1^2 \, . \, \sqrt{H}$$

where d_1=least diameter of water jet falling on turbine runner

$$\therefore \quad Q_{11} = \frac{Q}{d_1^2 \, . \, \sqrt{H}} \qquad \ldots(3.35a)$$

ii) **Specific Power—**

Power, $P \propto Q \, . \, H$

Since $Q \propto D_1^2 \cdot \sqrt{H}$ for a reaction turbine

$\therefore \quad P \propto D_1^2 \cdot H^{\frac{3}{2}}$

or $\quad P = k_7 \cdot D_1^2 \cdot H^{\frac{3}{2}}$

Now, by definition, specific power

$P_{11} = k_7 \cdot 1^2 \times 1^{\frac{3}{2}}$

Numerically,

$$P_{11} = k_7 = \frac{P}{D_1^2 \cdot H^{\frac{3}{2}}} \qquad \ldots(3.36)$$

Similarly for a Pelton Turbine,

$$P_{11} = \frac{P}{d_1^2 \cdot H^{\frac{3}{2}}} \qquad \ldots(3.36a)$$

iii) **Specific Force of Jet on Periphery of Runner—**

$F = \frac{w \cdot Q}{g}(v_{u_1} - v_{u_2})$

or $\quad F \propto Q \cdot v_u$

But $\quad Q \propto d_1^2 \cdot \sqrt{H}$ and $v_u \propto \sqrt{H}$

$\therefore \quad F \propto d_1^2 \cdot H \quad$ or $\quad F = k_8 \cdot d_1^2 \cdot H$

$\therefore$ By definition, specific force

$F_{11} = k_8 \cdot 1^2 \times 1$

or numerically,

$$F_{11} = k_8 = \frac{F}{d_1^2 \cdot H} \qquad \ldots(3.37)$$

iv) **Specific Torque—**

Torque=Peripheral Force × Radius of Runner

Symbolically, $\quad T \propto F$

or $\quad T \propto d_1^2 \cdot H$

or $\quad T = k_9 \cdot d_1^2 \cdot H$

By definition, specific torque,

$T_{11} = k_9 \cdot 1^2 \times 1$

Numerically, $\quad T_{11} = k_9 = \frac{T}{d_1^2 \cdot H} \qquad \ldots(3.38)$

Alternatively, $\quad T = \frac{P}{\omega}$

and since ω the angular velocity $\propto \sqrt{H}$

and $\quad P \propto D_1^2 \cdot H^{\frac{3}{2}}$

$\therefore \quad T \propto \frac{D_1^2 \cdot H^{\frac{3}{2}}}{H^{\frac{1}{2}}} \quad$ or $\quad T \propto D_1^2 \cdot H$

$\therefore$ Specific torque $T_{11} = \dfrac{T}{D_1^2 \cdot H}$

v) **Specific Speeds—**

a) **Turbine—** Unlike other specific quantities, specific speed is the speed of a geometrically similar turbine working under unit head and delivering unit brake horse-power.

It has been shown previously that

$$u_1 = \pi \cdot D_1 \cdot N$$

and $\quad u_1 \propto \sqrt{H}$

$\therefore \quad D_1 \propto \dfrac{\sqrt{H}}{N}$

Also it has been shown earlier that brake horse-power

$$P_t \propto Q \cdot H$$

where $\quad Q \propto D_1^2 \cdot \sqrt{H}$

$\therefore \quad P_1 \propto D_1^2 \cdot H^{\frac{3}{2}}$

Substituting for D_1,

$$P_t \propto \frac{H}{N^2} \cdot H^{\frac{3}{2}} \quad i.e., \quad P_t \propto \frac{H^{\frac{5}{2}}}{N^2}$$

or $\quad N \propto \sqrt{\dfrac{H^{\frac{5}{2}}}{P_t}} \quad$ or $\quad N = N_s \cdot \dfrac{H^{\frac{5}{4}}}{\sqrt{P_t}}$

where $\quad N_s = \dfrac{N \cdot \sqrt{P_t}}{H^{\frac{5}{4}}} \qquad \ldots(3.39)$

If $P_t = 1$ and $H = 1$, then numerically $N_s = N$

N_s therefore, is by definition, the specific speed of the turbine.

Problem 3.5 Each of the two Francis turbines installed at Talaiya Power House (*D V C*) works under a maximum head of 77 ft producing 2,800 HP at 250 rpm. Determine,

a) the HP and rpm of this turbine under a head of 1 ft,

b) the rpm of this turbine under 1 ft head and producing 1 HP,

and *c*) the HP of this turbine working under 1 ft head and running at 1 rpm.

Solution

$$H = 77 \text{ ft}, \qquad P_t = 2{,}800 \text{ HP}, \qquad N = 250 \text{ rpm}$$

a) (*i*) $\dfrac{P_t}{P_t'} = \left(\dfrac{H}{H'}\right)^{\frac{3}{2}} \qquad \ldots(H' = 1 \text{ ft})$

$\therefore \quad P_t' = P_t \cdot \left(\frac{H'}{H}\right)^{\frac{3}{2}} = 2{,}800 \times \left(\frac{1}{77}\right)^{\frac{3}{2}} = \mathbf{4{\cdot}14\ HP}$ *Answer*

ii) $\frac{N}{N'} = \left(\frac{H}{H'}\right)^{\frac{1}{2}}$ ...($H'=1$ ft)

or $N' = N \cdot \left(\frac{H'}{H}\right)^{\frac{1}{2}} = 250 \times \left(\frac{1}{77}\right)^{\frac{1}{2}} = \mathbf{28{\cdot}5\ rpm}$ *Answer*

b) $N_s = \frac{N \cdot \sqrt{P_t}}{H^{\frac{5}{4}}} = \frac{250 \times \sqrt{2{,}800}}{77^{\frac{5}{4}}} = \mathbf{58{\cdot}1\ rpm}$ *Answer*

c) N_s must be same—

$P_t = \left(\frac{N_s \cdot H^{\frac{5}{4}}}{N}\right)^2 = \left(\frac{58{\cdot}1 \times 1^{\frac{5}{4}}}{1}\right)^2 = \mathbf{3{,}380\ HP}$ *Answer*

b) **Pump—**

Specific speed of a pump is the speed of a geometrically similar pump delivering a unit quantity of water against a unit head.

For pumps also, $D_1 \propto \frac{\sqrt{H}}{N}$

and $Q \propto D_1^2 \cdot H^{\frac{1}{2}}$

Substituting for D_1,

$$Q \propto \frac{H}{N^2} \cdot H^{\frac{1}{2}}$$

or $N \propto \sqrt{\frac{H^{\frac{3}{2}}}{Q}}$

or $N \doteq N_s \cdot \frac{H^{\frac{3}{4}}}{\sqrt{Q}}$

$$\therefore \quad N_s = \frac{N \cdot \sqrt{Q}}{H^{\frac{3}{4}}} \qquad \text{...(3.40)}$$

When $Q=1$, and $H=1$, then numerically, $N_s = N$

$\therefore$ N_s is, by definition, the specific speed of a pump.

UNSOLVED PROBLEMS

3.1 Define dynamical similarity.

3.2 What is dimensional analysis ? State its uses.

3.3 What are fundamental and derived units ?

3.4 What are the two systems in which the derived units are expressed ?

3.5 What do you understand by "non-dimensional factors" and their use in model experiments? Derive any one, generally used in practice.

3.6 What is mechanical similarity? Define "Geometrical Similarity" and "Dynamical Similarity."

3.7 What are the laws of dynamical similarity and how would you apply them for the use in hydraulic machines?

3.8 What do you understand by geometrical, dynamical and kinematic similarities? Derive from first principles, ratios for model and prototype in terms of scale length for (*i*) velocity (*ii*) discharge and (*iii*) power. (*AMIE—May 1954*)

3.9 What is Reynolds' number and what is its significance in model studies?

3.10 Prove that the Reynolds' number of a model and that of the prototype are equal if the flow in the two cases is similar. Assume the forces acting are due to inertia and friction only.

3.11 State a few examples where Reynolds' number can be a sufficient criterian for similarity of flow.

3.12 Is Reynolds' number also used for considering similarity for turbines and pumps? If so, under what limitations?

3.13 What is relative roughness and state how it is applied for similar flows?

3.14 What is Froude's Model Law? (*Poona University—1958*)

3.15 Give some examples where Froude's Law determines the condition of similar flow.

3.16 Define the following "non-dimensional factors" and state where they are employed—Euler's number, Newton's number, Mach's number and Weber's number.

3.17 Describe Buckingham's Theorem. How is it applied for the determination of non-dimensional factors?

3.18 Define speed ratio in case of turbines and pumps.

3.19 What are "Unit Quantity" and "Specific Quantity"?

3.20 What is specific speed and how does it differ in definition from the other specific quantities?

3.21 Explain *a*) Unit speed, *b*) Unit discharge and *c*) Unit power of a hydraulic turbine. Derive expression for each of them. (*Jadavpur University—1954*)

3 22 Explain *a*) Specific flow, *b*) Specific power and *c*) Specific torque of a hydraulic turbine. Derive expression for each of them.

3.23 Deduce an expression for the specific speed of a hydraulic turbine and explain how it is useful in practice.

3.24 Deduce an expression for the specific speed of a pump. How does it differ from that of a turbine?

3.25 It is required to obtain dynamical similarity between 2 cusecs of water at 50°*F* flowing in a 6 in. pipe and linseed oil flowing at a velocity of 30 ft/sec at 90°*F*. What size of pipe is necessary for the linseed oil?

(ν for water at 50°F=0·0131 C.G.S. units,
ν for linseed oil at 90°F=0·051 C.G.S. units.)

(7·94 in. or 8 in.)

3.26 The flow about a 5 in. artillery projectile which travels at 2,000 ft/sec through still air at 90°F is to be modelled in high speed wind tunnel with a 1 : 5 model. If the wind tunnel air has a temperature of 0°F, what velocity is required ?

(ν for air at 0° and 90°F are 0·117 and 0·161 C.G.S. units respectively.) (7,270 ft/sec)

3.27 Find the speed of a model of an aeroplane which is fully submerged in water at 80°F and towed. The same model was tested at 68°F in air and gave the velocity at 150 ft per second. The ratio between model and prototype is 1 : 80.

Determine the drag of the prototype aeroplane in air if the resistance of the model in water is 1·2 lb.

Given :

ν for water at 80°F=0·00872 C.G.S. units.
ν for air at 68°E=0·15 C.G.S. units.
w for air at 68°F=0·077 lb/ft³.

(8·71 ft/sec ; 0·444 lb)

3.28 A full scale Torpedo moves fully submerged in water at 80°F with a speed of 20 ft per second. Its model is tested in a towing tank full of water at a speed of 80 ft per second.

Find the prototype model ratio.

Find the model speed if it is tested in a wind tunnel under a pressure of 20 atmospheres and under a constant temperature of 80°F. Take the absolute viscosity of air as $3{\cdot}85\times10^{-7}$ lb sec/ft² and density of air at 80°F as 0·0456 slugs/ft³ at 20 atmospheres. The kinematic viscosity for water at 80°F is $0{\cdot}938\times10^{-5}$ ft²/sec.

(4 ; 70·5 ft/sec)

3.29 A 20 ft model of a 500 ft ship was tested in a towing tank to ascertain the wave effect where the gravity forces are predominent. Calculate the speed at which the model must be towed to stimulate the wave effect of the ship when it is moving at a speed of 20 miles per hour. (5·86 ft/sec)

3.30 Water flowing with a depth of 2 ft in an open channel having a rectangular section, rises suddenly at a point in the flow to form a jump, the depth increases to 2·66 ft. If the velocity of flow before entering the jump be 10 ft/sec, what should be the corresponding velocity in another channel where the depth is 4 ft, if the jump of similar proportion is to occur. (14·14 ft/sec)

3.31 Define "Froude's number" and show that it may be expressed in terms of a ratio of stresses. The wave making resistance of the hull of a flying boat at a speed of 60 knots on sea water is to be estimated from experiment in fresh water tank on a model one-tenth full size. Determine the best speed for the test and calculate the amount of drag to be measured on the model if the estimated full scale wave making resistance is 1,200 lb. Take 1 knot=1·69 ft/sec. Sea water weighs 64 lb/cu ft.

(32·1 ft/sec, 1·17 lb) (*London University—July 1949*)

3.32 A boat is to be operated in sea water ($w=64$ lb/ft^3) at a relatively high speed, and its performance is estimated from a scale model $\frac{1}{12}$ full size operating in fresh water ($w=62{\cdot}4$ lb/ft^3). Find the ratio of the corresponding speed of model to the boat for geometrically similar free surface conditions. Also for this speed ratio, find the resistance of the boat due to wave formation in terms of that of model, and ratio of HP to overcome this compared with that of model. (0·289 ; 1,772 ; 6,130)

3.33 A model of a reservoir is drained in 6 minutes by opening the sluice gate. How long should it take to empty the prototype if the scale ratio is 1 : 256 ? (96 min) (*Poona University—1958*)

3.34 A model of spillway is built to a scale of 1 : 36. If the model velocity and discharge are 1·25 ft/sec and 2·5 cusecs respectively, what are the corresponding values for the prototype ?
(7·5 ft/sec ; 19,440 cusecs) (*Punjab University—1958A*)

3.35 A geometrical model of a surface vessel is tested in a laboratory. The linear scale of the model is $\frac{1}{49}$. It is observed that with a speed of 25 ft/sec the resistance of the model is 0·58 lb. The liquid used for the test is the same as that one on which the surface vessel is to sail. Calculate the corresponding speed and the resistance of motion of the surface vessel. Consider the effect of gravity only. (175 ft/sec ; 68,200 lb) (*Punjab University—1955A*)

3.36 Show by the application of dynamical similarity that the discharge of a fluid having density ρ, viscosity μ, from geometrically similar orifices is given by $Q=K\,.\,d^2\,.\,\sqrt{2gh}\,.\,\phi\left(\frac{Q\,.\,\rho}{d\,.\,\mu}\right)$ where K is a constant determined experimentally, h is the head on the orifice and d is its diameter while ϕ means "a function of."
(*Punjab University—1948A*)

3.37 Show that the drag or resistance per unit length of a long cylinder of diameter d moving with a velocity v in a direction perpendicular to its axis is given by—

$$s=\rho\,.\,v^2\,.\,d\,;\,\phi\left[\frac{\rho\,.\,v\,.\,d}{\mu}\right]$$

The resistance of a cylinder of 1 in. diameter when travelling in this way through air at 270 ft/sec is to be determined from an observation on a 0·25 in. diameter cylinder in a compressed air wind tunnel having a pressure of 18 atmospheres absolute. Find the required speed in the tunnel. If at this speed the measured resistance is 1·94 lb/ft of length of model, determine the resistance of the full-scale cylinder. Take the temperature and viscosity, the same in each case. (60 ft/sec, 8·73 lb/ft)

3.38 Show that for a Vee—notch having full notch angle $\theta=90°$, the flow for a fluid of kinematic viscosity ν will be

$$Q=H^{\frac{5}{2}}\,.\,g^{\frac{1}{2}}\,.\,\phi\left(\frac{H^{2}\,.\,g^{\frac{1}{2}}}{\nu}\right)$$

where H=head on the notch.

A Vee-notch is employed to measure the flow of a fluid which has kinematic viscosity 8 times that of water. If the measured

head is 10 in., calculate the head of water giving dynamical similarity.

From experiments on water, formula $Q=2{\cdot}48 \, . \, H^{2{\cdot}47}$ has been deduced where Q is in cfs. Calculate the flow of the fluid.

(2·5 in., 1·62 cfs.) (*Punjab University—Sept 1949*)

3.39 Prove by method of dimensions that in the rotation of similar discs in fluid, with turbulent motion, the frictional torque T of a disc of diameter D rotating at a speed N, in a fluid of viscosity μ and density ρ.

$$T=D^5 \, . \, N^2 \, . \, \rho \, . \, \phi \left(\frac{\mu}{D^2 \, . \, N \, . \, \rho} \right)$$

Hence determine the torque necessary to rotate a thin disc of 24 in. diameter at 3,000 rpm in air for which μ is $1{\cdot}8\times10^{-4}$ and $\rho=1{\cdot}2\times10^{-3}$ C.G.S. units, if the torque necessary to rotate a similar disc of 9 in. dia in water at corresponding speed is 0·079 ft lb. For water $\rho=1{\cdot}00$, and $\mu=0{\cdot}0101$ C.G.S. units.

(0·0565 ft lb) (*Punjab University—1949A*)

3.40 Fluid flows through similar pipes at a rate above critical velocity. Show that the drop in pressure per unit length is

$$\frac{p}{l}=\frac{\rho \, . \, v^2}{d} \, . \, \phi \, (R)$$

where $\frac{p}{l}$ = pressure drop per unit length,

d=diameter of pipe, ρ=density of fluid,
v=velocity of fluid, R=Reynolds' number.

(*Bombay University—1957*)

3.41 Show by method of dimensional analysis that the power P developed by a hydraulic turbine is given by—

$$P=\rho \, . \, N^3 \, . \, D^5 \, . \, \phi \left(\frac{N^2 \, . \, D^2}{g \, . \, H} \right)$$

where ρ=density of fluid, N=rotational speed,
D=rotor diameter, H=head,
ϕ means 'a function of.'

Hence deduce an expression for specific speed in terms of N, H and P. (*Punjab University—1953A*)

3.42 The quantity of water passing through a hydraulic turbine or a centrifugal pump is dependent upon the following quantities—

ω=Angular velocity of the runner, H=Head,

D=Diameter of runner, ρ=Density of fluid,

μ=Viscosity of fluid, g=Acceleration due to gravity.

Develop an expression for Q and show that Reynolds' and Froude's numbers based on peripheral velocity of the runner enter into the problem.

What other dimensionless ratios are involved ?

(*Punjab University—1958 Suppl*)

3.43 A 1 : 15 scale model of a propeller turbine 15 ft diameter is found to develop 3·21 HP with a flow of 10 cusecs under a head of 3·75 ft

while rotating at 500 rpm. Calculate corresponding HP, flow, head and speed of prototype, neglecting frictional effect.

(56·25 ft ; 129 rpm ; 8,700 cusecs ; 41,730 HP)

3.44 Derive an expression for the specific speed of a hydraulic turbine.

A turbine develops 5,000 horse-power under a head of 110 ft, at 100 rpm. What is its specific speed? What would be its normal speed and output under a head of 81 ft?

(19·85 ; 85·7 rpm ; 3,160 HP) (*Gujrat University—1958*)

3.45 Show that a rational formula for resistance of geometrically similar bodies moving partially submerged, in a liquid under conditions in which the formation of surface waves is the predominant feature is—

$$R=\rho \cdot L^2 v^2 \cdot \phi\ (F)$$

in which ρ denotes the density of the liquid, L is a linear dimension of the body, v is the speed of the body and F represents the Froude number.

Define the term *corresponding speed* used in connection with model experiments on ship resistance and show that for geometrically similar speed boats the resistance due to wave formation at corresponding speeds is proportional to the cube of the linear dimensions. [*A M I Mech E (Lond)—April 1957*]

3.46 Show, by applying the method of dimensions, that the resistance to the motion of a sphere having diameter D and moving with uniform velocity v through a fluid having density ρ and viscosity η may be expressed by

$$R=\frac{\eta^2}{\rho} \cdot \phi\left(\frac{\rho \cdot v \cdot D}{\eta}\right)$$

When the velocity is very small it is found that conditions are such that $\phi\left(\frac{\rho \cdot v \cdot D}{\eta}\right)=3\pi\left(\frac{\rho \cdot v \cdot D}{\eta}\right)$

Hence find the viscosity in poises (*i.e.* gm mass/cm sec) of a liquid through which a steel ball of diameter 0·10 cm falls, with uniform velocity, a distance of 20 cm in 10 sec. The specific gravity of the liquid is 0·90 and of the steel, 7·8.

[*AMI Mech E (Lond)—Oct 1958*]

3.47 *a*) Deduce an expression for the resistance experienced by a surface ship in terms of Reynolds and Froude numbers. Hence find the model scale necessary for dynamical similarity.

b) In a model test oil of kinematic viscosity $46{\cdot}5\times10^{-2}$ stokes (or cm²/sec) is used. What should be the model scale if the prototype liquid has a kinematic viscosity of 372×10^{-2} stokes?

(4) (*Punjab University—1960A ; converted to metric units*)

3.48 A submerged object is anchored in fresh water of kinematic viscosity $1{\cdot}13\times10^{-2}$ stokes (or cm²/sec) and density 992·5 kg/m³ which flows at the rate of 2·44 m/sec. The resistance of a 1 : 5 model in a wind tunnel as measured with air at 15°C at the atmospheric pressure and kinematic viscosity $14{\cdot}88\times10^{-2}$ stokes was 2·04 kg. What force acts on the prototype under dynamically similar conditions?

(9·63 kg) [*Punjab University—1960 S ; Converted to metric units*]

SECTION II

Hydraulic Prime Movers

CHAPTER 4

WATER WHEELS & DEVELOPMENT OF WATER TURBINES

4.1 History of the Development of Water Wheel and Water Turbine 4.2 Classification of Water Wheels 4.3 Overshot Water Wheel 4.4 Breast Water Wheel 4.5 Undershot Water Wheel or Impulse Wheel 4.6 Poncelet Water Wheel 4.7 Advantages and Disadvantages of Water Wheels 4.8 Jet Reaction Water Wheels 4.9 Outward Flow Reaction Turbine (Fourneyron Turbine) 4.10 Inward Flow Reaction Turbine (Francis Turbine) 4.11 Parallel Flow or Axial Flow Reaction Turbine (Jonval Turbine) 4.12 Axial Flow Impulse Turbine (Girard Turbine) 4.13 Further Development of Water Turbines.

4.1 History of the Development of Water Wheel and Water Turbine —The idea of using water as source of energy existed more than 2,200 years ago. The hydraulic energy was first produced in Asia (China and India) in the form of mechanical energy, by passing water through a water wheel The old type of water wheel made mainly from wood, still exists in India. Such type of prime-movers were taken from the Asian Continent to Egypt and then from there to European countries and America. It is estimated that the water wheel (undershot and overshot) was used in Europe 600 years after its origin in India. The actual design of water wheel was first made by Leonardo da Vinci (1452 to 1519 AD), the great Italian artist, which he did with hand sketches. The theory and the mathematical solutions of such a wheel were drawn by scientists Galileo Galilei and Descartes The practical experiments for the use of water wheels were carried out by Smeaton and Bossut in the year 1759. A French artillery Major Jean Victor Poncelet (1788 to 1867) first designed the water wheel taking in view the theory given by Borda in 1766. This kind of water wheel which the Major designed was mostly manufactured in England in 1828. The first book describing the theory and construction of water wheel came out in 1846. This book was written by Redtenbacher of Karlsruhe (Germany). A Swiss scientist Daniel Bernoulli (1700 to 1782) first wrote a theory for the conversion of water power into other forms of energy in his book "*Hydrodynamica*". Bernoulli's Theorem was given practical application by Segner in Goettingen (Germany) in 1750, who designed a water wheel. Then Leonard Euler (1707 to 1783) from Basle (Switzerland) wrote the theory of hydraulic machines in 1750, which is used even to this day showing the fundamentals of the subject. Bernoulli and Euler discovered most of the mathematical work concerning the problems. In 1824, a French scientist named Burdin designed a radial water wheel with a guide mechanism which could be used in practical field and was the first machine named as water turbine. Burdin could not make much fame of his work, and this turbine was further developed in designs by his student Fourneyron in 1827 which is the first water turbine. The Fourneyron turbines were built in 1843 in the USA. Further development of water turbine is given hereunder :

a) Henschel—Jonval Turbine [Axial Flow Turbine]	1837 (and 1850 in USA)

b) Girard (French Engineer) Turbine, and Howd and Swain (USA) Turbine [Inward Flow Turbine] 1850

c) James Bichens Francis [Inward Flow Turbine] 1865

The above development has been described for the reaction turbines which were mostly used for driving mills etc. The problem then arose to produce electrical energy with the help of water turbines by coupling the generator. This problem involved the factor concerning specific speed (*See* Art 5.14) of turbines, which was to be raised. The highest specific speed of Francis Turbine ($N_s = 117$) was obtained by Prof Dubs of Zurich (Switzerland) in 1914 and the Francis Turbine for specific speed between 63 and 117 was named as Dubs Turbine. The Francis Turbine is designed for a specific speed between 13·5 to 67·5. Prof Kaplan of Bruenn (Germany) worked out a new design in the year 1916 for higher specific speed, which is now known as Kaplan Turbine and works for specific speed between 67·5 to 225. The above reaction type of turbines are for low and medium heads of water. In a low head Kaplan turbine there are a number of bends at inlet, casing and draft tube, where the water has to turn, leading to head losses. In order to minimise these losses, Tubular Turbine has been developed. The new turbine uses Kaplan runner. It is becoming very popular for low head installations.

Development of Impulse Turbines—The undershot water wheel worked entirely with impulse of water and hence it was known as impulse wheel. Artillery Major Poncelet improved its design in the middle of 19th century. The later design was given by an American Engineer J. Pelton in 1880 with a tangential flow, which is still used as Pelton Turbine. This type of turbine is also called free jet turbine.

4.2 Classification of Water Wheels—Water wheels are classified in the following three groups according to driving action :

i) Wheels driven directly by the weight of water delivered to them,

ii) Wheels driven partly by weight and partly by impulse or dynamic action of moving water, and

iii) Wheels driven entirely by impulse.

Practically the water wheel consists of a central hub and a circular frame with a number of buckets or vanes mounted on the periphery. Water is delivered to the wheel at some point on its circumference, filling or striking one or more buckets at a time.

4.3 Overshot Water Wheel—Head race water is obstructed by a sluice gate and allowed to fall on the buckets through a small opening as shown in Fig 4.1. The sluice which regulates the flow of water according to need, is operated by hand or a governor connected to the water wheel with a belt. Water strikes the buckets near the crown of the wheel and drives it by its weight. As the buckets approach the heel, water is discharged into the tail race. Theoretically the diameter of the wheel should be equal to the height of fall. In practice, however, crown of the

wheel should be at a certain depth below the head race so that the water may strike the buckets with an initial velocity. Then the motive force is partly due to the impact of water on the buckets at the inlet. The heel of the wheel should also be at some height above the tailrace, in order to avoid splashing.

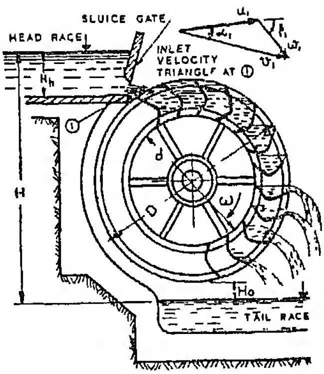


Fig 4.1 Overshot Water Wheel

Practical Data—

Diameter of wheel $D \approx H = 10$ to 70 ft (or 3 to 22 m)

Depth of shroud $\dfrac{D-d}{2} = 1{\cdot}5$ to 3 ft (or 0·5 to 1 m)

Speed $N = 4$ to 8 RPM

Number of buckets $z = 2\frac{1}{2}$ to 3 times D (in ft) [or 8 to 10 D (in m)]

Overall efficiency $\eta_t = 65$ to 85%

Peripheral velocity $u = \dfrac{\pi \cdot D \cdot N}{60} \approx 4$ to 8 ft/sec (or 1·2 to 2·5 m.s^{-1})

Generally $u \approx \frac{1}{2}\, v_1$

or $v_1 = u + (1{\cdot}5$ to $3)$ ft/sec [or $u + (0{\cdot}5$ to $1{\cdot}0)$ m.s^{-1})]

where v_1 is the velocity of incoming water.

Rate of Flow—

Quantity of water used per second

$$Q = (k_v \cdot \sqrt{2g \cdot H_h} + v_o)a \qquad \ldots(4{\cdot}1)$$

where k_v = velocity co-efficient,

H_h = head race height (*see* Fig 4.1),

v_o = velocity of approach,

and a = Cross-sectional area of sluice outlet,

$$\text{Water head efficiency } \eta_H = \frac{H_h + D}{H_h + D + H_o} \qquad \ldots(4.2)$$

where H_o is the overhung head above the tail race.

Horse-Power of the Wheel—

Let b be the breadth of wheel and c the depth of shroud.

Then capacity of each bucket $= b \cdot c \cdot \dfrac{\pi \cdot D}{z}$ nearly.

Number of buckets passing the stream per second $= \dfrac{z \cdot \omega}{2\pi}$

where ω = angular velocity of wheel in radians per second.

If a fraction λ of each bucket is filled with water, then

$$Q = \lambda \cdot \frac{b \cdot c \cdot \pi \cdot D}{z} \cdot \frac{z \cdot \omega}{2\pi} \qquad (\lambda = 0{\cdot}2 \text{ to } 0{\cdot}3)$$

$$= \frac{\lambda \cdot b \cdot c \cdot D \cdot \omega}{2}$$

or $$\lambda = \frac{2Q}{b \cdot c \cdot D \cdot \omega} \qquad \ldots(4.3)$$

Horse Power $$= \eta \cdot \frac{w \cdot Q \cdot H}{550}$$

$$\left[\text{or in metric units HP} = \eta \cdot \frac{w \cdot Q \cdot H}{75}\right]$$

$$= \eta \cdot \frac{w \cdot \lambda \cdot b \cdot c \cdot D \cdot \omega \cdot H}{2 \times 550}$$

Substituting, $$\omega = \frac{2\pi \cdot N}{60}$$

Horse Power $$= \eta \cdot \frac{\pi \cdot w \cdot \lambda \cdot b \cdot c \cdot D \cdot N \cdot H}{33{,}000} \qquad \ldots(4.4)$$

Problem 4.1 An overshot water wheel was installed to develop power. The canal approaching the wheel is 5 ft wide and the water which strikes the blades with a velocity of 10 ft per sec is ½ ft deep. The water fall is 30 ft. Find the effective horse power of the water wheel, if its efficiency is 75 percent.

Solution

Cross-sectional area of water head race$=5 \times 0{\cdot}5 = 2{\cdot}5$ sq ft

Velocity of head race water$=10$ ft/sec

$\therefore \quad Q = a \cdot v = 2{\cdot}5 \times 10 = 25$ cu ft/sec

$H = 30$ ft $\qquad \eta_t = 0\ 75$

$$\therefore \quad P_t = \frac{w \cdot Q \cdot H}{550} \cdot \eta_t = \frac{62{\cdot}4 \times 25 \times 30 \times 0{\cdot}75}{550}$$

$= \mathbf{63{\cdot}7\ HP}$ *Answer*

4.4 Breast Water Wheel—Breast wheel is a simple water wheel provided with an apron or breast from the head race to the tail race (See Fig 4.2). Purpose of the breast is to keep the buckets full of water until they reach the tail race. Water flows into the wheel through special openings in the circular plate covering a part of the wheel's periphery. The regulator plate slides along the cover with a rack and pinion arrangement. Thus the required number of holes can be thrown open, to pass the water to the bucket.

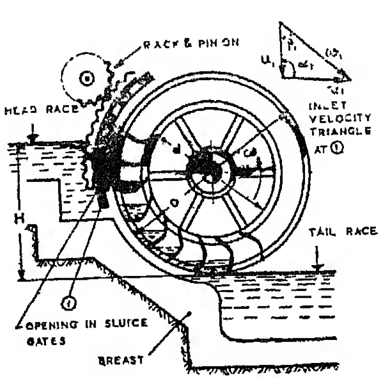


Fig 4.2 Breast Water Wheel

Practical data—

Head	$H = 3$ to 15 ft (or 1 to 5 m)
Diameter	$D = 12$ to 25 ft (or 4 to 8 m)

Depth of shroud $\frac{D-d}{2}=1$ to 1·75 ft (or 0·3 to 0·6 m)

Speed $N=3$ to 7 RPM

Peripheral velocity $u=\frac{\pi \cdot D \cdot N}{60}=4{\cdot}5$ to 8 ft/sec (1 5 to 2·5m.s^{-1})

Efficiency $\eta_t=50$ to 65%
Rate of flow $Q \leq 70$ cusecs (or $\leq 2m^3\,s^{-1}$)
Breadth of wheel $b \leq 10$ ft (or $\leq 3m$)

Power of a breast wheel—

Rate of flow = Area across flow × velocity of flow

$$Q=\lambda \cdot b \cdot c \cdot u$$

where λ = fraction of each bucket full of water
b = breadth of wheel
c = depth of shroud
and u = mean velocity of buckets.

Now if H be the net head *i.e.* gross head minus overhung head,

$$\text{Power} \quad =\eta \cdot \frac{w \cdot Q \cdot H}{500}\ \text{HP} \left(\text{or } \eta\ \frac{w \cdot Q \cdot H}{75}\ \text{metric HP}\right) \quad (4.5)$$

$$=\eta \cdot \frac{w \cdot \lambda \cdot b \cdot c \cdot u \cdot H}{550}\ \text{HP} \quad \ldots(4.5a)$$

Problem 4.2 A breast water wheel 20 ft in diameter and 6 ft wide working on a fall of 14 ft and having a depth of shroud $1\frac{1}{4}$ ft, has its buckets $\frac{5}{8}$ full. The mean velocity of the buckets is 5 ft per sec. Find the horse power of the wheel, assuming the efficiency as 70 percent.

Solution

$D=20$ ft $\quad b=6$ ft

$\frac{D-d}{2}=1{\cdot}25$ ft $\quad v_1=5$ ft/sec

Bucket $\frac{5}{8}$ full with water

$\therefore$ Discharge $Q=(1{\cdot}25\times6\times\frac{5}{8})\times5=23{\cdot}4$ cu ft/sec

$H=14$ ft $\quad \eta_t=0{\cdot}7$

$$\therefore \quad P_t=\frac{w \cdot Q \cdot H}{550} \cdot \eta_t=\frac{62{\cdot}4\times23{\cdot}4\times14}{550}\times0{\cdot}7$$

$=$**26·1 HP** *Answer*

4 5 Undershot Water Wheel or Impulse Wheel—The undershot water wheel revolves entirely by the impulse of water, *i.e.* the wheel utilises the kinetic energy of the stream. The whole of the available head H (See Fig 4.3) is converted into velocity before the water impinges on the buckets of the wheel.

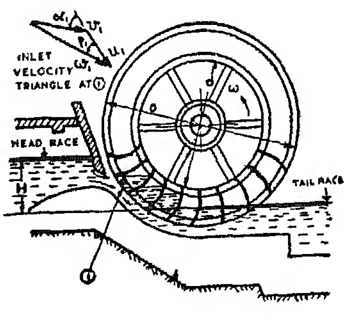


Fig 4.3 Undershot Water Wheel

The old type used to have straight radial blades (See Fig 1.5).

The improved type is shown in Fig 4.3.

Tangential force on the wheel (Eqn 1.24),

$$F_u=\frac{w \cdot Q}{g}(v_{u_1}-v_{u_2})$$

But $\quad v_{u_1}\approx v_1$ and $v_{u_2}\approx u$

$$\therefore \quad F_u=\frac{w \cdot Q}{g}(v_1-u)$$

$\therefore$ Theoretical Power $\quad P_H=F_u \cdot u = \dfrac{w \cdot Q}{g}(v_1 \cdot u-u^2) \quad \ldots(4.6)$

For maximum power, $\quad \dfrac{dP_H}{du}=0$

or $\quad \dfrac{w \cdot Q}{g}(v_1-2u)=0$, whence $u=\dfrac{v_1}{2}$

$$\therefore \text{ Max Power}=\frac{w \cdot Q}{g}\left(\frac{v_1^2}{2}-\frac{v_1^2}{4}\right)=\frac{w \cdot Q}{g}\cdot\frac{v_1^2}{4} \quad \ldots(4.7)$$

Available Power$=w \cdot Q \cdot \dfrac{v_1^2}{2g}$

$$\therefore \text{ Theoretical maximum efficiency}=\frac{\dfrac{w \cdot Q}{g}\cdot\dfrac{v_1^2}{4}}{\dfrac{w \cdot Q}{g}\cdot\dfrac{v_1^2}{2}}=\frac{1}{2} \text{ i.e. } 50\% \quad \ldots(4.8)$$

In practice, efficiency is only 20 to 30 per cent.

Practical data—

Head	$H\leq 5$ ft (or $\leq 1{\cdot}75$ *m*)
Diameter	$D=2$ to 4 times H
Speed	$N=2$ to 4 RPM
Efficiency	$\eta_t=35\%$ to 45% max.

4.6 Poncelet Water Wheel— It is an improvement on the straight blade type undershot water wheel. Water strikes the vanes of the wheel, practically without shock and drives it by impulse (See Fig 4.4). Rate of flow is regulated by sluice shaped to suit the curvature of vanes. Water is discharged from the blades almost vertically downwards.

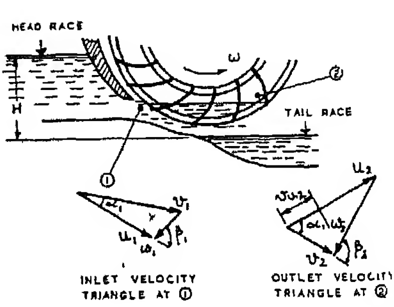


Fig 4.4 Poncelet Water Wheel

Practical data—

Head $\quad H=7$ ft max (or 2 *m*)

Diameter $D=2$ to 4 times H, to enable the stream to have some velocity.

Peripheral velocity $\quad u \approx \frac{v_1}{2}$

Angle $\quad \alpha_1 \approx 15^\circ$

Hydraulic efficiency $\quad \eta_h \approx 93\%$ $\quad$ but $\eta_t = 55\%$ to 65%

Inlet velocity $\quad v_1 = K_{v_1} \cdot \sqrt{2gH}$

Where velocity co-efficient K_{v_1} varies from 0·9 to 0·95

Velocity of water relative to blade at inlet is w_1 and neglecting shock and frictional losses, a water particle would rise along the blade, a vertical height,

$$h = \frac{w_1^2}{2g}$$

Theoretical hydraulic efficiency

$$\eta_H = \frac{\frac{v_1^2}{2g} - \frac{v_2^2}{2g}}{\frac{v_1^2}{2g}} = 1 - \frac{v_2^2}{v_1^2} \qquad \ldots(4.9)$$

Now η_H is maximum for a given value of v_1 when v_2 is minimum. This is the case when v_2 is perpendicular to u and then

$$u = \tfrac{1}{2} \cdot v_1 \cdot \cos\theta$$

and $\quad v_2 = v_1 \sin\theta$

$$\therefore \quad \eta_{H_{max}} = 1 - \frac{v_1^2 \cdot \sin^2\theta}{v_1^2} = \cos^2\theta \qquad \ldots(4.10)$$

$$\therefore \quad \eta_{H_{max}} = 1, \text{ when } \theta = 0$$

But this is a hypothetical case not realizable in practice without altering the construction of the wheel. The modified wheel then becomes a modern Pelton Turbine (*See* Chapter 6).

Problem 4.3 In a Poncelet Water Wheel, the direction of the jet impinging on the floats makes an angle of 15° with the tangent to the circumference and the tip of the floats makes an angle of 30° with the same tangent. Supposing the velocity of the jet to be 20 ft/sec, find

a) the proper velocity of the edge of wheel,

b) the height to which the water will rise on the float above the point of admission, and

c) the velocity and direction of motion of water leaving the float.

Solution

$\alpha_1 = 15^\circ \qquad \beta_1 = 30^\circ$

$v_1 = 20$ ft/sec

$v_{u_1} = v_1 \cdot \cos\alpha_1 = 20 \times 0{\cdot}9656 = 19{\cdot}3$ ft/sec

$v_{m_1} = v_1 \cdot \sin\alpha_1 = 20 \times 0{\cdot}2588 = 5{\cdot}176$ ft/sec

and $\quad v_{m_1} = w_1 \cdot \sin\beta_1$

$\therefore \quad 5{\cdot}176 = w_1 \cdot \sin 30^\circ$

or $\quad w_1 = \dfrac{5{\cdot}176}{0{\cdot}5} = 10{\cdot}35$ ft/sec

a) Velocity of the edge of wheel $= u_1 = u_2 = u$

The inlet velocity triangle shown in Fig 4.4 is an isosceles triangle, therefore $u_1 = w_1$

$= \mathbf{10{\cdot}35}$ **ft/sec** *Answer*

b) $w_1 = \sqrt{2gH}$ or $H = \dfrac{w_1^2}{2g} = \dfrac{10{\cdot}35^2}{64{\cdot}4} = \mathbf{1{\cdot}7}$ **ft/sec** *Answer*

c) From outlet velocity triangle (*See* Fig 4.4)

$w_1 = w_2 = 10{\cdot}35$ ft/sec. $\quad \beta_2 = 30^\circ$

$\therefore \quad w_2 \cos 30^\circ = 10{\cdot}35 \times 0{\cdot}866 = 8{\cdot}95$ ft/sec

$v_{u_2} = u_2 - w_2 \cdot \cos 30^\circ = 10{\cdot}35 - 8{\cdot}95 = 1{\cdot}4$ ft/sec

$v_{m_2} = w_2 \sin\beta_2 = 10{\cdot}35 \times 0{\cdot}5 = 5{\cdot}17$ ft/sec

$\therefore \quad v_2$, the velocity of water leaving the float

$= \sqrt{v_{u_2}^2 + v_{m_2}^2} = \sqrt{1{\cdot}4^2 + 5{\cdot}17^2}$

$= \mathbf{5{\cdot}36}$ **ft/sec** *Answer*

and α_2, the direction of motion of water at exit, is

$$\tan\alpha_2 = \frac{v_{m_2}}{v_{u_2}} = \frac{5{\cdot}17}{1{\cdot}4} = 3{\cdot}69$$

$\therefore \quad \alpha_1 = \tan^{-1} 3{\cdot}69 = \mathbf{74^\circ\ 50'}$ *Answer*

4.7 Advantages and Disadvantages of Water Wheels—

Advantages *i*) Simple and strong consruction,

and *ii*) High and consistent efficiency even if Q is not constant (specially for Overshot Wheel).

Disadvantages *i*) Power to weight ratio is very small *i.e.* the machinery is too heavy for the small power it produces,

ii) Poor governing ability,

and *iii*) Speed too slow.

The slow moving water wheels are totally incapable of driving any modern machinery and particularly the generators. It is for this reason that the water wheels have been superseded by water turbines which operate under any head (highest head so far 5,800 ft) and practically any desired speed enabling the generator to be coupled directly.

4.8 Jet Reaction Water Wheels—All the water wheels described so far operate in the open air, that is under atmospheric pressure. As described under Art 4 2, they are driven either by weight or by impulse of moving water. The latter type is known as impulse wheel in which case the whole of the head of moving water is converted to velocity head. The water, with high velocity, then, impinges on the buckets of the wheels in the open air.

The reaction wheels on the other hand, operate under pressure of water which is above atmosphere, and the wheels are either full of water or submerged under water. The principle of jet propulsion (Art 1.12 and Fig 1.25) or reaction of jet is applied to impart motion to such wheels. The simple application to this principle is *Barker's Mill* (Fig 4.5) which is an adaptation of hydraulic use of *Hero's Engine*. The Barker's Mill consists of a vertical pipe mounted in bearings To the lower end of this pipe are connected two or more radial pipes. The ends of the radial pipes are closed and holes are drilled in them as shown in Fig 4.5*b*. In some cases the radial pipes and bent at right angles at the ends (See Fig 4.5*a*). The central pipe is fed with water under pressure and the re - action of water escaping from the ports at the ends of the arms causes them to rotate. Barker's Mill is mostly used as rotating lawn sprinkler.

The Barker's Mill is also known as *Segner Water Wheel* in Germany. The modified form of this wheel which has the arms bent spirally, is sometimes known as *Whitelaws' Turbine* or *Scotch Turbine*.

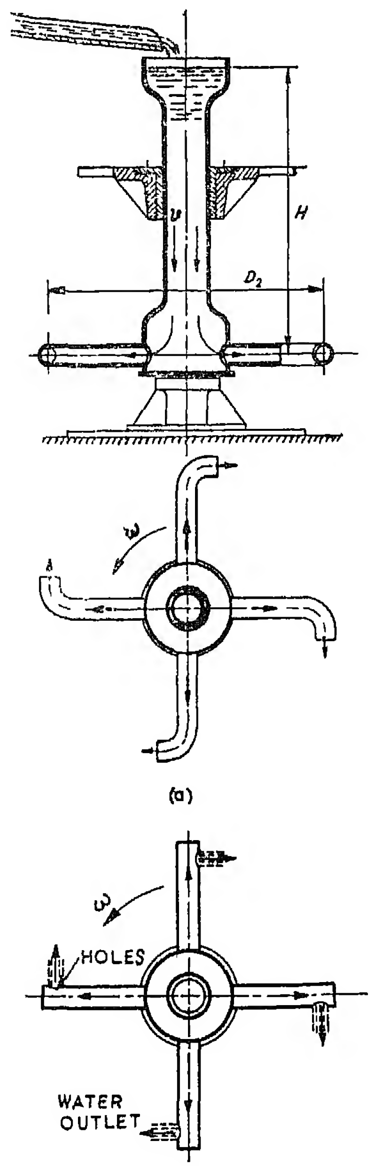


Fig 4.5 Barker's Mill

Peripheral force $P_u = \frac{w \cdot Q}{g} (v_{u_1} - v_{u_2})$ .. (See Eqn 1.24)

Draw the inlet and outlet velocity triangles (See Fig 4 6)

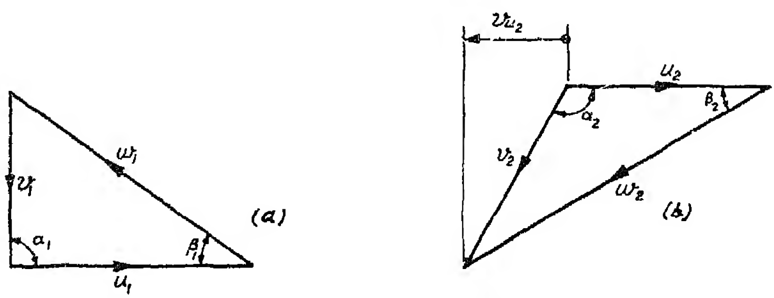


Fig 4 6 Inlet and Outlet Velocity Triangles of Barker's Mill

$$\alpha_1 = \frac{\pi}{2},$$

$$v_{u_1} = \cos \alpha_1 = 0,$$

and $\alpha_2 > \frac{\pi}{2}$,

$\therefore$ $\cos \alpha_2 < 0$ (*i e.* negative)

Substituting the values of α_1 and α_2,

$$P_u = \frac{w \cdot Q}{g} \cdot v_{u_2}$$

But $v_{u_2} = w_2 \cos \beta_2 - u_2$

where $u_2 = \frac{\pi \cdot D_2 \cdot N}{60}$ (For D_2, See Fig 4.5)

Taking the extreme case,

$\beta_2 = 0$, $\alpha_2 = \pi$ or $\cos \alpha_2 = -1$,

then $F_u = \frac{w \cdot Q}{g} (w_2 - u_2)$ .. (4.11)

Pressure exerted by water in the neighbourhood of the exit= Pressure due to the static head H (See Fig 4.5) + Pressure due to centrifugal force of revolving water in the arms.

or $\frac{w_2^2}{2g} = H + \frac{u_2^2}{2g}$ ...(4.12)

Theoretical power developed by the wheel

$$P = F_u \cdot u = \frac{w \cdot Q}{g} (w_2 - u_2) \cdot u_2$$ ...(4.13)

Theoretical efficiency of the wheel $\eta = \frac{\text{Output}}{\text{Input}}$

or $$\eta=\frac{\frac{w \cdot Q}{g}(w_2-u_2)\, u_2}{w \cdot Q \cdot H}=\frac{(w_2-u_2)\, u_2}{gH}$$

but $$H=\frac{w_2^2}{2g}-\frac{u_2^2}{2g} \qquad \text{...(See Eqn 4.12)}$$

$$\therefore \qquad \eta=\frac{(w_2-u_2)\, u_2}{g\left(\frac{w_2^2}{2g}-\frac{u_2^2}{2g}\right)}=\frac{2\,(w_2-u_2)\, u_2}{(w_2-u_2)\,(w_2+u_2)}$$

$$=\frac{2u_2}{(w_2+u_2)} \qquad \text{...(4.14)}$$

Some losses have to be taken into account when calculating the output.

4.9 Outward Flow Reaction Turbine (Fourneyron Turbine)— The Fourneyron turbine was the first hydraulic prime mover, which was named as "Turbine". The word turbine was used for the water wheel having numerous buckets or vanes which are all supplied with water simultaneously, whereas in the case of water wheel the number of buckets or vanes were less and the water was delivered on a part of its circumference, impinging only a few buckets at a time. In modern practice the above difference between the turbine and the water wheel is applied to differentiate the two main types of turbines *viz* reaction and impulse turbines respectively (See Art 5 12).

The Fourneyron Turbine was designed to receive its driving force from the reaction of water flowing radially outward from the central pipe and impinging on the blades of the wheel, thus rotating a shaft passing upward through the elbow (See Fig 4.7). The changing direction of radial flow, as influenced by the runner blades, provides the necessary reaction to operate the turbine. Since the flow in the turbine is from the centre to outward, it is known as outward flow or centrifugal reaction turbine. The Fourneyron turbine was the outcome of a prize competition of F_r 6,000 (Rs. 600/- approx) in France. The first set of turbines installed at the famous Niagra Falls for the generation of electric power, were of this type.

Discharge $$Q=\varphi_o\,(\pi \cdot D_o \cdot B_o)\, v_{m_o}$$
$$=\varphi_1\,(\pi \cdot D_1 \cdot B_1)\, v_{m_1}$$
$$=\varphi_2\,(\pi \cdot D_2 \cdot B_2)\, v_{m_2} \quad \text{etc}$$

where $$\varphi=\frac{(\text{area of flow})-(\text{area occupied by the blades})}{\text{area of flow}}$$

Peripheral force on turbine runner is given by Eqn 1.24—

$$F_u=\frac{w \cdot Q}{g}\,(v_{u_1}-v_{u_2})$$

Since $\alpha_2=\frac{\pi}{2}$, $\qquad v_{u_2}=v_2 \cos \alpha_2=0$

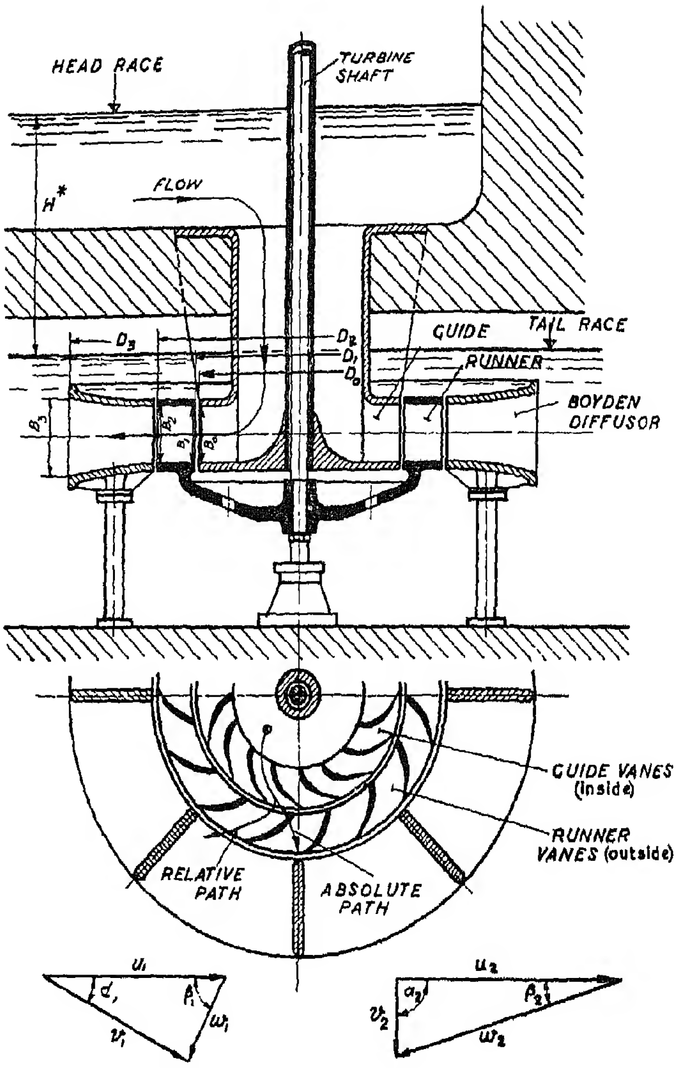


Fig 4.7 Outward Flow Reaction Turbine (Fourneyron Turbine)

$$\therefore \quad F_u = \frac{w \cdot Q}{g} v_{u_1} \qquad \ldots(4.15)$$

Theoretical power developed by the turbine

$$P = F_u \cdot u_1 = \frac{w \cdot Q}{g} v_{u_1} \cdot u_1 \qquad \ldots(4.16)$$

Theoretical efficiency $\eta = \frac{\text{Output}}{\text{Input}} = \frac{\frac{w \cdot Q}{g} \cdot u_{u_1} \cdot u_1}{w \cdot Q \cdot H}$

or $$\eta = \frac{v_{u_1} \cdot u_1}{gH}$$

η is between 75 and 80%.

In order to increase the efficiency of the turbine, Boyden suggested that the water should be discharged from the runner to the tail race through the diffuser (See Fig 4.7).

4.10 Inward Flow Reaction Turbine (Francis Turbine) (See Fig 4 8) —In this type of turbine the inflowing water enters the runner blades from the periphery towards the centre and discharged radially inward. Thus the motion of water in the turbine is centripetal. Therefore

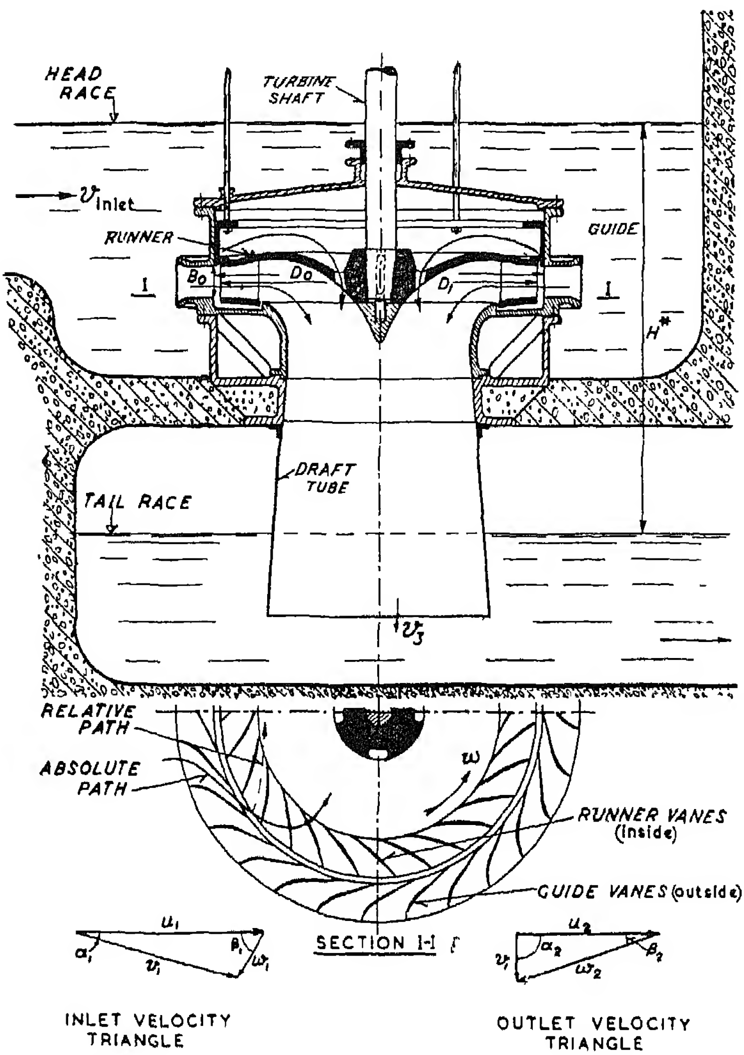


Fig 4 8 Inward Flow Reaction Turbine (Francis Turbine)

centrifugal force will act against the direction of flow and thus reduce the tendency of turbine to race, which is the advantage of this turbine over the outward flow reaction type.

Howd was the first to suggest the inward flow radial turbine in 1827 and Francis designed and constructed the first pure radial centripetal turbine after carrying out several experiments, in 1865. The modern Francis turbine (Chapter 7) discharges the water inward and down instead of inward and radial as explained above.

The same calculations for the determination of power and efficiency described under outward flow reaction turbine (Art 4.9) will apply.

4.11 Parallel Flow or Axial Flow Reaction Turbine (Jonval Turbine) (Fig 4.9)—The direction of flow of water in this type of turbine is parallel to the axis of the wheel. Jonval Turbine (1838—1850) which is a parallel flow reaction turbine consists of a horizontal runner into which the water enters through the guide vanes placed above it.

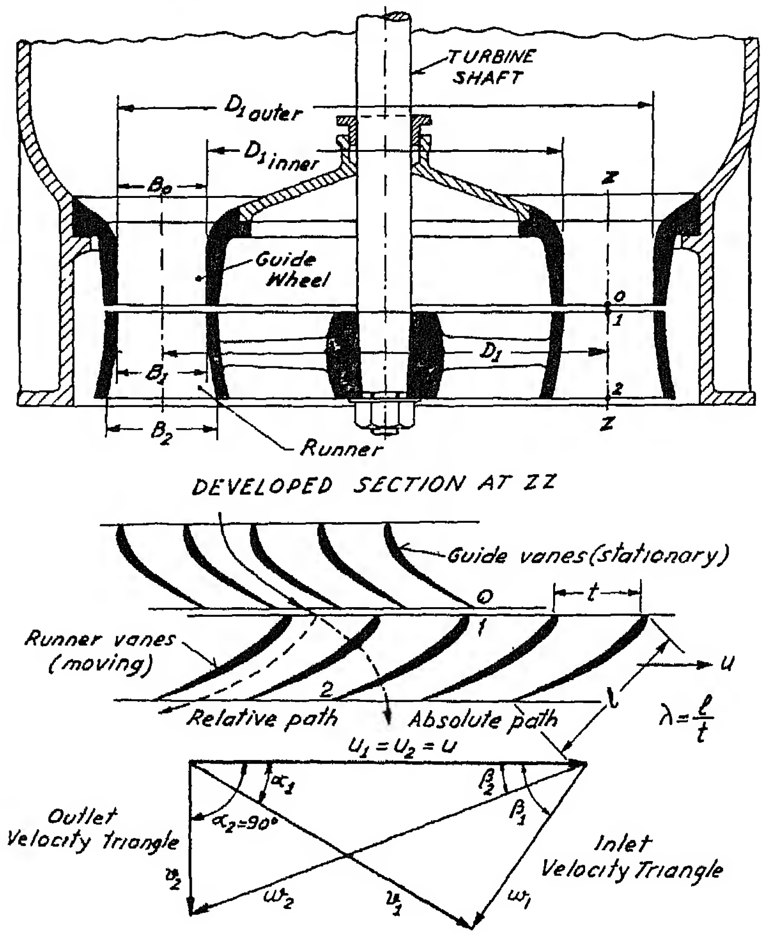


Fig 4.9 Parallel Flow or Axial Flow Reaction Turbine (Jonval Turbine)

The calculations for the determination of power and efficiency are the same as described under outward flow reaction turbine (Art 4.9).

Practical data—

If $\lambda = \frac{l}{t}$ (See Fig 4.9)

where l=length of blades

t=distance between two blades

then λ is known as relative blade length.

$\lambda_{outer} = 3, \qquad \lambda_{inner} = 5$

(In modern axial flow turbines of Kaplan type –Chapter 7, $\lambda = 1$)

$$\frac{B_2}{D_1} = \frac{1}{5} \text{ to } \frac{1}{6}$$

$D_1 = 1{\cdot}45\sqrt{Q_1}$ (D_1 and H in metres Q in m^3/sec)

$v_2 = 0{\cdot}18$ to $0\ 22\sqrt{2gH}$

4.12 Axial Flow Impulse Turbine (Girard Turbine)—The construction of Girard turbine is similar to that of Jonval type with the difference that guide vanes do not cover the whole of circumference, but only two opposite quadrants. Thus the turbine runner will not be full of water and it will move by the impulse of moving water. The velocity triangles are shown in Fig 4.10.

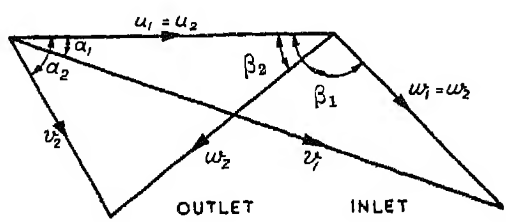


Fig 4.10 Velocity Triangles for Axial Flow Impulse Turbine (Girard Turbine)

In modern practice the water turbines are coupled to the electric generators to produce electric power. Almost all the water wheels and turbines described in this chapter are obsolete for this purpose. The modern water turbines will be dealt with in the next chapters.

Problem 4.4 The mean blade circle diameter of the runner of an axial flow impulse turbine is 5 ft. The guide blade angle is 25° and the vane angles at entry and exit are 54° and 26° respectively and the breadth of the moving blade at inlet is 5 in. If the working head is 312 ft, find the speed of the turbine. Also find the horse power developed if admission is over one half of the circumference. Allow 12 per cent loss of area due to vanes and take co-efficient of guide vanes 0 98.

(Punjab University—1960A)

Solution

$D_1 = 5$ ft	$\alpha_1 = 25°$
$\beta_1 = 180 - 54 = 126°$	$\beta_2 = 26°$
$B_1 = 5$ in.	$H = 312$ ft

Loss of area due to vanes=12%,

or $\phi = 1 - 0{\cdot}12 = 0{\cdot}88$

Co-efficient of guide vanes, $K_{v_1}=0.98$

Drawing inlet velocity triangle (See Fig 4.10)—

$$v_1=K_{v_1}\cdot\sqrt{2gH}=0.98\times\sqrt{64.4\times312}$$

$$=0.98\times8.02\times17.67=138.5 \text{ ft/sec}$$

$$v_{m_1}=v_1\sin\alpha_1=138.5\times\sin 25^\circ$$

$$=138.5\times0.4226=58.5 \text{ ft/sec}$$

$$v_{u_1}=v_1\cos\alpha_1=138.5\times0.9063$$

$$=125.5 \text{ ft/sec}$$

$$u_1=v_{u_1}-\frac{v_{m_1}}{\tan\beta_1}$$

$$=125.5-\frac{58.5}{1.3764}$$

$$=125.5-42.5$$

$$=83 \text{ ft/sec}$$

$$w_1=\frac{v_{m_1}}{\sin(180-\beta_1)}=\frac{58.5}{0.8090}=72.3 \text{ ft/sec}$$

Now taking the outlet velocity triangle (See Fig 4.10)—

$$u_2=u_1\ldots\ldots\text{(impulse turbine)}$$

$$w_2=w_1\ldots\ldots\text{(no loss on blade)}$$

$$v_{u_2}=u_2-w_2\cos\beta_2$$

$$=83-72.3\times0.8988$$

$$=83-65=18 \text{ ft/sec}$$

$$Q=\phi\{(\pi D_1\times\tfrac{1}{3})\times B_1\}\cdot v_{m_1}$$

$$=0.88\times\pi\times5\times\tfrac{1}{3}\times\tfrac{5}{12}\times58.5$$

$$=168.5 \text{ cfs}$$

$$\text{Horse Power developed}=\frac{w\cdot Q}{550\,g}\cdot\left(v_{u_1}-v_{u_2}\right)u_1$$

$$=\frac{62.4\times168.5}{550\times32.2}\times(125.5-18)\times83$$

$$=\mathbf{5290\ HP}\quad \textit{Answer}$$

ii) Speed of turbine—

$$u_1=\frac{\pi D_1 N}{60}$$

or $$83=\frac{\pi\times5\times N}{60}$$

or $$N=\frac{60\times83}{\pi\times5}=\mathbf{317\ rpm}\quad \textit{Answer}$$

4 13 Further Development of Water Turbines—Prior to World War I, the development of water power took place in regions far from urban areas as the hydro-power was available mostly in rural or mountainous districts. The industry had to be installed near the power station as the mills used to be driven by the water wheels and turbines directly. The prime movers had to run all the time even if some of the machines in the mill were to remain idle. The drive of the mills was slow and was difficult to control. In those days electricity was still regarded as something of a curiosity in many countries.

Then came the evolution in the use of electric power. Use of electricity for the drive of machines in the industry became very common as it proved very convenient to handle. To-day every individual machine in the shop of a mill is run by electric motor. With the further development in electric power, high speed machines were designed with which the size of the machine for the same power was reduced.

Due to the increase in the use of electric power it was considered necessary to carry the electricity from the remote hydro stations to the far distant industrial towns It was not economical to transmit the electric power for long distances, as the voltage loss was great. The high voltage transmission technique was then developed, with which the percentage voltage drop became less. The transformers are used to step up the voltage produced at the power station and then step it down after it has been transmitted to the places where it is to be consumed.

Due to the reasons described above, the water turbine has become an admirable prime mover for the central station power generating service. It has the following advantages over the other types of prime movers—

a) Simple, efficient, easily controlled and long lived.

b) Outstanding ability to act as a standby unit, as it can be started cold and assumes full load in a few minutes.

c) Can be made automatic and remote controlled.

d) May be combined with steam generating plant.

Thus where sufficient water power is available the hydraulic turbine has become the back bone of large power distribution systems.

In view of the above development in electrical engineering, the development of water turbine took place in the following spheres—

a) Increase in head, speed and capacity. *b*) Efficient governing.

To day there exist mainly three types of water turbines *viz*, Pelton, Francis and Kaplan which are explained in detail in Chapters 6 and 7.

UNSOLVED PROBLEMS

4.1 Write short notes on—

a) Overshot Water Wheel,

b) Breast Water Wheel,

c) Undershot Water Wheel, and

d) Poncelet Water Wheel.

4.2 Explain why water wheels have been replaced by turbines.

4.3 What is the difference between impulse wheel and jet reaction wheel ?

4.4 Describe Barker's Mill. Where is it used in modern practice ?

4.5 Differentiate between outward, inward and parallel flow turbines. Name their designers.

4.6 What is the difference between axial flow reaction and axial flow impulse turbines ?

4.7 Describe the development of modern water turbine.

4.8 An overshot wheel 32 ft diameter has shrouds 14 in. deep and is required to give 29 HP when making 5 rpm. Assuming the buckets to be one-third filled with water, find the width of the wheel when the total fall is 32 ft, and the efficiency 60%. (4.18 ft)

4.9 A Barker's Mill is supplied with water under a head of 10 ft. The combined areas of the orifices amount to 40 sq in. and the velocity at the centres of the orifices is 24 ft/sec. Find the horsepower if the net efficiency is 60%. Find also the hydraulic efficiency. (11.8 HP ; 68·7%)

4.10 An outward flow turbine wheel has an internal diameter of 5.249 ft and an external diameter of 6·25 ft. The head above the turbine is 141·5 ft. The width of the wheel at inlet is 10 in. and the quantity of water supplied per second is 215 cu ft. Assuming the hydraulic losses as 20%, determine the angles of tips of the vanes so that the water shall leave the wheel radially. Determine the horsepower of the turbine, then verify the work done per lb of water from velocity triangles. (134°—40′ ; 9°—10′ ; 2,760 HP)

4.11 A parallel flow reaction turbine runs at 52 rpm under a head of 12 ft. The radial velocity of flow through the wheel is 4 ft/sec. The diameters of outer and inner crowns of the wheel are 8 ft and 5 ft respectively. Determine the guide blade angle and vane angles if hydraulic efficiency of the wheel is 80%. (15°—46′ ; 152°—11′ ; 10°—21′)

4.12 An axial flow impulse turbine makes 144 rpm when working under a head of 100 ft. The mean diameter of the wheel is 6 ft. The angle of the guide blade at entrance is 30°, and the angle the vane makes with the direction of motion at exit is 30°. 8% of head is lost in the supply pipe and guide, and 10% of the total head is lost in friction and shock in the wheel. Determine the relative velocity of water and wheel at entrance, and the velocity with which the water leaves the wheel. Calculate the hydraulic efficiency of the turbine. (45·36, 77, 44, 36, 23 ft/sec ; 73·75%)

(*N.B.* Problems on inward flow reaction turbines are given in Chapters 1 and 7).

CHAPTER 5

HYDRO-ELECTRIC PLANTS

5 1 Introduction 5 2 Heads and Rate of Flow or Discharge 5 3 Essential Components of Hydro-Electric Power Plant 5 4 Classification of Water Power Plants 5 5 High Head Water Power Plants 5 6 Low Head Water Power Plants 5 7 Canal Water Power Plants 5 8 Medium Head Water Power Plants 5 9 Classification of Modern Water Turbines 5 10 Classification of Turbines Depending upon the Head and the Quantity of Water Available 5 11 Classification According to the Names of Originators 5 12 Classification According to Action of Water on Moving Blades 5 13 Classification According to Direction of Flow of Water in the Runner 5 14 Classification According to Disposition of Turbine Shaft 5 15 Classification According to Specific Speed 5 16 Classification According to Non-dimensional Factor K_s in Turbines 5.17 Relation between N_s and K_s

5 1 Introduction—The purpose of a Hydro-Electric Plant is to harness power from water flowing under pressure. As such it incorporates a number of water driven prime-movers known as Water Turbines.

Water flowing under pressure has two forms of energy—kinetic and potential. The kinetic energy depends on the mass of water flowing and its velocity, while the potential energy exists as a result of the difference in water level between two points, which is known as "head". The water or hydraulic turbine, as it is sometimes named, converts the kinetic and potential energies possessed by water into mechanical power. The hydraulic turbine is, thus, a prime-mover which when coupled to a generator produces electric power. The projects designed to produce electric power from water are known as "Hydro-Electric Projects". Hydro-Electric Projects may not be used exclusively for power generation. Sometimes, they are the off-shoot of flood control and irrigation projects in which case they are known as "Multipurpose Projects", for example, Bhakra Nangal and Damodar Valley Corporation (D.V.C.) projects. The former is useful mainly for power and irrigation whereas the latter is primarily concerned with flood control.

Hydro-electric power can be developed wherever water continuously flowing under pressure is available. Dams constructed across flowing rivers divert the riverine bounty through the turbines giving rise to such useful power. But this is not all. Water that collects in natural or artificial lakes in the high and huge mountains, due to the melting of snow and due to heavy monsoon rain, can be led down to the turbines through large pipes known as *penstocks*.

The following table gives the total energy consumption and the resources of hydro-electric power of some of the important countries of the world.

TABLE 5.1

Total Energy Consumption & Hydro-Electric Resources

Country	Energy Consumption: Population ÷ 16^6	Energy Consumption: Total Consumption ÷ 10^6 *kwh* per year	Electric Consumption: Total ÷ 10^6 *kwh* per year	Electric Consumption: *kwh* per head per year	Hydro-Electric Resources: Potential ÷ 10^6 *KW*	Hydro-Electric Resources: Developed ÷ 10^6 *KW*
U S.A.	123	8,800,000	544,000	4,400	100	18
U.S.S.R.	193	4,400,000	166,000	860	60	1·7
U K.	50	1,750,000	64,000	1,270	0·5	0·27
France	41	590,000	41,000	1,000	4·5	4·5
Norway	3·3	100,000	22,000	6,000	7·5	2·9
India	361	1,200,000	9,100	25	40	0·8

It shows that India has developed only 2% of its water power potential so far.

5 2 Heads and Rate of Flow or Discharge—Head is the difference in elevation between two levels of water. The head of a hydro-electric plant is entirely dependent on the topographical conditions. Head can be characterised as

a) gross head,

and *b*) net or effective head.

a) **Gross Head** is defined as the difference in elevation between the head race level at the intake and the tail race level at the discharge side, naturally, both the elevations have to be measured simultaneously. The gross head may vary as both the elevations of water do not remain the same at all times. It is essential to know the maximum and minimum as well as the normal values of the gross head. The normal value would be that for which the plant works most of the time. In rainy season the flood may raise the elevation of tail race, thus reducing the gross head. On the other hand at the time of draught the same may be increased.

b) **Net or Effective Head** is the head obtained by subtracting from gross head all the losses in carrying water from the head race to the entrance of the turbine. The losses are due to friction occurring in tunnels, canals and penstocks which lead the water into the turbine. Net or effective head is, therefore, the true pressure difference between

the entrance to the turbine casing and the tail race water elevation. The measurement of the pressure head at the entrance to the turbine casing must be corrected for the velocity head at that point, and the tail-water elevation corrected for residual velocity head.

Rate of Flow or Discharge of water is the quantity of water used by the water turbine in unit time and is, generally, measured in cu ft per sec (cusecs) or cu ft per min in *FPS* units and m^3/sec or lit/sec in metric units.

The maximum, minimum and the normal rates of flow and flow distribution throughout the various seasons ought to be known.

5.3 Essential Components of Hydro-Electric Power Plant—

a) Storage reservoir,

b) Dam with its Control Works,

c) Waterways with their Control Works,

d) Power house with turbo and other machinery,

e) Tail race,

and *f*) Generation and transmission of electric power.

a) **Storage Reservoir**—The water available from a catchment area is stored in a reservoir, so that it can be utilised to run the turbines for producing electric power according to the requirement throughout the year. The storage reservoir may be natural or artificial.

Natural reservoir or lake may be found in high and huge mountains. Water is taken off from one end of the reservoir through a tunnel built by cutting the mountain.

Artificial reservoir is made by constructing dam on one or two openings of a valley in order that considerable amount of water is stored.

b) **Dam with its Control Works**—Dam is a thick wall erected on suitable site to provide for the storage of water and to create head. Dam may be built to make an artificial reservoir from a valley or it may be erected in a river to control the flowing water. Dams may be of different types such as earth dam, masonry dam, wooden dam or steel dam. Masonry dams are very common.

Structures and appliances to control the supply of water from the storage reservoir through the dam, are known as *Control Works* or *Head Works*. The principal elements of control works are

i) *gates and valves* of various types and their operating mechanisms,

ii) structures necessary for their operation,

and *iii*) devices for the protection of gates and hydraulic machines.

The different types of gates are—

Plain sliding gates, wheeled gates, sluice gates, crest gates (*i.e.* flash boards or shutters), tainter gates, rolling or drum gates, radial gates etc.

The different types of valves are butterfly and needle valves.

The devices for the protection of gates and hydraulic machines consist of

i) *Trashracks* (See Fig 5.2 and 5.4).—They are made of a row of rectangular cross-sectional structural steel bars placed across the entire intake opening in an inclined position. They are used to obstruct debris from going into the intake.

ii) *Debris cleaning device* fitted on the trashracks.

iii) *Heating element* against ice troubles.

c) **Waterways with their Control Works**—Waterway is a passage through which the water is carried from the dam to the power house. It may consist of tunnel, canal, flumes, forebay, pipes (*i.e.* penstocks) etc. *Tunnel* (See Fig 5.1) is passage made by cutting the mountain to save distance. *Forebay* (See Fig 5.4) in an enlarged section of a canal spread out to accomodate the required width of intake. It serves the purpose of storage reservoir.

The control works for the tunnels, canals, flumes, forebay and pipes, may be different types of gates and valves described above. In addition to these, *surge tank* is used where water is carried through penstocks. A surge tank (See Fig 5.1) is a reservoir fitted at some opening made on a long pipe line to receive the rejected flow when the pipeline is suddenly closed by a valve at its steep end. The surge tank, therefore, controls the pressure variations resulting from the rapid changes in pipe line flow thus eliminating water hammer effects. For further details, See Chapter V of "Hydraulics".

d) **Power House** is a building to house the turbines, generators and other accessories for operating the machines.

e) **Tail Race** is a waterway to conduct the water discharged from the turbines to a suitable point where it can be safely disposed of or stored to be pumped back into the original reservoir.

f) **Generation and Transmission of Electric Power**—It consists of electrical generating machines, transformers, switching equipments and transmission lines.

5.4 Classification of Water Power Plants—Water power plants can be classified according to the heads under which they work, as follows :

a) **High Head Plants** working under heads ranging from several hundreds to a few thousands of feet.

b) **Low Head Plants** working under heads mostly below hundred feet.

c) **Medium Head Plants** working under heads lying between the above two and not covered by them.

5.5 High Head Water Power Plants—Water is usually stored up in lakes on high mountains during the rainy season or during the season when the snow melts.

The rate of flow should be such that water can last throughout the year. From one end of the lake, tunnels are constructed which lead the water into smaller reservoirs known as *forebays*. The forebays distribute the water to penstocks through which it is led to the turbines. These forebays help to regulate the demand of water according to the load on the turbines. In cases where it is not possible to construct forebay, *surge tanks* are built. When the load on the turbines decreases, the excess

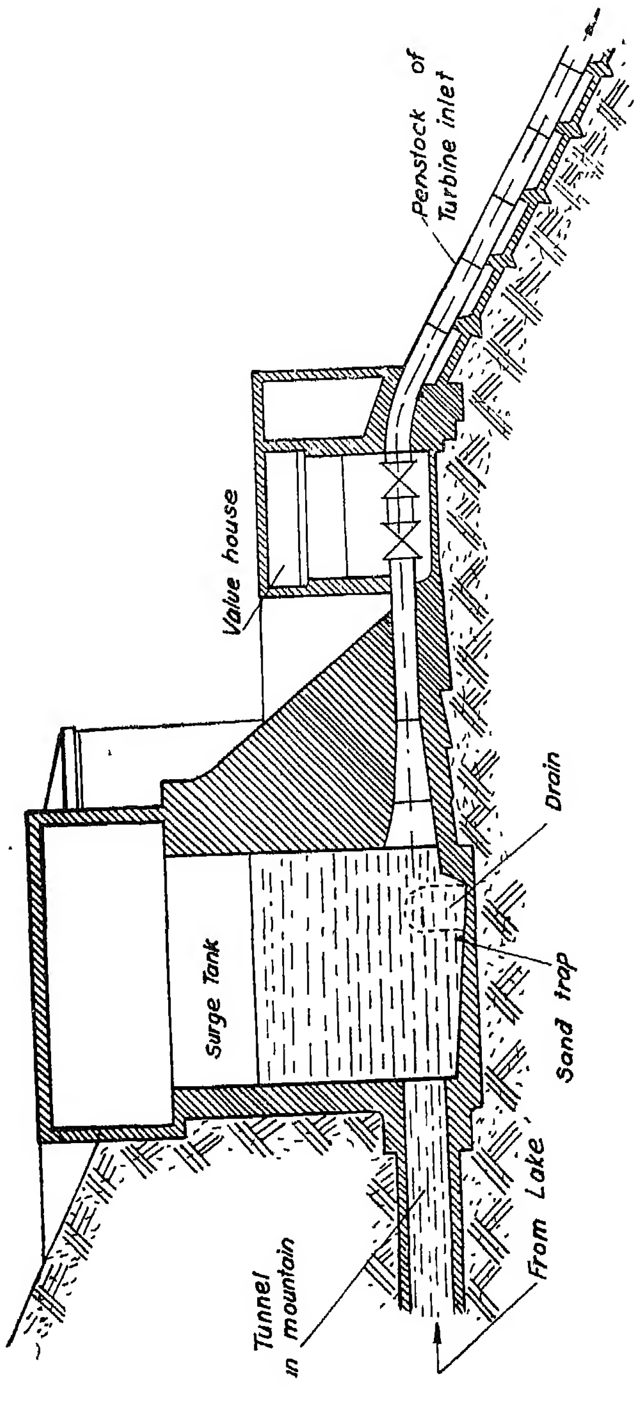


Fig 5 1 High Head Water Power Plant Layout

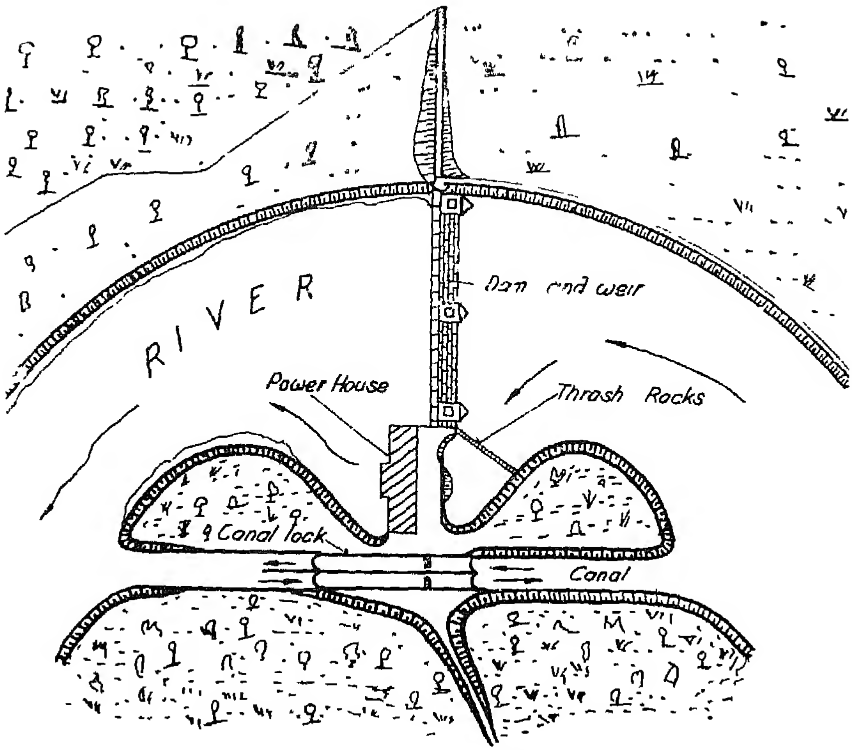


Fig 5.2 Low Head Water Power Plant Layout

quantity of water surges into the surge tank preventing a sudden pressure rise. Surge tank occupies much less space than a forebay. Fig 5.1 illustrates water flowing through the tunnel to the surge tank. Situated near the surge tank is a *Valve House* housing electrically driven sluice type valves meant for control of flow in the penstocks. The valves also help to close the penstocks and, thus, facilitate cleaning and repairing.

The total distance from the water source to the turbine inlet may, sometimes, be even 8 to 10 miles. The water is discharged after passing through the turbines into the tail race from where it can be taken for irrigation. It is sometimes fed to fish ponds.

5.6 Low Head Water Power Plants—These plants usually consist of a dam across a river. A sideway stream diverges from the river at the dam. Over this stream the power house is constructed. Later this channel joins the river further downstream. Fig 5 2 illustrates a typical low head station together with a canal-lock for easy navigation of ships from one side of the dam to the other.

5 7 Canal Water Power Plants—These also fall under low head plants. If there is a river flowing (*See* Fig 5.3) whose level A is higher than at B, a straight canal joining A and B is constructed. A dam near A diverts the water into the channel. The main sluice gates are erected near A to control the rate of flow of water in the canal as well as the forebay. The power house is shown on the right hand side of the forebay. On leaving the turbines, installed in the power house, the water is led back into the river by the canal at B. The river

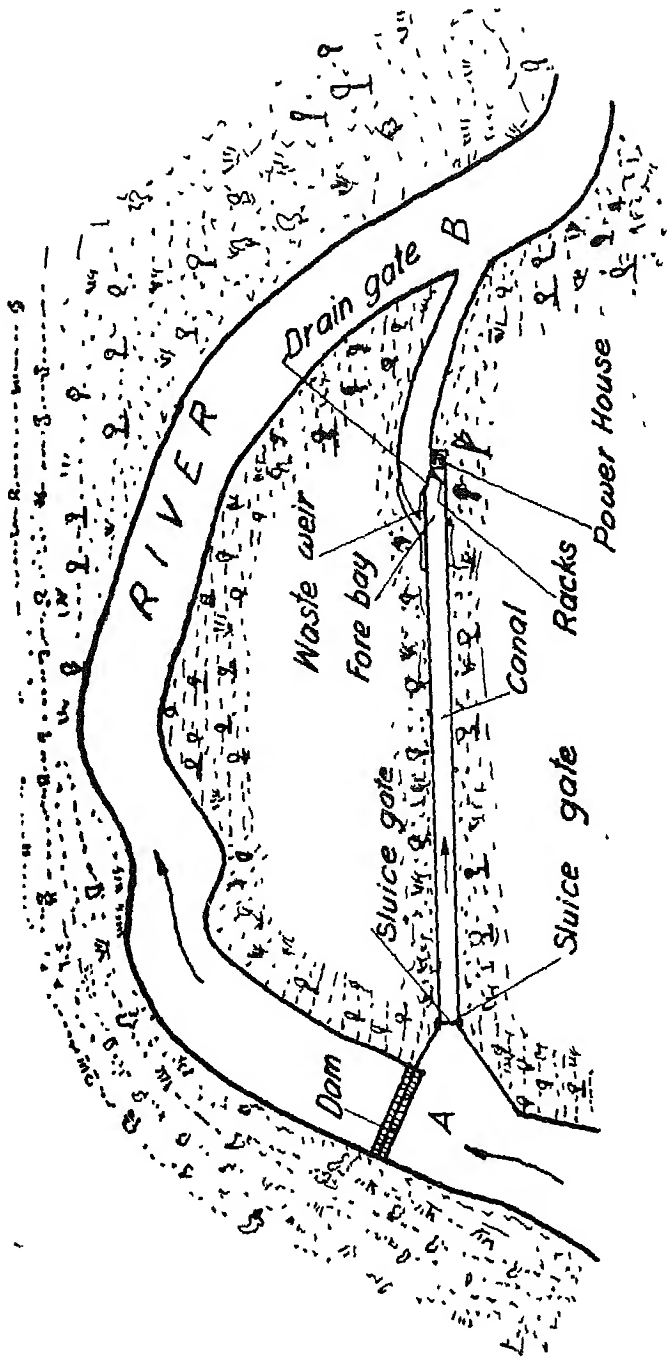


Fig 5 3 Canal Water Power Plant Layout

water generally contains floating or suspended matter like grass, leaves, or small twigs which may bring the turbine to a standstill if permitted to pass through. To separate such foreign bodies in the water, it is passed through the trashracks. Fig 5.4 shows the details of the plant of Fig 5.3.

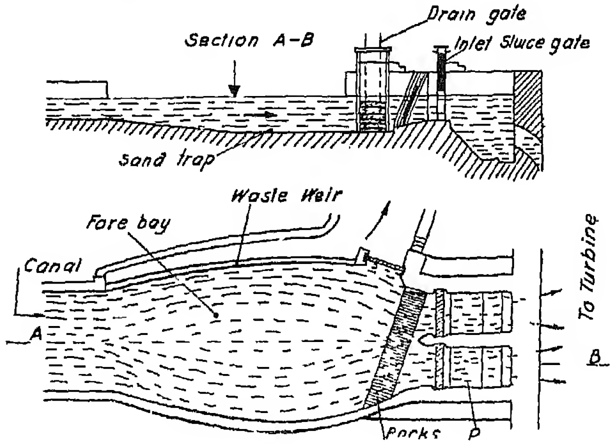


Fig 5.4 Details of Low Head Water Plant

When the turbine head decreases and there is less demand for water by the turbines, the excess flow instead of swelling the canal is diverted over the *waste weir* into the river, thus by-passing the turbines. A drain gate on the side of waste weir facilitates cleaning and repairing of canal.

5.8 Medium Head Water Power Plants—If the head available in low head plant is above one hundred feet, a penstock joining the forebay to the turbine inlet is built as shown in Fig 5.5 This plant would now be known as a medium head plant, which may also include a high head plant working under heads less than 1,000 feet.

The following figures give a rough idea of the heads under which the various types of plants work—

a) High head power plants—300 ft (about 100 m) and above,
b) Medium head power plants—100 to 1,500 ft (30 to 500 m),
c) Low head power plants—10 to 160 ft (3 to 50 m).

It is to be noted that the figures given above overlap each other. Therefore, it is difficult to classify the plants directly on the basis of head alone. The basis, therefore, technically adopted is the *specific speed* of the turbine as explained later.

5.9 Classification of Modern Water Turbines can be made according to :

i) the head and the quantity of water available,
ii) the name of the originator,
iii) the action of water on the moving blades,
iv) the direction of flow of water in the runner,
v) the disposition of shaft,
and *vi*) the specific speed or non-dimensional factor K_s.

The last one is the only scientific classification.

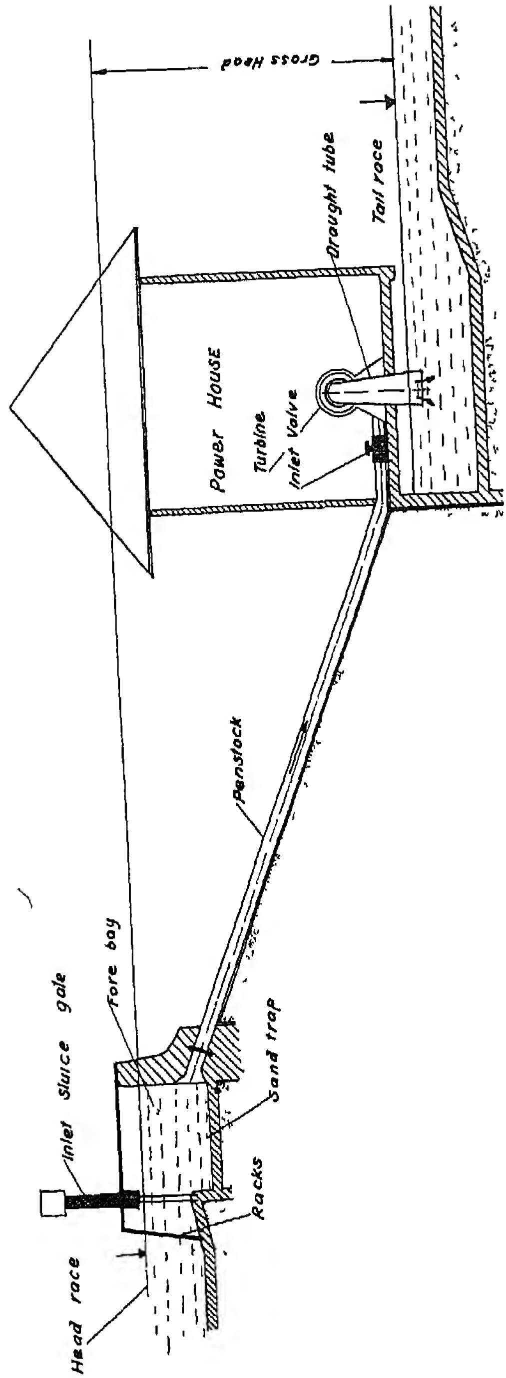


Fig 5 5 Medium Head Water Power Plant Layout

5.10 Classification of Turbines Depending Upon the Head and the Quantity of Water Available :

a) **Impulse turbine** requires high head and small quantity of flow.

b) **Reaction turbine** requires low head and high rate of flow.

Actually there are two types of reaction turbines, one for medium head and medium flow and the other for low head and large flow.

Similar to the classification of Water Power Plants (Art 5.4), it is quite difficult to classify the turbines according to head and discharge, as for the same head *or* discharge all types of turbines can be employed.

5.11 Classification After the Names of Originators—

a) **Pelton Turbine**—named in honour of Lester Allen Pelton (1829—1908) of California (USA) in an impulse type of turbine, used for high head and low discharge.

b) **Francis Turbine**—named after James Bichens Francis (1815-92) who was born in England and later went to USA., is a reaction type of turbine for medium high to medium low heads and medium small to medium large quantities of water.

c) **Kaplan Turbine**—named in honour of Dr. Victor Kaplan (1876—1934) of Bruenn (Germany) is a reaction type of turbine for low heads and large quantities of flow.

5.12 Classification According to Action of Water Moving Blades

Turbine
- Impulse Turbine
  - Pelton Turbine
- Reaction Turbine
  - Francis Turbine
  - Propeller and Kaplan Turbines

a) **Impulse Turbine**—The water is brought in through the penstock ending in a single nozzle. The whole pressure energy of water is transformed into kinetic energy. The water coming out of the nozzle in the form of a free jet is made to strike on a series of buckets mounted on the periphery of a wheel. The water is delivered to the wheel on a part of its circumference filling or striking only a few of the buckets at a time. The wheel revolves in open air *i.e.* there is no difference of pressure in the water at the inlet to the runner and at the discharge. Therefore the casing of an impulse turbine has no hydraulic function to perform. It is necessary only to prevent splashing and to lead the water to the tail race, and also as a safeguard against accidents.

This turbine is also known as a free jet turbine.

In such type of turbine, it can be written that

$$p_1 = p_2, \qquad v_1 >>> v_2$$

and $w_1 \approx w_2$ (neglecting losses in buckets).

b) **Reaction Turbine**—The reaction turbine operates with its wheel submerged in water. The water before entering the turbine has

pressure as well as kinetic energy. All pressure energy is not transformed into kinetic energy as in case of impulse turbine. The moment on the wheel is produced by both kinetic and pressure energies. The water leaving the turbine has still some of the pressure as well as the kinetic energy. The pressure at the inlet to the turbine is much higher than the pressure at the outlet. Thus, there is a possibility of water flowing through some passage other than the runner and escape without doing any work. Hence, a casing is absolutely essential due to the difference of pressure in reaction turbine.

For this type of turbine, it can be mentioned that

$$p_1 >> p_2, \quad v_1 > v_2 \quad \text{and} \quad w_1 < w_2.$$

Propeller turbines are mainly Kaplan Turbines but Moody, Nagler, and Bell turbines may be found in the market. The main difference between Kaplan Turbine and the other type of propeller turbines is that the former has adjustable runner blades.

5.13 Classification According to Direction of Flow of Water in the Runner

Turbine
- Tangential flow Turbine [Pelton Turbine]
- Radial flow Turbine
  - Outward radial flow Turbine (centrifugal type) [Fourneyron Turbine] (Art 4.9)
  - Inward radial flow Turbine (centripetal type) [Old Francis Turbine] (Art 4.10)
- Axial flow Turbine [Kaplan Turbine]
- Mixed (Radial and Axial) flow Turbine [Modern Francis Turbine]

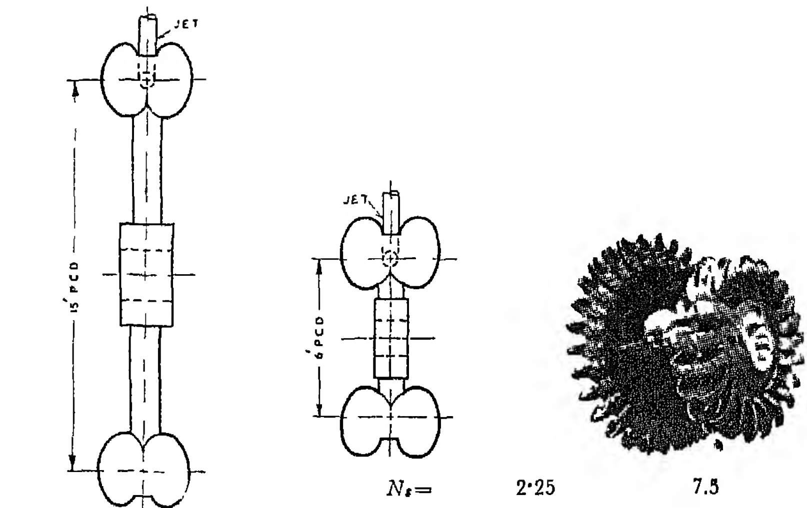
In tangential flow turbine of Pelton type the water strikes the runner tangential to the path of rotation. This path is the centre line of buckets which is, sometimes, known as pitch circle diameter or mean diameter of wheel.

Radial flow has already been explained in Chapter I. In axial flow turbine water flows parallel to the axis of the turbine shaft. In inward flow turbine, the water flows from the periphery towards the centre of the wheel.

In mixed flow turbine, water enters radially and emerges out axially, so that the discharge is parallel to the axis of the shaft. Modern Francis turbines have mixed flow runners.

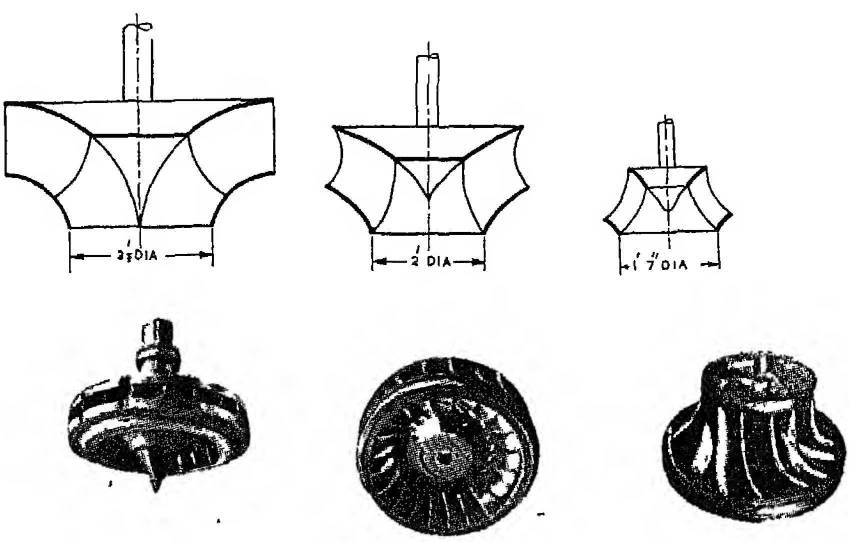
5.14 Classification According to Disposition of Turbine Shaft—Turbine shaft may be either vertical or horizontal. In modern turbine practice, Pelton turbines usually have horizontal shafts whereas the rest, especially the large units, have vertical shafts.

5.15 Classification According to Specific Speed—Turbines fall under a natural order when classified according to their specific speeds. The specific speed of a turbine is the speed of a geometrically



$N_s =$ 2·25 7.5

a) PELTON RUNNERS



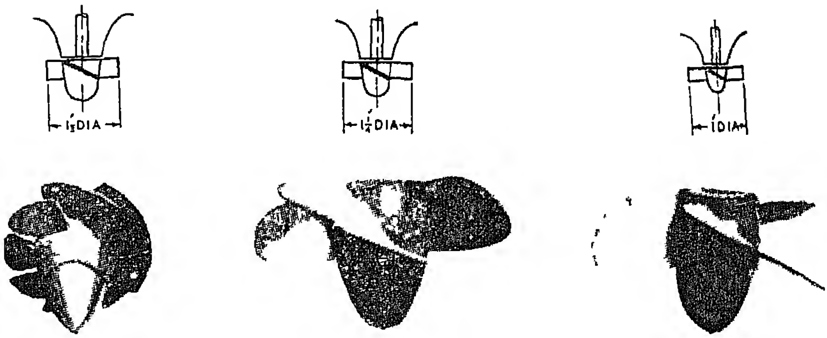
$N =$ Slow 17 Normal 34 Fast 67·5

b) FRANCIS RUNNER

$N_s = 90$ Extremely Fast

c) DUBS RUNNER

(Fig 5.6 *Contd*)



8—Blades
$N_s = 100$

5 Blades
135

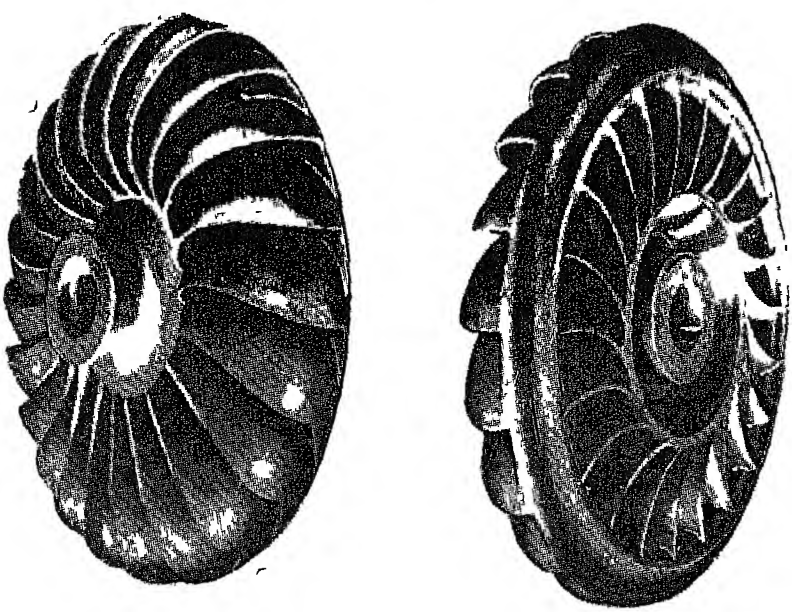
3—Blades
225

d) KAPLAN RUNNERS

Different Types of Water Turbine Runners

Comparison of dimensions is made on the basis that each runner develops 100 BHP under 40 ft net head, when running at a speed equal to 10 N_s (British).
(Manufactured by Escher Wyss & Co. Ltd., Zurich, Switzerland)

Fig 5.6 Classification of Water Turbines According to Specific Speeds N_s



(Manufactured by Gilbert Gilkes & Gordon Ltd , Kendal—England)
Fig 5.7 Turgo-Impulse Water Turbine Runner

similar turbine that would develope one Brake Horse Power (BHP) under a head of one foot or considering metric units, one metric HP under a head of one metre.

$$\text{Specific speed}\quad N_s = \frac{N \cdot \sqrt{P_t}}{H \cdot \sqrt[4]{H}} = \frac{N \cdot \sqrt{P_t}}{H^{\frac{5}{4}}} \qquad \text{(See Eqn 3.39)}$$

where N=normal working speed of turbine in RPM,
P_t=turbine output in BHP or metric HP
H=net or effective head in ft or metres

$$N_s\ (\text{metric}) = 4{\cdot}45\ N_s\ (\text{British}) \qquad ..\ (5.1)$$

TABLE 5.2

Specific Speed of Turbines

Type of Turbine	Type of Runner	Specific Speed	
		British Units	Metric Units
Pelton	Slow	2·5 to 4·5	10 to 20
	Normal	4·5 to 6·5	20 to 28
	Fast	6·5 to 8	28 to 35
Francis	Slow	13·5 to 27	60 to 120
	Normal	27 to 40·5	120 to 180
	Fast	40·5 to 67·5	180 to 300
Kaplan	—	67·5 to 225	300 to 1,000

Pelton wheels having a specific speed of 2·5 to 8 British Units (or 10 to 35 Metric Units) are those designed for one nozzle. The gap between the specific speed relating to Pelton and Francis turbines *i.e* 8 to 13·5 British Units (or 35 to 60 Metric Units) is bridged by equipping Pelton wheel with more than one nozzles which, of course, is the general practice. Messrs Gilbert Gilkes and Gordon Ltd., Kendal (England) have patented another turbine known as *Turgo-Impulse Wheel* (See Fig 5·7) which covers the required range of specific speed.

M/s Komplex, Budapest, Hungary manufacture **Banki** turbines (See Art 6.18) having a specific speed of 4·5 to 18 British Units (or 20 to 80 Metric Units). It is an impulse turbine based on the patent of Prof. Dohat Banki (1859-1922) of Budapest Technical University.

Francis turbines could be designed for a specific speed greater than 67·5 in which case it will have a special runner as designed by Prof Dubs, N_s of which ranges from 63 to 117. British Units (or 284 to 520 Metric Units.)

Special types of runners such as screw runners, are used for specific speed more than 225. British Units (or 1000 Metric Units.)

Problem 5.1 Find the specific speed of a turbine installed at Khapoli Power House belonging to Tata Hydro-Electric Co., Bombay,

which develops 17,100 HP under a head of 1,685 ft when running at 300 rpm. Specify the type of turbine employed.

Solution

$P_t = 17{,}100$ HP $\quad H = 1{,}685$ ft $\quad N = 300$ rpm

$$\therefore \text{ Specific speed } N_s = \frac{N \cdot \sqrt{P_t}}{H^{\frac{5}{4}}} = \frac{300 \times \sqrt{17{,}100}}{1{,}685^{\frac{5}{4}}}$$

$$= \frac{300 \times 131}{1{,}685 \times 6{\cdot}41} = 3\ 63 \qquad \textit{Answer}$$

According to Table 5 2 the type of turbine employed is *Pelton.*

Problem 5 2 The turbine installed at Ganguwal Power House (Punjab) develops 33,500 HP under a head of 98 ft. Find the specific speed of this turbine if it runs at 166 7 rpm. Knowing the specific speed, what type of runner would you select for such a turbine ?

(*AMIE – Nov 1954*)

Solution

$P_t = 33{,}500$ HP $\quad H = 98$ ft $\quad N = 166\ 7$ rpm

$$\therefore \text{ Specific speed } N_s = \frac{N \cdot \sqrt{P_t}}{H^{\frac{5}{4}}} = \frac{166\ 7 \times \sqrt{33{,}500}}{98^{\frac{5}{4}}}$$

$$= \frac{166{\cdot}7 \times 183}{98 \times 3\ 15} = 98{\cdot}7 \qquad \textit{Answer}$$

Therefore, according to Table 5·2 *Kaplan* runner is to be selected for the above turbine.

5.16 Non-Dimensional Factor K_s in Turbines—The specific speed N_s is a dimensional quantity, but a non-dimensional factor K_s could be deduced for turbines in place of N_s.

$$K_s = \frac{Q \cdot N^2}{v^3} \qquad \ldots(5.2)$$

where Q=rate of flow in cu ft/sec (or in $m^3 s^{-1}$)

N=speed in *rpm*

and v=velocity of water in ft/sec (or in $m\ s^{-1}$)

$= \sqrt{2gH}$

To prove K_s as a dimensionless factor—

$$K_s = \frac{\frac{\text{ft}^3}{\text{sec}} \times \left(\frac{1}{\text{sec}}\right)^2}{\left[\frac{\text{ft}}{\text{sec}^2}\right]^{\frac{3}{2}} \times (\text{ft})^{\frac{3}{2}}} = \frac{\text{ft}^3 \times \text{sec}^3}{\text{ft}^3 \times \text{sec} \times \text{sec}^2} = \text{a pure number}$$

Most modern method to classify the water turbines would, therefore, be according to the non-dimensional factor K_s. The values of factor K_s are given in Table 5.3.

TABLE 5.3

The Values of Non-dimensional Factor K_s

Type of Turbine	Type of Runner	K_s
Pelton	Slow Normal Fast	0·098 to 0·39 0·39 to 0·76 0·76 to 1·2
Francis	Slow Normal Fast	3·5 to 14 14 to 31·5 31·5 to 88
Kaplan	—	88 to 980

Problem 5.3 Find the specific speed of a turdine installed at Pykara Power House (South India), which develops 19,000 HP under a head of 2,800 ft. The speed of the turbine is 600 rpm. State the type of turbine that would be required. If this turbine uses 67 cusecs of water, what is the value of K_s?

Solution

$P_t = 19{,}000$ HP $\quad H = 2{,}800$ ft $\quad N = 600$ rpm

a) Specific Speed $N_s = \dfrac{N \cdot \sqrt{P_t}}{H^{\frac{5}{4}}} = \dfrac{600 \times \sqrt{19{,}000}}{2{,}800^{\frac{5}{4}}} = \mathbf{4{\cdot}06}$ *Answer*

∴ This turbine is of Pelton type, using a slow runner.

b) $K_s = \dfrac{Q \cdot N^2}{v^3}$ where $v = \sqrt{2g\,H}$

$$= \frac{67 \times 600^2}{(\sqrt{2 \times 32{\cdot}2 \times \sqrt{2{,}800}})^3} = \mathbf{0{\cdot}315} \quad \textit{Answer}$$

5.17 Relation between N_s and K_s—

a) **In FPS Units—**

$$N_s = \frac{N \cdot \sqrt{P_t}}{H^{\frac{5}{4}}} = \frac{N}{\sqrt{H}} \cdot \sqrt{\frac{P_t}{H \cdot \sqrt{H}}}$$

where $P_t = \dfrac{w \cdot Q \cdot H}{550} \cdot \eta_t$

$$\therefore \quad N_s = \frac{N}{\sqrt{H}} \cdot \sqrt{\frac{w \cdot Q \cdot H \cdot \eta_t}{550 \cdot H \cdot \sqrt{H}}}$$

$$= 0{\cdot}336 \cdot \frac{N}{\sqrt{H}} \cdot \sqrt{\frac{Q}{\sqrt{H}}} \cdot \sqrt{\eta_t}$$

Dividing both sides by $\sqrt{2g} \cdot \sqrt[4]{2g}$

$$\frac{N_s}{\sqrt{2g} \cdot \sqrt[4]{2g}}=0{\cdot}336 \sqrt{\eta_t} \cdot \frac{N}{\sqrt{2gH}} \cdot \sqrt{\frac{Q}{\sqrt{2gH}}}$$

but $\sqrt{2gH}=v$

$$\therefore \frac{N_s}{\sqrt{2g} \cdot \sqrt[4]{2g}}=0{\cdot}336 \sqrt{\eta_t} \cdot \frac{N}{v} \cdot \sqrt{\frac{Q}{v}}$$

or $$N_s=7{\cdot}62 \cdot \sqrt{\eta_t} \cdot \frac{N \cdot \sqrt{Q}}{v^{\frac{3}{2}}}$$

but $$K_s=\frac{Q \cdot N^2}{v^3}$$

$$\therefore \quad N_s=7{\cdot}62 \cdot \sqrt{\eta_t} \cdot \sqrt{K_s}$$

or $$K_s=\left(\frac{N_s}{7{\cdot}62 \cdot \sqrt{\eta_t}}\right)^2=\left(\frac{N_s}{7{\cdot}62}\right)^2 \cdot \frac{1}{\eta_t} \qquad \text{...(5.3)}$$

Assuming an average value of $\eta_t=0{\cdot}9$ and substituting in the above equation

$$K_s=\left(\frac{N_s}{7{\cdot}62}\right)^2 \times \frac{1}{0{\cdot}9}=\left(\frac{N_s}{7{\cdot}25}\right)^2 \text{ or } K_s=\frac{N_s^2}{52{\cdot}5} \qquad \text{...(5.3}a)$$

Apparently N_s also seems to be dimensionless from the above equation, but it must be borne in mind that value 52·5 includes factors such as g etc.

b) **In Metric Units**—

$$N_s=\frac{N \cdot \sqrt{P_t}}{H^{\frac{5}{4}}}=\frac{N}{\sqrt{H}} \sqrt{\frac{P_t}{H \cdot \sqrt{H}}}$$

where $$P_t=\frac{w \cdot Q \cdot H}{75} \cdot \eta_t$$

$$\therefore \quad N_s=\frac{N}{\sqrt{H}} \cdot \sqrt{\frac{w \cdot Q \cdot H \cdot \eta_t}{75\, H \cdot \sqrt{H}}}$$

$$=3{\cdot}65 \frac{N}{\sqrt{H}} \cdot \sqrt{\frac{Q}{\sqrt{H}}} \cdot \sqrt{\eta_t}$$

Dividing both sides by $\sqrt{2g} \cdot \sqrt[4]{2g}$

$$\frac{N_s}{\sqrt{2g} \cdot \sqrt[4]{2g}}=3{\cdot}65 \sqrt{\eta_t} \cdot \frac{N}{\sqrt{2gH}} \cdot \sqrt{\frac{Q}{\sqrt{2gH}}}$$

but $\sqrt{2gH}=v$

$$\therefore \quad \frac{N_s}{\sqrt{2g} \cdot \sqrt[4]{2g}}=3{\cdot}65 \sqrt{\eta_t} \cdot \frac{N}{v} \cdot \sqrt{\frac{Q}{v}}$$

$$N_s=34\tfrac{1}{2}\sqrt{\eta_t} \cdot \frac{N \cdot \sqrt{Q}}{v^{\frac{3}{2}}}$$

but $$K_s = Q \; N^2 \;\; {}^3$$

$$\therefore \qquad N_s = 34 \sqrt{\eta_t} \cdot \sqrt{K_s}$$

or $$K_s = \left(\frac{N_s}{34 \cdot \sqrt{\eta_t}}\right)^2 = \left(\frac{N_s}{34}\right)^2 \cdot \frac{1}{\eta_t} \qquad (5\ 4)$$

Assuming average value of $\eta_t = 0\ 9$ and substituting in the above equation

$$K_s = \left(\frac{N_s}{34}\right)^2 \times \frac{1}{0{\cdot}9} = \left(\frac{N_s}{32{\cdot}2}\right)^2$$

or $$K_s = \frac{N_s^2}{1{,}040} \qquad (5.4a)$$

Problem 5 4 It is proposed to develop 2,000 HP at a site where 480 ft of head is available. What type of turbine—impulse or a low, medium, or high head reaction turbine would be employed if it had to run at 300 rpm? If the same turbine is now used under a head of 100 ft, find the power developed and its rpm.

Solution

$P_t = 2{,}000$ HP $\qquad H = 480$ ft $\qquad N = 300$ rpm

a) Specific speed of the turbine

$$N_s = \frac{N \sqrt{P_t}}{H^{\frac{5}{4}}} = \frac{300 \times \sqrt{2{,}000}}{480^{\frac{5}{4}}} = 5\ 99$$

The specific speed of a Pelton (impulse) type turbine ranges from 2 5 to 8 0, therefore Pelton turbine would be selected for the above project and according to Table 5 2 a normal runner would be employed.

b) According to Art 3.13(*iii*) $$\frac{P_{t_1}}{P_{t_2}} = \left(\frac{H_1}{H_2}\right)^{\frac{3}{2}}$$

or $$P_{t_2} = P_{t_1} \left(\frac{H_2}{H_1}\right)^{\frac{3}{2}}$$

$$\therefore \qquad P_{t_2} = 2{,}000 \times \left(\frac{100}{482}\right)^{\frac{3}{2}} = \mathbf{190\ HP} \qquad \textit{Answer}$$

c) $$\frac{N_2}{N_1} = \left(\frac{H_2}{H_1}\right)^{\frac{1}{2}}$$

or $$N_2 = N_1 \cdot \left(\frac{H_2}{H_1}\right)^{\frac{1}{2}} \qquad \text{...(See Eqn 3.31)}$$

$$\therefore \qquad N_2 = 300 \times \left(\frac{100}{480}\right)^{\frac{1}{2}} = \mathbf{137\ rpm} \qquad \textit{Answer}$$

Problem 5 5 An impulse turbine develops 2,500 HP under a head of 220 ft What could be the maximum and minimum speeds of the turbine with a single nozzle? What speed would be the best for coupling it to an alternator? How high a speed could a reaction turbine give?

Solution

$P_t = 2{,}500$ HP $\qquad H = 220$ ft

The specific speed of a turbine is $N_s = \dfrac{N . \sqrt{P_t}}{H^{\frac{5}{4}}}$

The range of the specific speed for the impulse (Pelton) turbine is 2·5 to 8 0

∴ Minimum speed of the impulse turbine is given by

$$2\,5 = \frac{N \sqrt{2{,}500}}{220^{\frac{5}{4}}} \text{ or } N = \frac{(220 \times 3\,85) \times 2\,5}{50} = \mathbf{42\,3\ rpm} \quad \textit{Answer}$$

Maximum speed of impulse turbine :

$$N = \frac{H^{\frac{5}{4}} . N_s}{\sqrt{P_t}} = \frac{(220 \times 3\,85) \times 8}{50} = \mathbf{135\ rpm} \qquad \textit{Answer}$$

The best synchronous speed for 50 cycles frequency can be **125 rpm**

b) The maximum speed which a reaction turbine of Francis type could give :

$$N = \frac{(220 \times 3{\cdot}85) \times 67\,5}{50} \quad \text{...(as 67·5 is the maximum specific speed)}$$

$= \mathbf{1{,}142\ rpm}$ *Answer*

The maximum speed which could be obtained by a Kaplan turbine under the same conditions :

$$N = \frac{(220 \times 3\,85 \times 225)}{50} = \mathbf{3{,}812\ rpm} \qquad \textit{Answer}$$

Problem 5 6 A horizontal Francis turbine has the following specifications—

$H = 98$ m $\qquad Q = 10{\cdot}8$ m^3/sec

Find the minimum synchronous speed of the turbine, if its efficiency is 87%.

Solution

Brake Horse Power of the turbine $P_t = \dfrac{w . Q . H}{75} . \eta_t$

$$P_t = \frac{1{,}000 \times 10\,8 \times 98}{75} \times 0{\cdot}87$$

$= 1{,}228$ HP (metric)

The minimum and maximum specific speeds for a Francis turbine are 60 and 300 (metric units) respectively

∴ Minimum speed of Francis turbine—

$$N_s = \frac{N \cdot \sqrt{P_t}}{H^{\frac{5}{4}}} \quad \text{or} \quad 60 = \frac{N \cdot \sqrt{1,228}}{98^{\frac{5}{4}}}$$

or $$N = \frac{60 \times (98 \times 3 \cdot 15)}{35} = 529 \text{ rpm}$$

The nearest synchronous speed for 50 cycles per second is **600 rpm** *Answer*

UNSOLVED PROBLEMS

5.1 What are "Multi-Purpose Projects" ? Give examples.

5.2 Define head and discharge. What is the difference between the gross head and net head ?

5.3 What are the essential components of hydro-electric power plant ?

5.4 Define "Head Works". For what purpose are they installed ?

5.5 Name the different types of gates and valves.

5.6 What are the trashracks ? State their uses. How are the debris removed from the trashracks ?

5.7 What are water ways ?

5.8 Classify Water Power Plants. Why is it difficult to classify according to heads under which they work ?

5.9 Describe with the help of sketch,

a) High head water power plant, *b*) Canal water power plant.

5.10 Write an essay on the importance of high head and low head hydro-electric power projects. (*Punjab University—Sept 1954*)

5.11 Illustrate diagrammatically a general layout for a water power scheme comprising a Francis turbine supplied by a penstock and discharging through its draught tube into tail race. Indicate the effective head on turbine, after taking into account the losses upto the turbine intake and the final velocity in the tail race. (*Punjab University—1948 Annual*)

5.12 What means are provided to save the penstock from water hammer, when the gates or valves of a turbine are quickly closed ?

5.13 What is impulse turbine and why is it so named ?

5.14 Name the different water turbines which are used now-a-days. How are these turbines classified ?

5.15 What is the difference between the impulse and the reaction water turbines ?

5.16 Are Pelton turbines mostly vertical or horizontal and why ?

5.17 Is the modern Francis turbine a purely radial flow turbine ?

5 18 Define specific speed of a turbine. How does this figure help in the design of a turbine ? *(AMIE—Nov 1953)*

5 19 Determine the specific speed and the type of each of the following water turbines. State the type of turbine runner used in each case :—

a) *Tilaiya* Power House (D.V.C.) having two turbines, each developing 2,800 BHP, when running at 250 rpm under a maximum head of 77 ft.

b) *Pathri* Hydro Station on the Ganga Canal in U P. equipped with three Water Turbines, manufactured by Voith (Germany), each developing 9,500 BHP, when running at 125 rpm under a net working head of 32 ft.

c) *Reisseck* (Austria) Power Station having two turbines manufactured by Charmilles (Switzerland), each rated at 31,000 BHP, when running at 750 rpm, under an effective head of 5,800 ft, the highest head so far used to develop power any where in the world upto date.

d) *Aluminium* Co Power Plant (Canada) having two turbines, manufactured by Dominion (Canada), each rated at 140,000 BHP, when running at 327·5 rpm, under a rated head of 2,485 ft.

[*a*) 58·2, Francis, fast *b*) 160, Kaplan *c*) 2·61, Pelton, Slow *d*) 6·98, Pelton, fast].

5.20 It is proposed to build a turbine to operate at 106 rpm and develop 13,500 HP under a head of 317 ft. What type of turbine would you suggest ? (Multi-nozzle, Pelton Turbine or Turgo Impulse Turbine)

5.21 Can a Pelton wheel with single nozzle be obtained to utilize 10 cfs under a head of 225 ft, and run at a speed of 600 rpm ? If not, what may be done ?

(No, either two nozzle with one wheel or two units each with a single nozzle may be used).

5 22 The Pelton turbine at Seera de Cubatao (Brazil) is a double over-hung, two nozzle, horizontal type and develops, 85,000 HP under a head of 710 m Determine its specific speed in metric units, if it runs at a synchronous speed of 50 cycles per second.

$$\left[\textit{Hint}: \; P_t \text{ per runner per nozzle} = \frac{85{,}000}{4} = 21{,}250 \text{ HP} \right]$$

(250 rpm)

CHAPTER 6

PELTON TURBINE

6.1 Development of Pelton Turbine—In Girard Turbine the water was directed through the guide vanes to the runner vanes as shown in Fig 4.9 of Jonval Turbine. As already stated in Art 4.12, the difference between the Girard Turbine and the Jonval Turbine was that in the former type the guide vanes did not cover the whole circumference of the runner as in the latter type. The reason being the Girard turbine was an impulse turbine and water used to strike the runner under atmospheric pressure, while the Jonval turbine was a reaction turbine in which the runner was full of water and worked under pressure more than atmospheric. When applied to high heads, the Girard Turbine was supplied by a single nozzle regulated by a slide covering the rectangular orifice. The large-sized Girard turbines were horizontal-shaft type. This was the development of *tangential wheel*, which helped by the construction of Poncelet (undershot) water wheel (See Fig 5.4), appeared in the market as "*Spoon Wheel*" or "*Hurdy-Gurdy Wheel*," having spoon-shaped blades or buckets runner. This wheel was run by impinging jet from a rectangular orifice placed outside the runner.

In 1880 Pelton discovered the importance of split elliptical bucket. This idea was put into practice and the runner was patented as "Pelton Wheel". Such wheels were used in the mining industry of California.

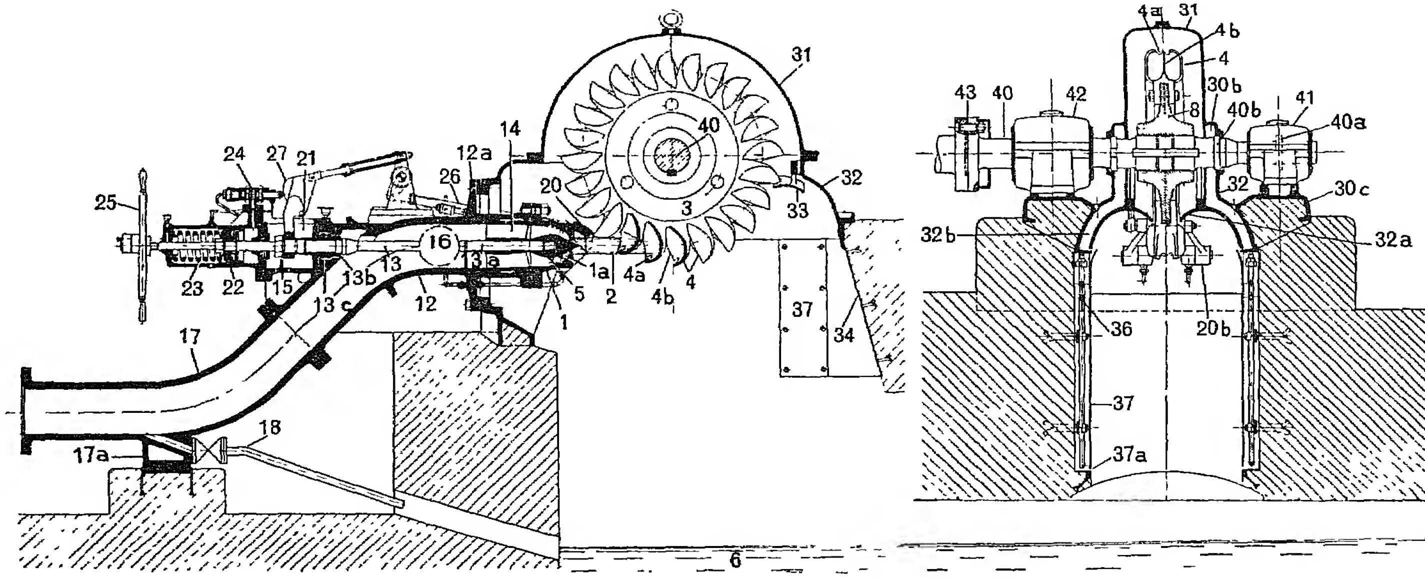


Fig 6.1 Pelton Turbine (Manufactured by Messrs Escher Wyss & Co Ltd, Zurich-Switzerland)

List of Parts

1 Nozzle 1a Inset piece 1b Holder 1c Eyes 1d Protecting roof 2 Water jet 3 Runner wheel 4 Runner Buckets 4a Shaped-out portion of bucket 4b Centre partition 5 Spear head 5a Spear Point 5c Stud bolt 5d Key 5e Spear body 5f 5g Cylindrical holes with pins 6 Tail race D_1 Runner diameter d_0 Jet diameter 7 Lugs of bucket 8 Runner disc 8a Runner boss 9 Fixing bolts with nuts 9a and 9b Bolt head 10 Tensioning bolts 11 Radial keys 12 Inlet bend 12a Flange 13 Spear rod 13a Bronze sleeve 13b Balancing piston 13c Leather packing 14 Guide cross 14a Guide ribs 14b Cylindrical member for foregoing 15 Support for spear rod 16 Cleaning hole 17 Lower bend 17a Anchoring foot 18 Emptying pipe 19 Turbine valve 20 Jet deflector 20a Intercepting piece 20b Levers 21 Relay lever 22 Servomotor piston for spear adjustment 23 Closing spring for spear 24 Distributing valve for nozzle 25 Handwheel for spear adjustment 26 Deflector rod 27 Cam 30 Casing 30a Opening for the inlet bend 30b Lateral chambers 30c Cast-iron bearing supports 31 Casing hood 32 Lower part of casing 32a Guide walls 32b Drainage channels for spray water from turbine shaft 33 Spray water catcher 34 Lining 36 Cooling coils 37 Cover for protection of latter 37a Hole for discharging cooling water 40 Turbine shaft 40a Shaft collar 40b Ring 41 External turbine bearing 42 Internal turbine bearing 43 Shaft coupling 44 Speed governor 45 Flywheel.

Later Pelton improved upon his invention such that the buckets were designed with divergent sides with which the water escaping smoothly utilized its full reactionary force in addition to the direct force due to the momentum of the impinging jet. The water jet after impinging on the buckets is deflected through an angle of about 165°, instead of 180°, so that it may not strike the back of the incoming bucket and retard the motion of the wheel.

6.2 **Modern Pelton Turbine**—Pelton turbine, also called free jet turbine, operates under a high head of water and, therefore, requires a comparatively less quantity of water. Water is conveyed in penstocks from the head race in the mountains to the turbine in the power house. The penstock is joined to a branch pipe or lower bend fitted with a nozzle at the end. Water comes out of the nozzle in the form of a free and compact jet. The number of nozzles required for a turbine depends on its specific speed. All the pressure energy of water is converted into velocity head. The water having a high velocity is made to impinge, in air, on buckets fixed round the circumference of a wheel, the latter being mounted on a shaft (See Fig 6.1). The impact of water on the surface of the bucket produces a force which causes the wheel to rotate, thus, supplying a torque or mechanical power on the shaft. The jet of water strikes the double hemispherical cup-shaped buckets at the centre and is deviated on both sides, thus eliminating an end thrust (see Art 1.5). After performing work on the buckets water is discharged into the tail race. The wheel must be so located that the buckets do not splash into the tail race water when the wheel revolves.

Pelton Turbine Main Components

6.3 **Guide Mechanism**—This mechanism controls the quantity of water passing through the nozzle and striking the buckets, thus meeting the variable demand for power. It maintains the speed of the wheel constant even when the head varies. The mechanism essentially consists of a spear fixed to the end of a shaft which is operated by governor (See Fig 6.1). When the speed of the wheel increases, the spear is pushed into the nozzle thereby reducing the quantity of water striking the buckets. If the speed of the wheel falls, the spear is drawn back allowing a greater quantity of water to pass through the nozzle. Sometimes a sudden decrease in load takes place. Consequently water requirement of the turbine suddenly falls. This necessitates the immediate closure of the nozzle with the spear, which may cause the pipe to burst due to the sudden increase in the pressure of water. In order to avoid such an occurrence, an additional nozzle is used. Water can pass through this nozzle, without striking the buckets. This nozzle is called the *Bypass Nozzle* and is opened when the main nozzle is being closed on a reduction in load taking place.

The modern practice is to provide the guide mechanism with a *deflector* (See Fig 6.1 and 6.2). The deflector is a plate connected to the spear rod by means of levers 20*b* (See Fig 6.1) the spear rod 5 (See Fig 6.2) being operated by the governor. The plate is located between the nozzle and the buckets. When a sudden reduction in load takes place, the governor brings the deflector in front of the buckets thus deflecting the water jet and preventing it from striking the buckets.

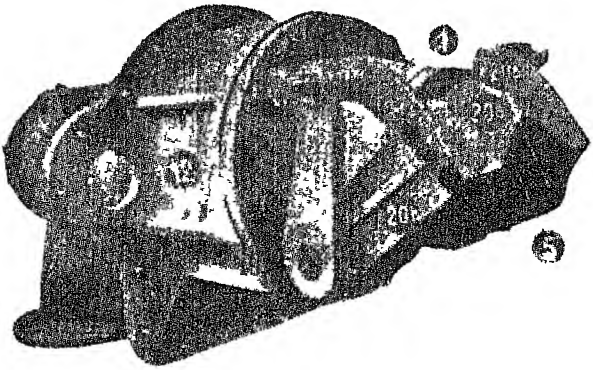


Fig 6.2 Deflector (Manufactured by Escher Wyss)

English Electric Co have provided some of the Pelton turbines with a *Seewer Jet Diffusor* (See Fig 6 3), which disperses or diffuses the jet by means of a number of slightly slanting flat blades surrounding the needle and forming a part of spear head. With the sudden fall in load,

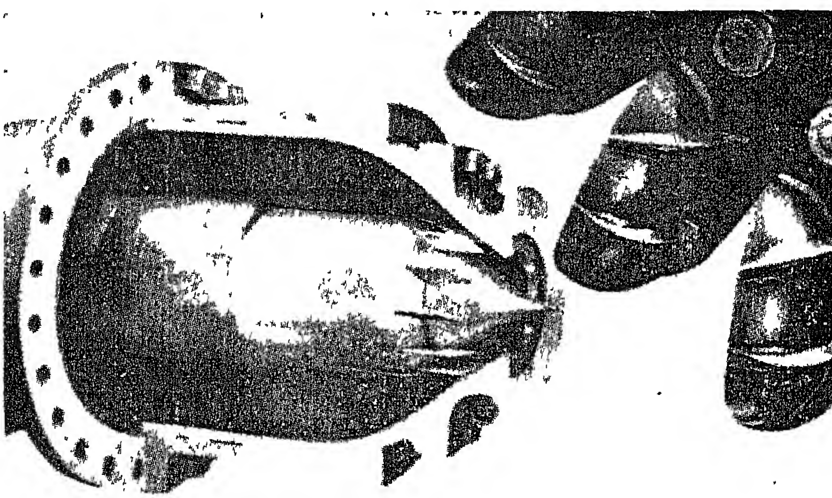


Fig 6 3 Seewer Jet Diffusor (Manufactured by English Electric Co.)

the blades instantly go forward into the stream, causing it to whirl and converting the compact jet into a hollow cone broken into spray, thus destroying the energy by diffusion. For the normal operation, the blades are withdrawn and their ends flushed with the cover, leaving a clear passage for the water to go out through the nozzle.

6.4 **Buckets and Runner**—Each bucket is divided vertically into two parts by a sharp edge at the centre, thus having the shape of a double hemispherical cup (See Fig 6.4). The sharp edge helps the jet to divide, without shock, into two parts moving side-ways in opposite directions (See Fig 6.5). The rear of the buckets should be so shaped as not to interfere with the passage of water to the bucket preceding in order of

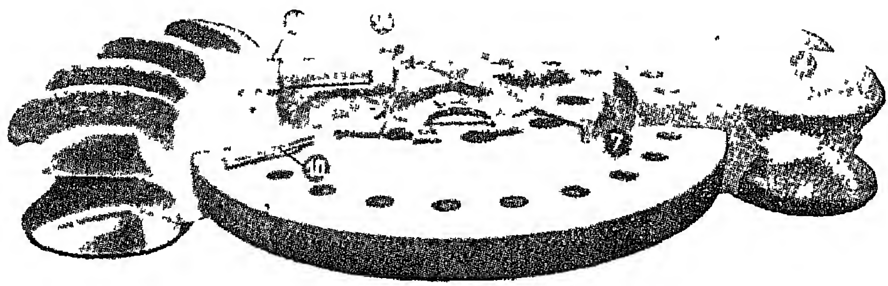


Fig 6.4 Buckets and Runner (Manufactured by Escher Wyss)

rotation. The jet should be deflected backwards when leaving the buckets. The angle of deflection being about 160°. The buckets form the most important part of the turbine and should be designed to withstand the full force of the jet when the turbine is shut off. It is important to select a suitable material for the buckets, so that they should not crack under the considerable force of the jet. Cast Iron is used for low heads but for higher heads bronze or, better still, stainless steel is used. The

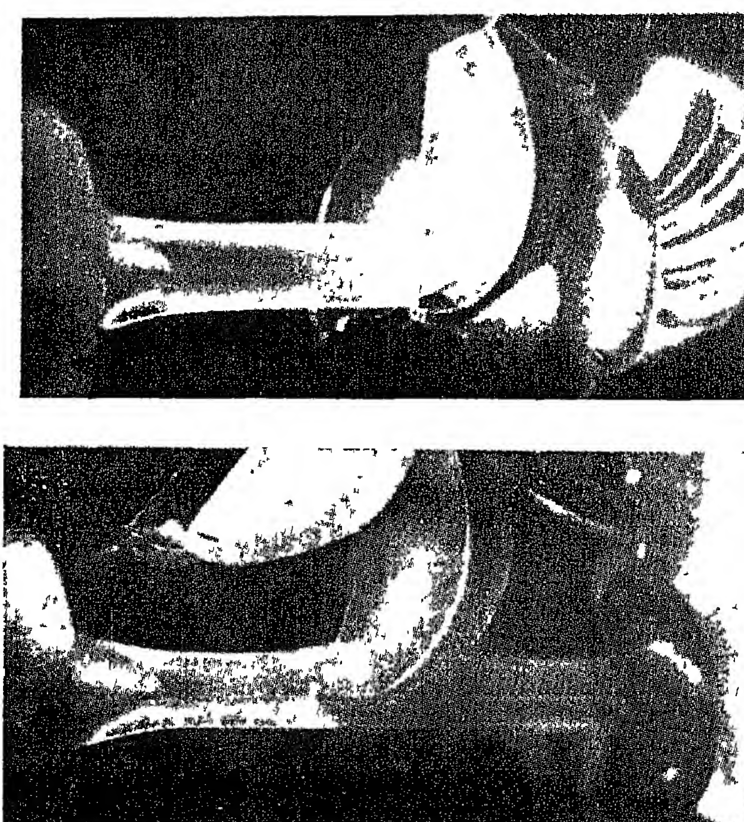


Fig 6.5 Jet Impinging on Pelton Buckets (Escher Wyss)

buckets should be properly polished in order to avoid undue stresses under the action of the jet, which may lead to cracking at sections which are incorrectly shaped.

The buckets are bolted on to the circumference of a round disc forming the runner of the Pelton turbine (See Fig 6 6). The buckets and the disc can be cast as a single unit which will be more economical. Many turbine manufacturers believe that all the buckets wear equally in a given time. But it is found in practice that buckets may break

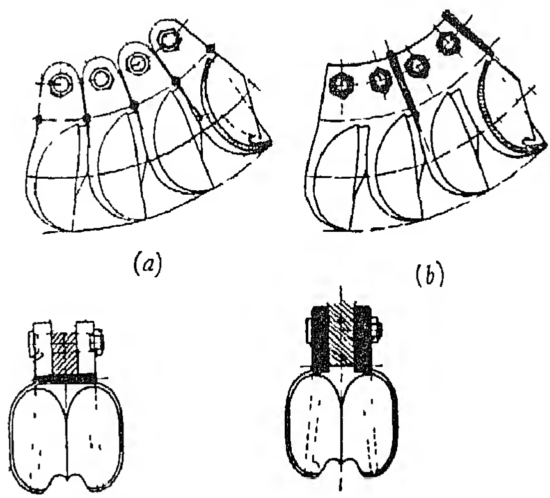


Fig 6 6 Methods of Bucket Attachment (Escher Wyss)
a) Single Bucket
b) Pair of Buckets

down one at a time. It is easy to replace a broken bucket unless the runner is cast as a single unit. The runner is made of Cast Iron, Cast Steel or Stainless Steel. Cast Iron is used to reduce cost in turbines designed for low heads.

6 5 Casing—As already mentioned in Art 5.12 the casing of a Pelton turbine has no hydraulic function to perform. It is necessary only to prevent splashing and to lead the water to the tail race, and also as a safeguard against accidents.

The casing is made of cast or fabricated parts. It has to take a force of the jet projecting beyond the runner in the event of overspread. In horizontal shaft units, the casing is split into bed plate and cover, so that the runner can be lowered into place or lifted out by a crane. The lowest part of the bed plate is embedded and anchored in the lower house floor. The escape of water along the shaft is prevented by a seal. Baffles (See Fig 6.1, No 33) are arranged to reduce windage loss or to protect the runner and jet from interference by splash.

6.6 Hydraulic Brake—After shutting down the inlet valve of turbine, the large capacity runner will go on revolving for a considerable period, due to its inertia. This has necessitated the development of a brake to bring the turbine to a standstill in the shortest possible time. The brake consists of a small nozzle fitted in such a way that on being

opened, it directs a jet on the back of the buckets to bring the revolving runner quickly to rest (See Fig 6.7*a*). The least diameter of the brake jet has been found to be equal to 0·6 times* the least diameter of the main jet.

6.7 Different Layouts of Pelton Turbine—

a) **Arrangements of Jets**—Most of the Pelton turbines have a single jet and a horizontal shaft, as shown in Fig 6.7*a*. However the number of the jets depends upon the specific speed. It is, therefore, possible that more than one jet may be employed. Fig 6.7*b* and *c* show the arrangements of double jet with horizontal shaft and four jets with vertical shaft respectively.

b) **Arrangements of Runner**—The runner of the turbine as well as the rotor of the generator, to be driven by the turbine, are keyed on the same shaft. The rotor of the generator is generally heavier than the Pelton runner, and is, therefore, supported in two bearings while the turbine runner is keyed on the length of the shaft overhanging beyond one of the bearings. This is known as *overhung* arrangement of runner (Fig 6.8*a*).

If the turbine is designed for greater power or higher speed, two turbine runners are installed. They may be arranged together on one side of the generator and each of them having its own bearing as shown in Fig 6.8*b* ; or one on each of the projecting ends of the shaft (Fig 6.8*c*). The latter arrangement is known as *double overhung* type.

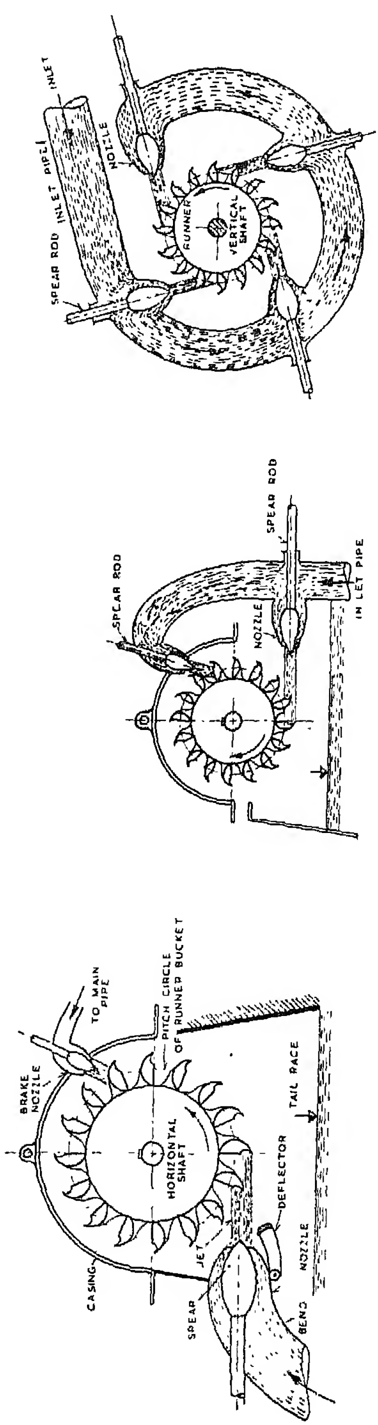
6.8 Notable Pelton Turbine Installations of the World—The largest Pelton turbine of the world till today is at Cimego (Italy) producing 150,000 BHP under a head of 2,365 ft when running at 300 rpm. It is a double overhung type, having one jet per runner.

The next largest Pelton turbine, manufactured by Dominion (Canada), is installed at Kemano (Canada) in an underground power station, meant for aluminium reduction works at Kitimat. This is a vertical-shaft, four-jet turbine producing 140,000 BHP at 2,485 ft net head and 327·5 rpm.

The highest head used for a Pelton turbine anywhere in the world up-to-date is 5,792 ft net. This turbine is installed at Reisseck (Austria) producing 33,800 BHP at 750 rpm. It is manufactured by Charmilles (Switzerland). The next turbine using largest head is installed at Chandoline (Dixence), Switzerland. It delivers 50,000 BHP with two runners, when working under 1748 m($\approx$5709 ft) gross head and running at 500 rpm. It has a discharge of 3·1 to 10·25 m^3/sec and net head of 1622 m.

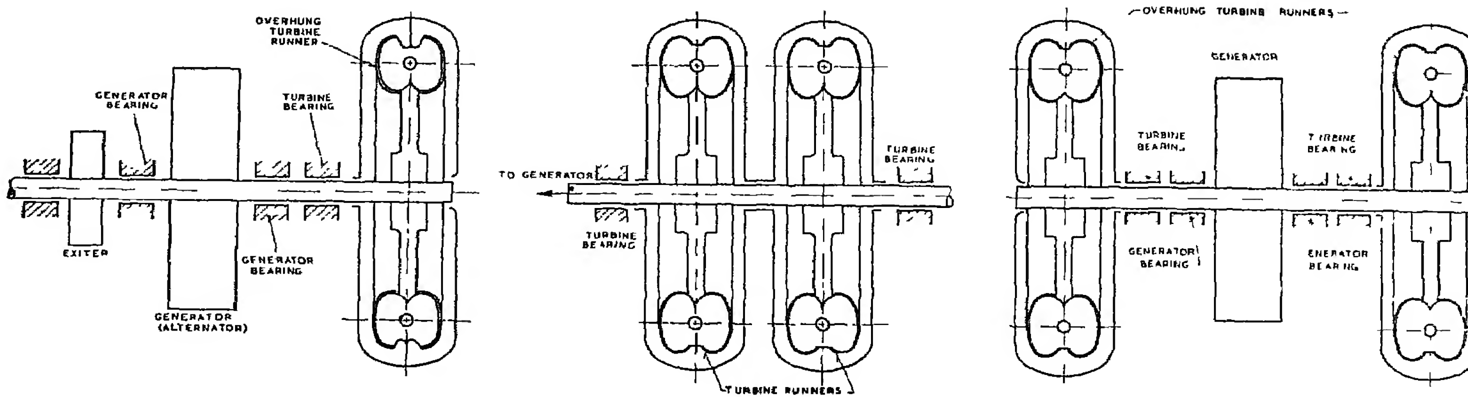
At Seera de Cubatao (Brazil), a double overhung horizontal turbine, developing 85,000 HP (*i.e* 42,500 HP per wheel per jet) under a head of 710 m($=$2,330 ft) has been installed by Voith, Heidenheim (West Germany). The turbine runner has a mean diameter of 3·5 m($\approx 11\frac{1}{2}$ ft) and has 21 buckets, each bucket weighing 420 kg ($\approx$925 lb). Eight such turbines have been installed in the above power house.

*See "Characteristics of Free Jet Water Turbine in Working and Brake Regions" (in German), by Dr Jagdish Lal, Published by Springer-Verlag, Vienna, 1952.



a) Single-Jet, Horizontal Shaft Pelton Turbine *b*) Double-Jet, Horizontal Shaft Pelton Turbine *c*) Four-Jet, Vertical Shaft Pelton Turbine

Fig 6.7 Arrangements of Jets for Pelton Turbine



a) Overhung Runner shown with Generator and Exciter with their Bearings (Most Common Arrangement)

b) Double-Runner Arrangement

c) Double Overhung Runner Arrangement

Fig 6.8 Arrangements of Runners for Pelton Turbine

6.9 Pelton Turbine Installations in India—

The Tata Hydro-Electric Companies Ltd, Bombay: Under the Tatas Management there are three firms, namely

a) The Andhra Valley Hydro-Electric Co Ltd,

b) The Tata Hydro-Electric Co Ltd,

and c) Tata Power Co Ltd,

a) **The Andhra Valley Hydro-Electric Co Ltd**, which came into operation in 1921 has got its Power House situated at Bhivpuri. Local trains connect Bombay Victoria Terminus (V T) to Bhivpuri Road (58 miles). The Power House is 9 miles from the Bhivpuri Road Railway Station and the only conveyance to reach the destination is the petrol driven trolley run by the Company on the broad gauge.

The Power House is fed by a 14 miles long × 1 mile wide Andhra Lake made up from a Valley by erecting 1,875 ft long dam at Tokarwadi to block Andha river. The average rainfall in this area is 99 inches per year.

Penstocks—Eight penstocks of which six are installed emerge from a three miles tunnel connecting the Andhra Lake. Each penstock has a diameter of 42 inches which reduces to 32 inches at the turbine inlet.

Turbines—There are six identical Pelton Turbines with governors manufactured by Pelton Water Wheel Co Ltd San Francisco, each designed for—

Power = 15,000 BHP

Head = Max 1,740 ft and Min 1,660 ft

Speed = 300 RPM

$\eta_t = 83\%$

Runner Dimensions: $D_1 = 9$ ft$-$2 in., $D_{ext} = 12$ ft$-$7 in.

$d_o = 9\frac{1}{2}$ in, $z_2 = 18$

Each turbine is equipped with an auxiliary relief (i e. bypass) nozzle.

Generators: There are six identical Alternators manufactured by G E. (USA), each developing 10,000 KVA at 5,000 V at 50 cycles frequency. The exciter for each of the alternators is driven by a small separate Pelton turbine.

b) **The Tata Hydro-Electric Co Ltd,** which came into operation in 1915 has its Power House situated at Khapoli (Bombay-Poona main Road). This place is also connected by rail through a branch line from Karjat (Bombay-Poona section), 63 miles from Bombay.

The Power House is fed from three lakes namely Shirawta, Walwhan and Lonavla. The first two are joined by a tunnel and the water flows from the former towards latter. From Walwhan Lake, a 22,840 ft long duct line carries water to a forebay at Khandalla. Another 1,595 ft long duct joins Lonavala Lake to the main duct. The average rainfall in this area is 163 inches a year.

Penstocks—Khandalla Forebay connects the Power House firstly by two pipes each 8,196 ft long having a dia of $82\frac{1}{2}$ inches reducing to 72

inches and then by six pipes each 4,479 ft long having a dia of $42\frac{3}{4}$ inches reducing to 38 inches at the turbine inlet.

Turbines : The Power House consists of six Pelton Turbines, five of which are manufactured by Escher Wyss & Co Ltd, Zurich and the sixth by the English Electric Co Ltd, Rugby. Five Escher Wyss Turbines are identical and are equipped with governors. Each of these turbines (new 1954 units) is designed for :

Power = 17,100 HP

Head = 513 m (1,685 ft),

Q = 2,940 lit/sec (104 cusecs),

Speed = 300 RPM

$\eta_t = 86\%$

Runner Dimensions : D_1 = 2,900 mm, D_{ext} = 3,615 mm

d_0 = 200 mm, $z_2 = 23$

s = 158 mm

Each turbine is equipped with a deflector.

The Pelton Turbine manufactured by English Electric Co has the following design data :

Power = 15,000 BHP, Head = 1,655 ft (net), 1,720 ft (max)

Speed = 300 RPM, $\eta_t = 84\%$

Runner Dimensions : $z_2 = 24$

The turbine is fitted with a diffusor (Seewer design) instead of deflector.

Generators : There are four Siemens and two G E. (USA) make Alternators, each developing 10,000 KVA, 5,000 V at 50 cycles frequency. The exciters for each of the alternators are separately run by small Pelton turbines.

c) **The Tata Power Co Ltd,** which came into operation in 1927 has its Power Station situated at Bhira which is approachable by the Company's trolley from Kolad 18 miles away. Kolad is 91 miles from Bombay and is connected by bus. Dharamtar which is 25 miles from Kolad also connects by bus. One can reach Dharamtar from Bombay by ferry. The nearest Railway Station for Bhira is Lonavla (Bombay-Poona line) about 54 miles from Kolad approachable only by Company's car.

The Power Station is fed by Mulshi Lake which is about 28 miles from Poona. The catchment area of the lake is 95·6 sq miles. Average rainfall is 100 inches to 250 inches a year at different places of lake.

Penstocks : An 8 miles long tunnel connects Mulshi Lake to two 112 ft long pipes of 82 inches diameters. These two pipes are then connected to five pipes each 3,760 ft long having a dia of $58\frac{1}{2}$ inches reducing to 51 inches towards bottom. Finally ten pipes each 2,050 ft long and of average 36 inches diameter connect the five pipes leading to the turbine-inlets.

Turbines : There are six Pelton Turbine sets of double overhung type, consisting of—

i) Five identical English Electric Pelton turbines (new units erected in 1954), each developing

Power $=25{,}500$ to $30{,}700$ BHP (by two runners),

Head $=1{,}440$ ft to $1{,}555$ ft

Speed $=375$ RPM

$\eta_t=86\%$

Runner Dimensions : $D_1=7$ ft-6 in., $d_o=10{\cdot}25$ in.

$s=5{\cdot}313$ in., $z_2=20$

Each turbine is equipped with Seewer type diffuser.

ii) One Pelton Turbine manufactured by Morgan Smith (USA) having the following data ;

Power $=35{,}500$ BHP (by two runners),

Head $=1{,}440$ ft (net)

Speed $=375$ RPM

$\eta_t=85\%$

Runner Dimensions : $D_1=7$ ft-3 in., $D_{ext}=9$ ft$-3\frac{1}{4}$ in.

$d_o=10{\cdot}07$ in , $z_2=20$

$s=9{\cdot}02$ in.

The turbine is equipped with deflector.

Generators : Five English Electric Alternators developing 21,875 KVA, 9,400 to 11,000 V at 50 cycles and one G E (USA) make. The exciters are run by smaller size Pelton turbines.

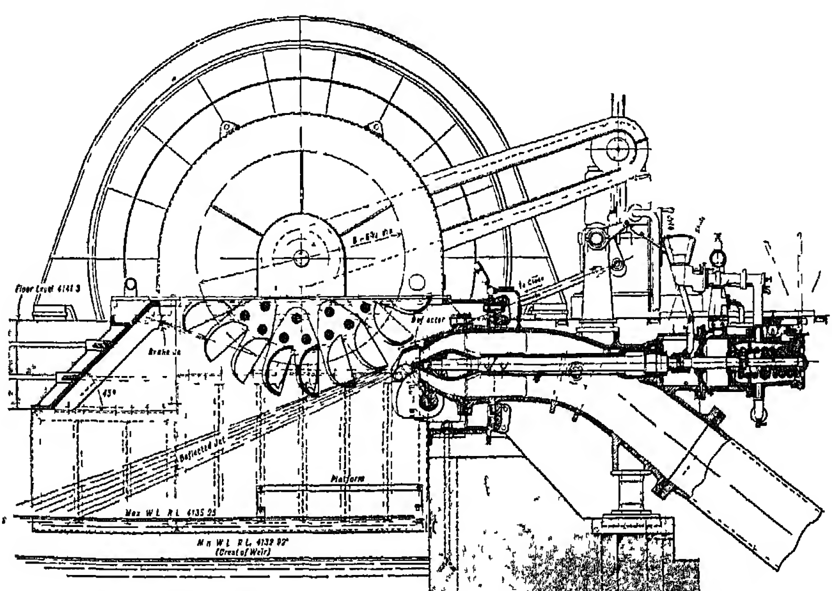


Fig 6.9 Pelton Turbine (17,000 HP, 1,668 ft head and 428·5 RPM) installed at Jogindar Nagar (Mandi Hydro-Electric Scheme). Supplied by Boving & Co Ltd, London ("The Engineer" dated 8-1-1932). (See Unsolved Prob 6,38)

SOME MORE PELTON TURBINE

Sl No.	Scheme	Site of Power House	Source of Water
1	Mandi Hydro-Electric Scheme (Fig 6 9) (P.W.D Punjab)	Joginder Nagar (Shanan)	Uhl River (Reservoir at Brot)
2	Simla Hydro-Electric Scheme (Simla Municipality)	Chaba	Nauti (tributary of Sutlaj River)
3	Nainital Hydro-Electric Scheme Kumaun Hills, U.P.	Nainital	Natural lake
4	Mussoorie Hydro-Electric Ltd (Mussoorie Municipality)	Mussoorie (6 miles from city)	Kiarkuli River
5	Shillong Hydro-Electric Ltd (Assam Govt.)	Shillong	Stream nearby
6	Darjeeling District Hydro-Electric Power Scheme (Darjeeling Municipality)	Sedrapong (4 miles from Darjeeling)	Mountain Streams
7	Govt of Mysore, Electrical Department	Cauvery	Cauvery Falls
8	The Satara Hydro-Electric Scheme	Satara	Pykara River
9	Pykara Hydro-Electric Scheme The Glen Morgan Scheme	Pykara	Pykara River
10	Mahatma Gandhi Hydro-Electric Works (Jogfalls, Mysore)	Sharavathi (on Bangalore Honnavar Road)	Sharavathi River
11	Pallivasal Power Station (Kerala Govt.)	Pallivasal (7 miles from Munnar)	Mudirapuzha River (Tributary of River Periyar)
12	Mahora Power Station Kashmir	Mahora (55 miles from Srinagar)	Jehlum River
13	Konya Hydro-Electric Project (Bombay State)	Konya	—
14	Kundah Hydro-Electric Project, Madras (Two Stages)	Kundah	Kundah River (Nilgiris)

INSTALLATIONS IN INDIA

Manufacturers or Suppliers	No of Turbines	Turbine Specifications				
		Power HP Each	Head ft	Water Quantity Cusecs, each	Speed rpm	Turbine efficiency per cent
Boving & Co.	4	17,000	1,668 (av) 2,001 (max)	100	428 5	88 6
Boving & Co.	3 2	375 590	540 540	7·7 12	— —	88 —
—	3	172	950	2 17	—	73 5
Boving & Co.	2	675	1,000	8 5	750	—
James Gordon	2	375	1,000	—	600	—
Voith, Germany	2	135	550	—	1,000	80
Escher Wyss	2	368	567	6 7	1,000	85
—	2	290	650	4 9	715	80
Escher Wyss	6	1,450	384	—	—	—
Voith	2	70	—	—	1,000	—
Voith	2	10,900	3,080	—	600	—
a) Escher Wyss	3	10,900	2,800	—	—	—
b) Escher Wyss	2	19,000	2,800	67	600	89 2
c) Escher Wyss	2	20,000	2,800	—	—	—
Escher Wyss	4 4	17,500 32,500	1,205 ft (gross)	— —	428 5 428 5	— —
a) Escher Wyss	3	6,000	1,880	—	750	—
b) Boving & Co.	3	10,600	1,880	—	600	—
A. Double (USA) (Two-nozzles, 16 buckets per runner)	4	800	400	27.6	500	—
Neyrpic (France)	4	87,000	1,560 (net)	—	300	—
Dominion Engg (Canada)	1	28,750 (5 jet)	1,187	—	—	—
	1	50,000 (5 jet)	2,470	—	—	—

Design of Component Parts of Pelton Turbine

6.10 Turbine Power—The power available from water can be calculated for a specified net head and rate of flow of water, as follows

$$P_a = \frac{w \cdot Q \cdot H}{550} \text{ (in FPS units) or} = \frac{w \cdot Q \cdot H}{75}$$

(in Metric units) .. (6.1)

where P_a=Power available in HP or metric HP,

w=Specific weight of water in lb/cu ft or kg/m_3,

Q=Rate of flow of water in cfs or m^3/sec

H=Net head in ft or m.

This formula gives the theoretical available power as the losses in the turbine are not taken into account. To obtain the power supplied on the turbine shaft, P_a is multiplied by the efficiency of the turbine, approximate values of which are given in Table 6.1.

TABLE 6.1

Practical Data for η_t

η_t in %	85	88	89	90
P_a in HP	100	1,000	10,000	1,00,000

$\therefore \quad P_t = P_a \cdot \eta_t$ ...(6.2)

where P_t=Power supplied by the turbine in BHP.

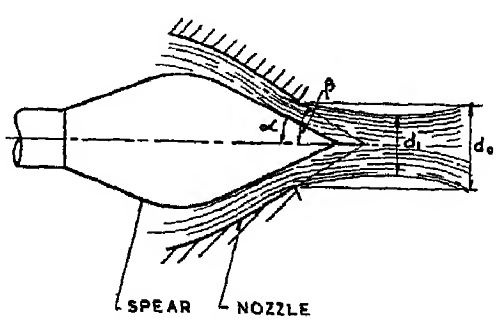


Fig 6.10 Spear, Nozzle and Jet

6.11 Nozzle and Jet Diameters—The least diameter of the free jet (Fig 6.10) which occurs at the vena contracta is given by the relation

$$Q = \frac{\pi}{4} \cdot d_1^2 \cdot v_1$$

where d_1=the least dia of the jet in ft (or in m),

v_1=velocity of jet in ft/sec (or in m/sec) measured at vena contracta

Q=discharge through jet in cusecs (or m^3/sec)

Now $v_1 = K_{v_1} \cdot \sqrt{2gH}$

K_{v_1}=co-efficient of velocity,

H=net head in ft (or in m)

Value of K_{v_1} varies from 0·98 to 0·99

or $$d_1=\sqrt{\frac{4Q}{\pi \cdot K_{v_1} \cdot \sqrt{2g} \cdot \sqrt{H}}} \qquad ...(6.3)$$

Substituting average value of $K_{v_1}=0{\cdot}985$,

a) Least diameter of jet in FPS units—

$$d_1=\sqrt{\frac{4}{\pi\times0{\cdot}985\times8{\cdot}02}} \cdot \sqrt{\frac{Q}{\sqrt{H}}}$$

$$=\sqrt{0{\cdot}161} \cdot \sqrt{Q_1}$$

$$=0{\cdot}401 \cdot \sqrt{Q_1} \qquad ..(6.3a)$$

b) Least diameter of jet in metric units—

$$d_1=\sqrt{\frac{4}{\pi\times0{\cdot}985\times\sqrt{2\times9{\cdot}81}}} \cdot \sqrt{\frac{Q}{\sqrt{H}}}$$

$$=0{\cdot}5415 \cdot \sqrt{Q_1} \qquad ...(6.3b)$$

Let diameter of the nozzle at the outlet be d_o (See Fig 6.10) which is always greater than d_1. It can be determined as follows—

$$Q=a_o \cdot v_o$$

where $$v_o=K_{v_o} \cdot \sqrt{2gH}$$

The experimental value of K_{v_o} is 0·81 to 0·83

$$\therefore \qquad a_o=\frac{Q}{K_{v_o} \quad \sqrt{2gH}} \qquad ...(6.4)$$

Value of a_o is obtained from the following experimental relation

$$a_o=A \cdot s-B \cdot s^2 \qquad ...(6.5)$$

Where A and B are constants for a particular nozzle and their values are as follow

$$A=2\pi \cdot r_o \cdot \sin\alpha \cdot \frac{\beta-\alpha}{\sin(\beta-\alpha)} \qquad ...(6.6)$$

$$B=\frac{\pi \cdot (\beta-\alpha)(\sin\beta-\sin\alpha) \cdot \sin^2\alpha}{\sin^2(\beta-\alpha)} \qquad ...(6.7)$$

Also s=maximum spear travel

$=0{\cdot}8\ d_o$

2α=angle of spear

and 2β=angle of nozzle $\quad$ (See Fig 6.10)

The dimensions of the nozzle and the branch pipe are generally given in terms of d_1 and can be determined when d_1 is known.

These values are calculated for

$\alpha=25°$

$\beta=42°$

These values of α and β are mostly used by European Continental Turbine Manufacturers. In UK, however, the values of α and β are taken as $22\frac{1}{2}°$ and $28\frac{1}{2}°$ respectively.

6.12 Multi-jets—The specific speed of a Pelton turbine is calculated for a single jet. Thus

$$N_s = \frac{N \cdot \sqrt{P_t}}{H^{\frac{5}{4}}}$$

Introducing dimensionless quantity K_s in place of N_s

$$K_s = \frac{Q \cdot N^2}{v^3}$$

or $$K_s = \left(\frac{N_s}{7{\cdot}25}\right)^2, \text{ assuming } \eta_t = 90\% \qquad \text{(See Eqn 5.3)}$$

Considering $N_{s_{max}}$ for a Pelton Turbine $\approx 7{\cdot}25$

$K_s = 1$ for a single jet.

If for a single wheel two nozzles and, therefore, two jets are provided, then total power $= 2P_t$

and specific speed $N_s = \dfrac{N \cdot \sqrt{2P_t}}{H^{\frac{5}{4}}} = N_s \cdot \sqrt{2}$

This shows that for specific speed greater than 7·25 a multi-nozzle Pelton turbine must be employed. Explaining in terms of dimensionless factor, for two nozzles K_s lies between 1 and 2, for three nozzles K_s lies between 2 and 3 and so on.

Maximum number of nozzles so far used in a turbine is six in a vertical installation and two in a horizontal installation.

Though multi-jets are essential when the specific speed of runner is more than 7·25, but such an arrangement makes the governing of turbine complicated and more expensive.

6.13 Mean Diameter of Pelton Runner—Let the mean diameter of Pelton runner be D_1 and the speed be N rpm when the net head is H.

Circumferential velocity of a bucket,

$$u_1 \propto \sqrt{2gH} = K_{u_1} \cdot \sqrt{2gH}$$

The velocity co-efficient K_{u_1} is known as *Speed Ratio, i.e.* the ratio of the peripheral or linear velocity of the buckets at their mean diameter to the theoretical spouting velocity of water under the head acting on the turbine,

or $$K_{u_1} = \frac{u}{\sqrt{2gH}}$$

Sometimes the speed ratio K_{u_1} is denoted by ϕ.

Circumferential velocity can also be given as

$$u_1 = \frac{\pi \cdot D_1 \cdot N}{60} \text{ ft/sec (or } m\text{/sec)}$$

The centre line of the jet is always a tangent to the wheel pitch circle drawn with diameter D_1, at its lowest point, as shown in Fig 6.11.

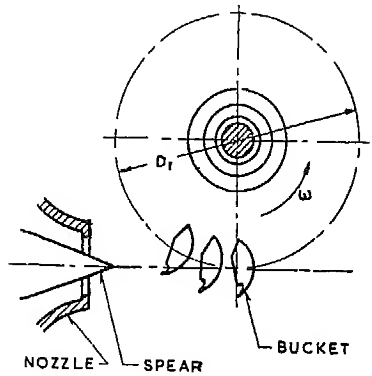


Fig 6.11 Mean Diameter of Pelton Runner

$$\text{Now } K_{u_1} \cdot \sqrt{2gH} = \frac{\pi \cdot D_1 \cdot N}{60}$$

$$\text{or } D_1 = \frac{60 \cdot K_{u_1} \cdot \sqrt{2gH}}{\pi \cdot N} \text{ ft (or } m) \quad (6.8)$$

$$= \frac{153 \cdot K_{u_1} \cdot \sqrt{H}}{N} \text{ ft}$$

(in FPS units) ...(6.8a)

$$= \frac{84{\cdot}5 \; K_{u_1} \cdot \sqrt{H}}{N} \; m \text{ (in Metric Units)} \quad \ldots(6.8b)$$

TABLE 6.2

Experimental Values for K_{u_1} Corresponding to Maximum Efficiency

N_s	2 25	...	...	7·5	(FPS Units)
N_s	10	...	...	32	(Metric Units)
K_{u_1}	0·46	...	...	0·44	

Taking an average value for $K_{u_1} = 0{\cdot}45$,

$$D_1 = 153 \times 0{\cdot}45 \times \frac{\sqrt{H}}{N} \text{ (in FPS Units)}$$

$$\left[\text{or } 84{\cdot}5 \times 0{\cdot}45 \times \frac{\sqrt{H}}{N} \text{ (in Metric Units)} \right]$$

$$= 68{\cdot}9 \cdot \frac{\sqrt{H}}{N} \text{ (in FPS units)} \quad \ldots(6.8a)$$

$$\left[\text{or } 38{\cdot}06 \; \frac{\sqrt{H}}{N} \text{ (in Metric Units)} \right]$$

Working speed, N (RPM), of the turbine depends on the nature of the driven unit. If an electrical generator (alternator) is directly coupled to the turbine, N will be determined by the relation

$$f = \frac{p \cdot N}{60} \quad \ldots(6.9)$$

where f is the frequency of the alternator in cycles/sec and p is the number of pair of poles.

If $f=50$, as is usually the case in this country, the speed

$$N=\frac{3,000}{p} \qquad \ldots(6.9a)$$

Cost of a high speed generator being less, it is economical to select a high speed wherever possible. Generally for a Pelton Turbine, the speed is 500, 375, 300 or 250 RPM, depending upon the size of the unit.

After N and K_{u_1} have been fixed, D_1 can be easily calculated from Eqn 6 8.

6 14 Jet Ratio of a Pelton Turbine—

Definition : Jet Ratio $=\dfrac{\text{Mean diameter of runner}}{\text{Least diameter of jet}}$

Symbolically, $m=\dfrac{D_1}{d_1}$

Jet ratio is an important feature of a Pelton turbine and influences its characteristics. It is, generally, used instead of specific speed in the selection and design of Pelton Turbines.

Relation between m and N_s :

$$N_s=\frac{N\,.\,\sqrt{P_t}}{H^{\frac{5}{4}}}$$

Where $N=\dfrac{60\,.\,K_{u_1}\,.\,\sqrt{2gH}}{\pi\,.\,D_1}$ RPM

a) **Now in FPS Units—**

$$P_t=\frac{w\,.\,Q\,.\,H}{550}\,.\,\eta_t \text{ HP}$$

But since $Q=\frac{\pi}{4}\,.\,d_1^2\,.\,K_{v_1}\,.\,\sqrt{2gH}$ cusecs

$$P_t=\frac{62{\cdot}4\times\frac{\pi}{4}\,.\,d_1^2\,.\,K_{v_1}\,.\,\sqrt{2gH}\,.\,H}{550}\,.\,\eta_t$$

$$\therefore\ N_s=\frac{\dfrac{60\,.\,K_{u_1}\,\sqrt{2g}\,.\,\sqrt{H}}{\pi\,.\,D_1}\times\sqrt{\dfrac{62{\cdot}4\times\pi\times\sqrt{2g}\,.\,K_{v_1}}{4\times550}\,.\,d_1^2\,.\,H^{\frac{3}{2}}\,.\,\eta_t}}{H^{\frac{5}{4}}}$$

$$=\frac{60\,.\,\sqrt{2g}\,.\,\sqrt[4]{2g}\,.\,\sqrt{62{\cdot}4}}{\sqrt{\pi}\,.\,\sqrt{4\times550}}\,.\,K_{u_1}\,.\,\sqrt{K_{v_1}}\,.\,\frac{d_1}{D_1}\,.\,\sqrt{\eta_t}$$

$$=129{\cdot}5\times K_{u_1}\,.\,\sqrt{K_{v_1}}\,.\,\frac{1}{m}\,.\,\sqrt{\eta_t} \qquad \ldots(6.10)$$

Assuming mean values,

$$K_{u_1}=0.45 \text{ and } K_{v_1}=0.985$$

$\therefore$ Specific speed $N_s=57.8\dfrac{\sqrt{\eta_t}}{m}$ .. (6.10a)

b) **In Metric Units—**

$$P_t=\frac{w \,.\, Q \,.\, H}{75}\,\eta_t \text{ (Metric) HP}$$

But since $Q=\dfrac{\pi}{4}\,d_1^2 \,.\, K_{v_1}\sqrt{2gH}$ m³/sec

$$P_t=\frac{1{,}000\times\frac{\pi}{4}\,d_1^2 \,.\, K_{v_1} \,.\, \sqrt{2gH} \,.\, H}{75}\,.\,\eta_t$$

$$N_s=\frac{\dfrac{60 \,.\, K_{u_1} \,.\, \sqrt{2g} \,.\, \sqrt{H}}{\pi \,.\, D_1}\times\sqrt{\dfrac{1{,}000\times\pi\times\sqrt{2g} \,.\, K_{v_1}}{4\times 75}\,d_1^2 \,.\, H^{\frac{3}{2}} \,.\, \eta_t}}{H^{\frac{5}{4}}}$$

$$=\frac{60 \,.\, \sqrt{2g} \,.\, \sqrt[4]{2g} \,.\, \sqrt{1{,}000}}{\sqrt{\pi} \,.\, \sqrt{4\times 75}} \,.\, K_{u_1} \,.\, \sqrt{K_{v_1}} \,.\, \frac{d_1}{D_1} \,.\, \sqrt{\eta_t}$$

$$=579 \,.\, K_{u_1} \,.\, \sqrt{K_{v_1}} \,.\, \frac{1}{m} \,.\, \sqrt{\eta_t}$$ ...(6.10b)

Assuming mean values,

$$K_{u_1}=0.45 \text{ and } K_{v_1}=0.985$$

$\therefore$ Specific speed $N_s=260\times\dfrac{\sqrt{\eta_t}}{m}$ ...(6.10c)

In modern turbines the value of jet ratio ranges from 10 to 30 depending on the specific speed and efficiency of the turbine.

TABLE 6.3

Experimental Values of *m*

m	6·5	7·5	10	20
N_s	7·75	7·2	5·4	2·25 (FPS units)
N_s	34·5	32	24	10 (Metric units)
η_t	0·82	0·86	0·89	0·9

For maximum efficiency m should be from 11 to 14. Highest jet ratio in the world is 110 for the five turbine units of Kt. Glaraus Power House in Switzerland. Specifications of each of these Pelton turbines are ;

Power=3,000 HP, Speed=500 RPM

D_1=211 inches (or 5,360 mm) d_1= 1·92 inches (or 48·77mm)

H_{gross}=5,412 ft (or 1,650 m) H_{net}=4,850 ft (or 1,480 m)

6·15 Selection of Speed—For given conditions, Pelton turbines ha·· a wide range of speed. There is a definite speed yielding maximum efficiency as seen from the table above. If the speed of the turbine is made higher, then

a) the specific speed will increase.

Its advantages are—

i) The size of the turbine will become smaller and hence it will be less costly.

ii) The jet diameter will decrease. Reduction in jet diameter will raise the jet ratio and enhance the turbine efficiency (See Table 6.3).

The disadvantage of having higher specific speed is that it necessitates multi-jets with which the governing becomes complicated and more expensive.

b) The speed of directly coupled generator will increase. This means that smaller number of pair of poles are required and hence the generator will also be less costly.

c) Material employed for high-speed machines (turbine and generator) will be costly, as high speed causes great stresses in revolving parts.

All points noted above should be kept in mind and conflicting interests compromised in the selection of a proper speed.

6.16 Buckets of a Pelton Runner—

Minimum Number of Buckets :

Let the number of buckets be z. Then the angular separation between two buckets *i.e.*, the angular pitch,

$$\theta=\frac{2\pi}{z} \text{ radians}=\left(\frac{360}{z}\right)^{\circ}$$

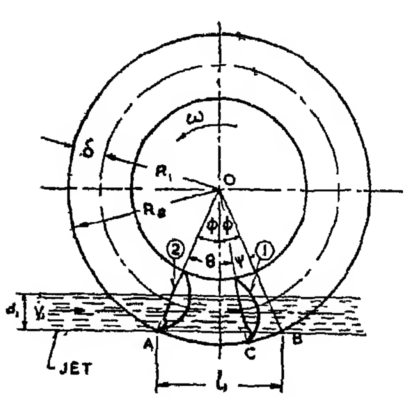


Fig 6.12 Calculation of Minimum Number of Buckets

Assume that the jet is running horizontally with its centre line touching the pitch circle at a point directly under the centre (See Fig 6.12). A and B are points in which the straight horizontal path of the lowest water particles in the jet intersects the circular path of the bucket tips. Let the angular distance of A or B from the vertical centre line of runner be ϕ.

Consider the instant when buckets are in the position shown in Fig 6.12. Bucket (2) has just arrived at A and the preceding bucket (1) is at C at an angle θ

ahead of A. This can be said to be a limiting position. The portion of the jet to the right of bucket (2) is meant to strike bucket (1). To reduce volumetric loss, not a single particle of jet should escape without striking bucket (1). Hence for maximum utilisation of jet with a minimum number of buckets, it is essential that a water particle at A, which has escaped from striking bucket (2), should be just able to overtake the bucket (1) before the latter has crossed the point B.

$\therefore$ Time t_1 taken by a water particle in the jet to traverse a horizontal distance l_1 should be equal to (for slightly less than) time t_2 taken by a bucket to move an angular distance ψ.

Now $\quad t_1=\dfrac{l_1}{v_1}$ and $t_2=\dfrac{\psi}{\omega}$

If $\quad R_e$ = extreme radius of runner,

R_1 = mean radius of runner,

d_1 = diameter of jet (assuming that it remains constant),

$\delta=R_e \quad R_1$ = half length of bucket

$=k \cdot d_1$ (See Fig 6.13)

then

$$\cos\phi=\frac{R_1+\dfrac{d_1}{2}}{R_e}=\frac{R_1+\dfrac{d_1}{2}}{R_1+\delta}=\frac{R_1+\dfrac{d_1}{2}}{R_1+k \cdot d_1}$$

$$=\frac{1+\dfrac{d_1}{2R_1}}{1+\dfrac{2k \cdot d_1}{2R_1}}=\frac{1+\dfrac{1}{m}}{1+\dfrac{2k}{m}} \qquad \ldots(6.11)$$

From Fig 6.12

$$l_1=2R_e \cdot \sin\phi=2(R_1+k \cdot d_1) \cdot \sin\phi$$

also $\quad \omega=\dfrac{u_1}{R_1}$

where u_1 is the tangential velocity of the centre of bucket

$$\therefore \quad \frac{2(R_1+k \cdot d_1) \cdot \sin\phi}{v_1}=\frac{\psi}{\dfrac{u_1}{R_1}}$$

$$\therefore \quad \psi=\frac{2u_1}{v_1}\left(1+\frac{2k}{m}\right) \cdot \sin\phi$$

But $\quad u_1=K_{u_1} \cdot \sqrt{2gH}$

and $\quad v_1=K_{v_1} \cdot \sqrt{2gH}$

$$\therefore \quad \psi=\frac{2K_{u_1}}{K_{v_1}} \cdot \left(1+\frac{2k}{m}\right) \cdot \sin\phi$$

Further, $\sin\phi=\sqrt{1-\cos^2\phi}$

$$\therefore \quad \psi = \frac{2K_{u_1}}{K_{v_1}} \cdot \left(1+\frac{2k}{m}\right) \cdot \sqrt{1+\left\{\frac{1+\frac{1}{m}}{1+\frac{2k}{m}}\right\}^2}$$

$$= \frac{2K_{u_1}}{K_{v_1}} \cdot \sqrt{\left(1+\frac{2k}{m}\right)^2-\left(1+\frac{1}{m}\right)^2} \qquad \ldots(6.12)$$

Since $\phi = \cos^{-1}\left\{\dfrac{1+\frac{1}{m}}{1+\frac{2k}{m}}\right\}$

both ψ and ϕ can be evaluated.

Now $\quad \theta = 2\phi - \psi \qquad \ldots(6.13)$

and minimum number of buckets required

$$Z = \frac{2\pi}{\theta_{rad}} = \frac{360}{\theta^{\circ}} \qquad \ldots(6.14)$$

From the above deduction, it is seen that values of ψ and ϕ depend on K_{u_1}, K_{v_1}, k and m. Also K_{u_1} itself depends upon m ; and K_{v_1} and k depend on d_1 which again is a function of m. Thus the minimum number of buckets obtained above ultimately depends on m.

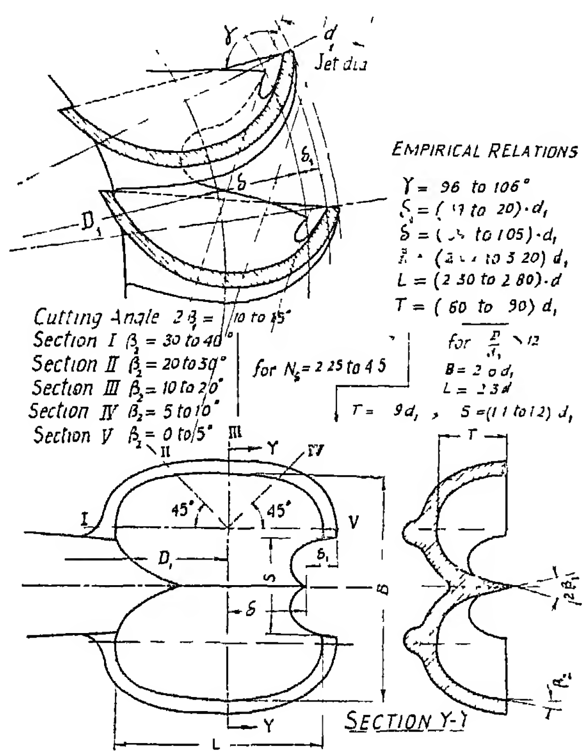


Fig 6.13 Construction of Pelton Runner Blade

Dr. Taygun* has suggested the following empirical relation for determination of the number of buckets.

$$Z = 0.5\ m + 15 \quad (6.15)$$

This equation holds good for all values of m from 6 to 35.

If the actual number differs from the value given by the above relation, the efficiency will not be a maximum.

Construction of the Bucket—

Main dimensions of the bucket are given in Fig 6 13.

Practical data :

Length	$L = 2.3$ to 2.8 times d_1
Width	$B = 2.8$ to 3.2 times d_1
Depth	$T = 0.6$ to 0.9 times d_1
Angle	$\beta_1 \approx 5^\circ$ to 8°
Angle	$\beta_2 \approx 10^\circ$ to 20° at centre.

6.17 Turgo-Impulse Turbine exclusively manufactured by Messrs Gilbert Gilkes & Gordon Ltd, Kendal (England) is a medium-head free jet impulse water turbine. It bridges the gap of specific speed (See Table 5.2) between the Pelton runner and the Francis runner, which is normally done by employing multi-jets for Pelton runner. The runner is made in one-piece single-discharge buckets (half Francis and half Pelton type), held inwardly by the hub and outwardly by a peripheral band. The jet impinges the buckets obliquely to one side of the runner and discharges at the other. The striking of jet on the buckets is similar to Girard turbine of impulse type (See Art. 4.12). Fig 6.14 shows the difference between the water jet impinging the Pelton

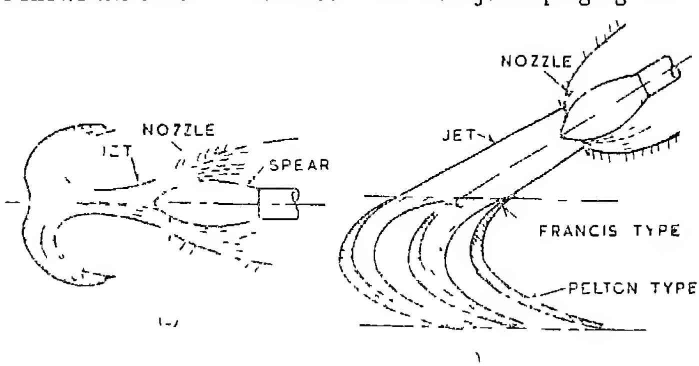


Fig 6 14 Difference between the Water Jet Impinging (a) the Pelton Bucket and (b) the Turgo Bucket

bucket and the Turgo bucket. Fig 6.15 shows the section through a Turgo-Impulse Turbine with hand-regulated spear. The number of jets to the runner is usually one and at most two. The end thrust of the jet is taken by bearings, therefore the overhung arrangement is not employed for such wheels.

***Taygun, H. F.** The influence of number of buckets on the efficiency of Pelton runner ; Dissertation of Swiss Federal Institute of Technology, Zurich 1946 (Published in German).

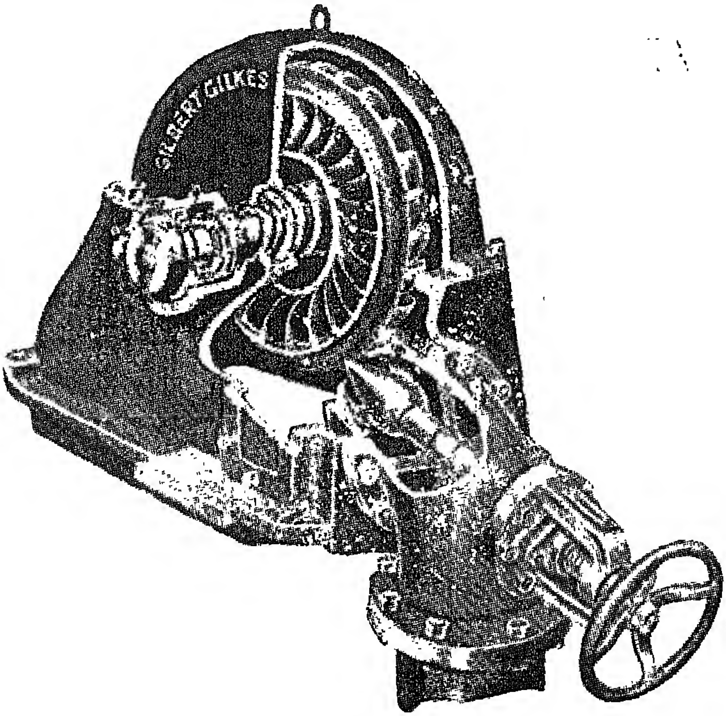


Fig 6.15 Section through a Turgo-Impulse Turbine with Hand Regulated Spear

The Power House at Poonch (Jammu and Kashmir State), 200 miles from Jammu, fed by Betar River is equipped with two Turgo-Impulse turbines, each developing 40 HP at 670 rpm.

6.18 Banki Turbine (See Fig 6.16 and 6.17)—As already described in Art. 5.15, Banki turbine is a free jet, impulse turbine which like Turgo-impulse turbine, bridges the gap of specific speed (See Table 5.2) between Pelton and Francis runner. The water jet striking the runner has a rectangular cross-section which can be varied lengthwise as well as breadthwise. The runner has a drum shape, connected to a shaft which is always horizontal. The drum-shaped runner can be made of any length with which large amount of discharge can be used.

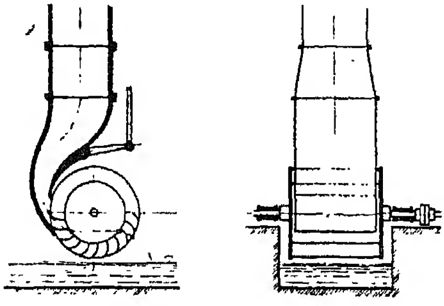


Fig 6.16 Banki Turbine

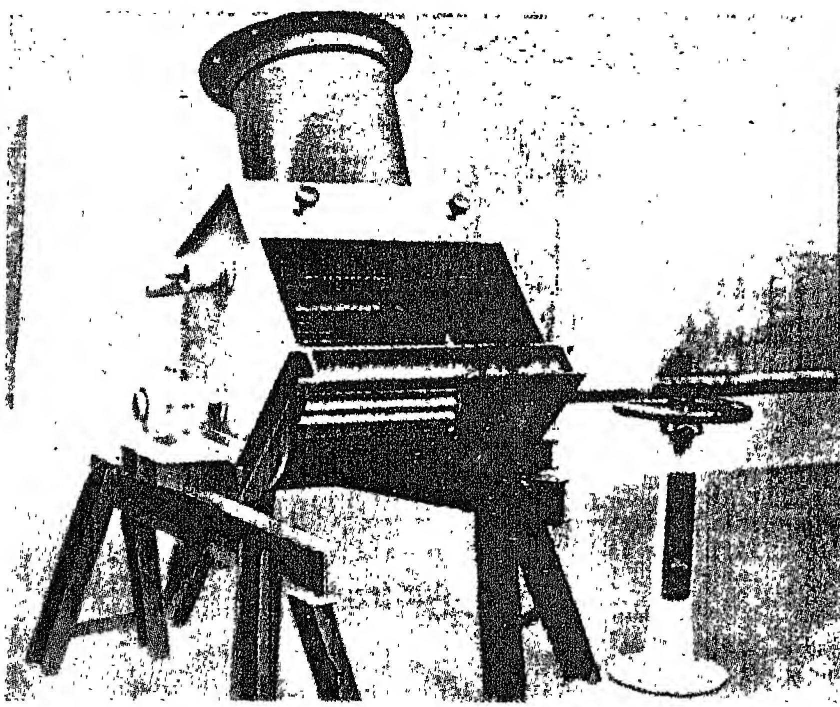


Fig. 6.17 Banki Turbine in a Laboratory

Problem 6.1 The Pykara (South India) Power House is equipped with impulse turbines of Pelton type. Each turbine delivers a maximum horsepower of 19,000 when working under a head of 2,800 ft and running at 600 rpm. Find the least diameter of the jet and the mean diameter of the wheel. What would be the approximate diameter of orifice of the nozzle tip ? Determine the value of the jet ratio and state if it is within the limits. Specify the number of buckets for the wheel. Take the overall efficiency of the turbine as 89·2%.

Solution

$P_t = 19{,}000$ HP $\qquad H = 2{,}800$ ft

$N = 600$ rpm $\qquad \eta_t = 0{\cdot}892$

a) Least diameter of the jet d_1—

$$P_t = \frac{w \cdot Q \cdot H}{550} \cdot \eta_t \qquad \text{(See Eqn 6.2)}$$

or $$19{,}000 = \frac{62{\cdot}4 \times Q \times 2{,}800}{550} \times 0{\cdot}892$$

$$19{,}000 \times 550$$

Assuming $K_{v_1}=0.988$

$$67=\frac{\pi}{4}\ d_1^2\times 0.988\times\sqrt{62.4\times 2{,}800}$$

or
$$d_1=\sqrt{\frac{67\times 4}{0.988\times 8.02\times 53\times\pi}}=\sqrt{0.205}$$

$$=0.452 \text{ ft}$$

or $\mathbf{d_1=5.42}$ **in.**

b) Mean diameter of wheel D_1—

$$u_1=K_{u_1}\ .\ \sqrt{2gH}\ =\frac{\pi\ .\ D_1\ .\ N}{60}$$

Assume $K_{u_1}=0.45$ (for maximum efficiency)

$$\therefore\qquad D_1=\frac{60\ .\ K_{u_1}\ .\ \sqrt{2gH}}{\pi\ .\ N}$$

$$=\frac{60\times 0.45\times 8.02\times 53}{\pi\times 600}$$

$$=6.1 \text{ ft}$$

or $\mathbf{D_1=6}$ **ft—1¼ in.**

c) Diameter of nozzle d_o—

$$d_o=1.25\ d_1=5.4\times 1.25=6.75 \text{ inches}$$

or $\mathbf{d_o=6\frac{3}{4}}$ **in.**

d) Jet ratio $m=\frac{D_1}{d_1}=\frac{6.1}{0.452}=\mathbf{13.5}$

As the jet ratio for maximum efficiency is between 11 and 14, this value is, therefore, within the limits.

e) Number of buckets Z—

$Z=0.5\ m+15$ (See Eqn 6.15)

$=0.5\times 13.5+15$

$=$**22 buckets**

Least diameter of jet	**=5.42 in.**	*Answers*
Mean diameter of wheel	**=6 ft—1¼ in.**	
Diameter of orifice	**=6¾ in.**	
Jet ratio	**=13.5**	
Number of buckets	**=22**	

Problem 6.2 A double overhung Pelton wheel unit is to operate at 30,000 KW generator under an effective head of 1,000 ft (or 304.8 *m*) at the base of the nozzle. Find the size of jet, mean diameter of runner, synchronous speed and specific speed of each wheel. Assume generator efficiency 93%. Pelton wheel efficiency 85%, co-efficient of nozzle velocity 0.97, speed ratio 0.46 and jet ratio 12.

Solution

There are two runners keyed on the two ends of the shaft, and the generator lies between them. Each runner is to be taken as one complete turbine. Thus the generator is fed by two Pelton turbines.

$\therefore$ Horsepower developed by each turbine,

$$P_t = \frac{30{,}000}{2 \times 0{\cdot}746} \times \frac{1}{0{\cdot}93} = 21{,}600 \text{ BHP}$$

Available horsepower of each turbine,

$$P_a = \frac{21{,}600}{0{\cdot}85} = 25{,}400 \text{ HP}$$

but $$P_a = \frac{w\,.\,Q\,.\,H}{550} \left[\text{or } P_a = \frac{w\,.\,Q\,.\,H}{75} \text{ in Metric Units} \right]$$

$$\therefore \quad Q = \frac{P_a \times 550}{w\,.\,H} = \frac{25{,}400 \times 550}{62{\cdot}4 \times 1{,}000} = 224 \text{ cusecs}$$

$$\left[\text{or } Q = \frac{P_a \times 75}{w\,.\,H} = \frac{25{,}400 \times 75}{1{,}000 \times 304{\cdot}8} = 6{\cdot}25\ m^3/\text{sec} \right]$$

Velocity of jet $\quad v_1 = K_{v_1} \cdot \sqrt{2g\,.\,H}$

$$= 0{\cdot}97 \times \sqrt{64{\cdot}4 \times 1{,}000} = 246 \text{ ft/sec}$$

$$[\text{or } v_1 = 0{\cdot}97 \times \sqrt{2 \times 9{\cdot}81 \times 304{\cdot}8} = 75\ m/\text{sec}]$$

Area of jet $$a_1 = \frac{Q}{v_1} = \frac{224}{246} \times 144 = 131 \text{ sq in.}$$

$$\left[\text{or } a_1 = \frac{Q}{v_1} = \frac{6{\cdot}25}{75} = 0{\cdot}0833\ m^2 \right]$$

$\therefore$ Diameter of jet, $d_1 = \sqrt{\dfrac{131}{\frac{\pi}{4}}} = \mathbf{12{\cdot}92\ inches}$

$$\left[\text{or } d_1 = \sqrt{\frac{0{\cdot}0833}{\frac{\pi}{4}}} = 0{\cdot}326\ m \quad \text{or } \mathbf{326\ mm} \right]$$

Diameter of wheel, $D_1 = d_1 \times \text{jet ratio}$

$= 12{\cdot}92 \times 12 = 12{\cdot}92$ ft [or $0{\cdot}326 \times 12 = \mathbf{3{\cdot}91\ m}$]

$\approx$ **13 ft** [or **3,910 mm**]

Peripheral velocity of the wheel $u_1 = K_{u_1} \cdot \sqrt{2g\,H}$

$$= 0{\cdot}46 \times 8{\cdot}02 \times \sqrt{1{,}000} = 117 \text{ ft/sec}$$

$$[\text{or } 0{\cdot}46 \times 4{\cdot}43 \times \sqrt{304{\cdot}8} = 35{\cdot}5\ m/\text{sec}]$$

But $$u_1 = \frac{\pi\,.\,D_1\,.\,N}{60} = \frac{\pi \times 13 \times N}{60}$$

$$\therefore \qquad N=\frac{60\times 117}{\pi\times 13}=172 \text{ rpm}$$

$$\left[\text{or } N=\frac{60\times 35.5}{\pi\times 3.91}=172 \text{ rpm}\right]$$

Assuming f, the frequency of generator as 50 cycles per sec,

$$f=\frac{p\,.\,N}{60}, \text{ where } p=\text{number of pair of poles}$$

$$\therefore \qquad N_{syn}=\frac{60\,.\,f}{p}=\frac{3{,}000}{p}$$

Assuming $\quad p=18, \quad N_{syn}=\frac{3{,}000}{18}=\mathbf{166{\cdot}7\ RPM}$

Revised Diameter of the wheel,

$$D_1=\frac{172\times 13}{166{\cdot}7}=13{\cdot}42 \text{ ft}$$

$$\approx \mathbf{13\ ft-5\tfrac{1}{8}\ in.}$$

$$\left[\text{or } D_1=\frac{172\times 3.91}{166.7}=4{\cdot}06\ m \quad \text{or } \mathbf{4{,}060\ mm}\right]$$

Specific speed $N_s=\dfrac{N\,.\,\sqrt{P_t}}{H^{\frac{5}{4}}}$

$$=\frac{166{\cdot}7\times\sqrt{21{,}600}}{1{,}000\times 1{,}000^{\frac{1}{4}}} \quad \left[\text{or } \frac{166{\cdot}7\times\sqrt{21{,}600}}{304{\cdot}8\times 304{\cdot}8^{\frac{1}{4}}}\right]$$

$$=4{\cdot}36 \ [\text{or } 19{\cdot}2]$$

Size of jet $\quad d_1=$ **12.92 in.** (or **326 mm**)
Mean diameter of each wheel $D_1=$ **13 ft $-5\frac{1}{8}$ in.** (or **4,060 mm**)
Synchronous speed $\quad N_{syn}=$ **166·7 RPM**
Specific speed of each runner $N_s=$ **4·36 British units** (or **19·2 Metric units**)
} *Answers*

FORCE, POWER AND EFFICIENCY

6.19 Velocity Triangles—In a Pelton wheel the jet simultaneously strikes a number of buckets. It commences to strike the bucket before it has reached a position directly under the centre of the wheel (Fig 6.12). The angle which the striking jet makes with the direction of motion of the bucket is denoted by symbol α_1 and in practice it varies from 8° to 20°. As shown earlier in Chapter 1 the force exerted by the jet can be calculated with the help of velocity triangles at inlet and at outlet.

In drawing the typical velocity triangles (Fig 6.18) for a Pelton runner the following points should be kept in mind :

$u_1=u_2$......(since $r_1=r_2$)

$w_1=w_2$......(Assuming there is no friction at the blade)

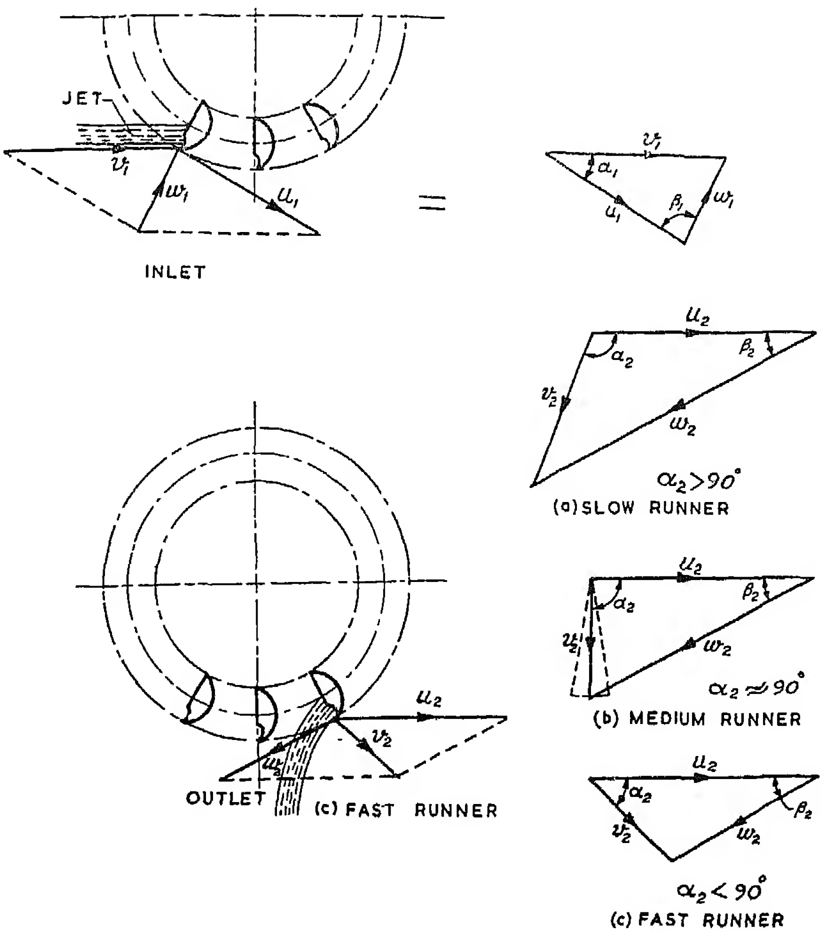
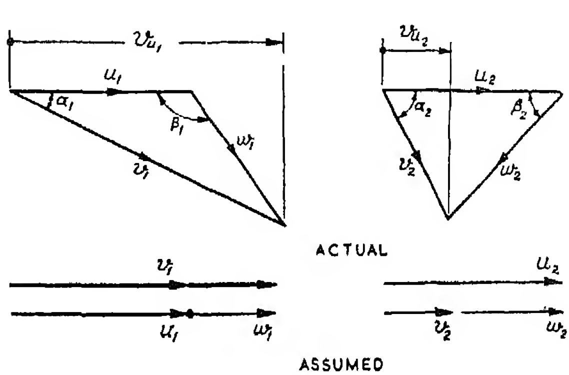


Fig 6.18 Typical Velocity Triangles of Pelton Runner



6.19 Ideal Velocity Triangles

$\alpha_1 < \beta_1,$ $v_1 >>> v_2$

$u_1 < v_1$ $v_2 < w_2,$ $\alpha_2 > \alpha_1$

6.20 Force Exerted by the Jet—For the calculation of the force exerted by the jet it is assumed that $\alpha_1 = 0$ *i.e.*, the bucket face is perpendicular to the jet.

If $\alpha_1 = 0°$, $\beta_1 = 180°$ (Fig 6.19)

then $v_{u_1} = v_1 \cos \alpha_1 = v_1 = u_1 + w_1$

or $w_1 = v_1 - u_1$

From velocity triangle at outlet

$$v_{u_2} = v_2 \cos \alpha_2 = u_2 - w_2 \cos \beta_2$$

For ideal case,

$\beta_2 = 0°$ *i.e.*, water is deflected back by 180°

$\therefore \quad v_{u_2} = u_2 - w_2 \qquad \ldots(\cos 0° = 1)$

But $u_1 = u_2$, and $w_1 = w_2$ (assuming for ideal case)

$$\therefore \quad v_{u_2} = u_1 - w_1$$

$$= u_1 - (v_1 \quad u_1) = 2u_1 - v_1$$

Force exerted by the jet in the direction of u,

$$F_u = \frac{w \cdot Q}{g}\left(v_{u_1} - v_{u_2}\right) \text{ lb (or kg)} \qquad \text{(See Eqn 1.24)}$$

(assuming that the total quantity Q strikes the bucket)

$$= \frac{w \cdot Q}{g}\left\{ v_1 - (2u_1 - v_1) \right\}$$

$$= \frac{2 \cdot w \cdot Q}{g} (v_1 - u_1)$$

Also, $Q = \frac{\pi}{4} \cdot d_1^2 \cdot K_{v_1} \cdot \sqrt{2gH}$

$v_1 = K_{v_1} \cdot \sqrt{2gH}$

and $u_1 = K_{u_1} \cdot \sqrt{2gH}$

Substituting these values in the expression for F_u,

$$F_u = \frac{2w}{g}\left(\frac{\pi}{4} \cdot d_1^2 \cdot K_{v_1} \cdot \sqrt{2gH}\right)\left[K_{v_1} - K_{u_1}\right] \cdot \sqrt{2gH} \text{ lb (or kg)}$$

$$= w \cdot \pi \cdot K_{v_1} \cdot (K_{v_1} - K_{u_1}) \cdot d_1^2 \cdot H \text{ lb (or kg)} \qquad \ldots(6.16)$$

Hence, force for unit head and unit diameter,

$$F_{u_{11}} = \frac{F_u}{d_1^2 \cdot H} = \pi \cdot K_{v_1} \cdot (K_{v_1} - K_{u_1}) \cdot w \qquad \ldots(6.17)$$

lb (or kg) per unit head and per unit jet dia.

Force will be maximum when $K_{u_1}=0$ *i.e.*, wheel is at rest.

$$(F_{u_{11}})_{max}=\pi \cdot K_{v_1}^2 \cdot w$$

lb (or kg) per unit head and per unit jet dia .. (6.17*a*)

Su bstituting average values $K_{u_1}=0{\cdot}985$ and $w=62{\cdot}4$ lb per cu ft (or 1,000 kg/m^3)

a) **In FPS Units—**

$$(F_{u_{11}})_{max}=\pi \cdot (0{\cdot}985)^2\times 62{\cdot}4$$
$$=190 \text{ lb per unit head and per unit jet dia.}$$

b) **Metric Units—**

$$(F_{u_{11}})_{max}=\pi \cdot (0{\cdot}985)^2\times 1{,}000$$
$$=3{,}050 \text{ kg per unit head and per unit jet dia.}$$

Under normal working conditions $K_{u_1}\approx 0{\cdot}45$, whence,

a) **In FPS Units—**

$$F_{u_{11}}=\pi\times(0{\cdot}985)\times(0{\cdot}985-0{\cdot}45)\times 62{\cdot}4$$
$$=\pi\times 0{\cdot}985\times 0{\cdot}535\times 62{\cdot}4$$
$$=103{\cdot}2 \text{ lb per unit head and per unit jet dia.}$$

or $F_{u_{11}}\approx 100$ lb ...(6.17*b*)

b) **Metric Units—**

$$F_{u_{11}}=\pi\times 0\ 985\times 0{\cdot}535\times 1{,}000$$
$$=1{,}657 \text{ kg per unit head and per unit jet dia.}$$

For runaway speed,

$K_{u_1}=K_{v_1}$ (See Fig 6.20)

then $F_{u\ 1}=0$

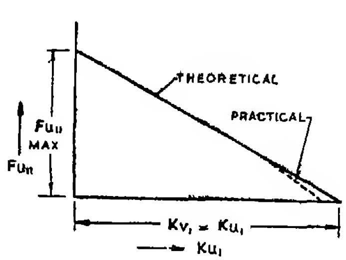


Fig 6.20 Force $(F_{u_{11}})$ vs Speed Ratio (K_{u_1})

Theoretically for maximum efficiency,

$$\frac{K_{v_1}}{K_u}=2$$

But in practice, on account of losses,

$$\frac{K_{v_1}}{K_{u_1}}=1{\cdot}8$$

Therefore the runaway speed of a Pelton turbine is 1·8 times the normal working speed.

6.21 Work Done and Power Developed by the Jet—

ower, $P_H=F_u \quad u$ ft lb sec

$$=2\cdot\frac{w}{g}\cdot\frac{\pi}{4}\cdot d_1^2\cdot K_{v_1}\cdot\sqrt{2gH}\cdot(K_{v_1}-K_{u_1})\sqrt{2gH}\cdot(K_{u_1}\sqrt{2gH})$$

$$=w\cdot\pi\cdot d_1^2\cdot K_{v_1}\cdot K_{u_1}\cdot(K_{v_1}-K_{u_1})\cdot\sqrt{2gH}\cdot H \qquad ...(6.18)$$

Power developed per unit head and unit jet diameter,

$$P_{H_{11}}=\frac{P_H}{d_1^2\cdot H\cdot\sqrt{H}}=w\cdot\pi\cdot K_{v_1}\cdot K_{u_1}\cdot(K_{v_1}-K_{u_1})\cdot\sqrt{2g} \qquad ...(6.19)$$

Substituting average values $K_{v_1}=0{\cdot}985$ and $K_{u_1}=0{\cdot}45$

(a) $P_{H_{11}}$ **in FPS Units—**

$$P_{H_{11}}=\pi\times62{\cdot}4\times0{\cdot}985\times0{\cdot}45\times(0{\cdot}985-0{\cdot}45)\times8{\cdot}02$$

$=372$ ft lb/sec/unit head and unit jet dia.

or $\quad P_{H_{11}}=\dfrac{372}{550}=0{\cdot}676$ HP/unit head and unit jet dia.

i.e., $\quad P_{H_{11}}=\frac{2}{3}$ HP/unit head and unit jet dia. ...(6.19a)

(b) $P_{H_{11}}$ **in Metric Units—**

$$P_{H_{11}}=\pi\times1{,}000\times0{\cdot}985\times0{\cdot}45\times(0{\cdot}985-0{\cdot}45)\times4{\cdot}43$$

$=3{,}333$ kg m/sec/unit head and unit jet dia.

or $\quad P_{H_{11}}=\dfrac{3{,}333}{75}=44{\cdot}5$ metric HP/unit head and unit jet dia.

6.22 Turbines Efficiencies—

Jet Efficiency or Head Efficiency—By definition the Jet Efficiency Head Efficiency.

$$\eta_H=\frac{P_H}{P_a}=\frac{\pi\cdot w\cdot d_1^2\cdot K_{v_1}\cdot K_{u_1}\cdot(K_{v_1}-K_{u_1})\cdot\sqrt{2gH}\cdot H}{w\cdot\frac{\pi}{4}\cdot d_1^2\cdot K_{v_1}\cdot\sqrt{2gH}\cdot H}$$

$$=4\,K_{u_1}\cdot(K_{v_1}-K_{u_1}) \qquad ...(6.20)$$

For maximum efficiency, assuming K_{v_1} as constant,

$$\frac{d\eta_H}{d\,K_{u_1}}=0 \quad \text{or} \quad 4\,(K_{v_1}-2\,K_{u_1})=0$$

i.e. $\quad K_{u_1}=\frac{K_{v_1}}{2} \quad \text{or} \quad u_1=\frac{v_1}{2}$

Also $\left(\eta_H\right)_{max}=4\times\frac{K_{v_1}}{2}\left(K_{v_1}-\frac{K_{v_1}}{2}\right)$

$$=K_{v_1}^2 \qquad \text{...(6 20a)}$$

Taking an average value of $K_{v_1}=0\ 985$,

$$\left(\eta_H\right)_{max}=0\ 985^2=0\ 97$$

In the ideal case $\left(\eta_H\right)_{max}=1$ but actually it is 0 96 to 0 98

Volumetric Efficiency—

So far the force, power and efficiency of the jet have been dealt with. Now the total quantity of water contained in the jet does not strike the bucket and always there is some amount of water that slips and falls in the tail race without doing any useful work. Thus, a new factor called *Volumetric Efficiency* is introduced.

If $\triangle Q$ be the quantity of water lost on account of slip,

$$\eta_Q=\frac{Q-\triangle Q}{Q} \qquad \text{...(6.21)}$$

Actual value of η_Q is between 0 97 and 0 99

Hydraulic Efficiency—Considering the hydraulic losses of the turbine, the hydraulic efficiency can be written as—

$$\eta_h=\left(\frac{H-\triangle H}{H}\right)\left(\frac{Q-\triangle Q}{Q}\right)=\eta_H\cdot\eta_Q \qquad \text{...(6 22)}$$

Mechanical Efficiency—There are always some mechanical losses in the transmission of power by the turbine and mechanical efficiency $\eta_{mech}=0{\cdot}97$ and 0 995 depending upon the size and capacity of the unit which may vary from 100 to 1,00,000 HP.

Final Power Output from Turbines—

If P_a be the natural available power, power produced by jet,

$$P_H=P_a\cdot\eta_H$$

Hydraulic Power generated by turbine,

$$P_h=P_H\cdot\eta_Q=P_a\cdot\eta_H\cdot\eta_Q$$

Net Brake Horse Power developed by the turbine shaft,

$$P_t = P_h \cdot \eta_{mech} = P_a \cdot \eta_H \cdot \eta_Q \cdot \eta_{mech} \qquad \ldots(6.23)$$

Hence, *Final* or *Overall Efficiency* of turbine,

$$\eta_t = \frac{P_t}{P_a} = \frac{P_a \cdot \eta_H \cdot \eta_Q \cdot \eta_{mech}}{P_a}$$

$$= \eta_H \cdot \eta_Q \cdot \eta_{mech} \qquad \ldots(6.24)$$

Substituting values for η_H, η_Q and η_{mech}

$$\eta_t = 0{\cdot}97 \times 0{\cdot}98 \times 0{\cdot}982$$
$$= 0{\cdot}932$$

It must be remembered that in calculating the above values of force, power and efficiencies it was presumed that

$$\beta_1 = 180°, \qquad \beta_2 = 0°, \qquad w_1 = w_2$$

In practice,

$\beta_1 = 95°$ to $110°$, $\beta_2 = 10°$ to $20°$ and $w_2 = (0{\cdot}96$ to $0{\cdot}98) \cdot w_1$

More accurate calculations for force, power and efficiencies can be made by taking into account these facts and making the necessary corrections.

Problem 6.3 The mean bucket speed of a Pelton turbine is 45 ft/sec. The rate of flow of water supplied by the jet under a head of 140 ft is 175 gallons per second. If the jet is deflected by the buckets at an angle of 165°, find the HP and the efficiency of the turbine.

Solution

$u_1 = u_2 = 45$ ft/sec $\qquad Q = 175$ gps $= \dfrac{175 \times 10}{62{\cdot}4} = 28$ cfs

$H = 140$ ft $\qquad \beta_2 = 180° - 165° = 15°$

Velocity of jet $\qquad v_1 = K_{v_1} \cdot \sqrt{2gH}$

Assuming $\quad K_{v_1} = 0{\cdot}985$

$$v_1 = 0{\cdot}985 \times \sqrt{64{\cdot}4 \times 140} = 93{\cdot}5 \text{ ft/sec}$$

Assuming $\quad \beta_1 = 180°$

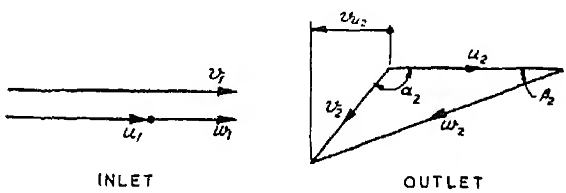


Fig 6.21

Draw the inlet velocity triangle (*see* Fig 6.21)

$$v_{u_1} = v_1 = 93{\cdot}5 \text{ ft/sec}$$

$$w_1 = v_1 - u_1 = 93{\cdot}5 - 45 = 48{\cdot}5 \text{ ft/sec}$$

For outlet velocity triangle, assume $u_1=u_2$

and $w_1=w_2$ (neglecting loss on buckets)

$\therefore$ $w_2=48{\cdot}5$ ft/sec

$u_2=45$ ft/sec

$\therefore$ $v_{u_2}=v_2 \cos \alpha_2=u_2-w_2 \cos \beta_2=45-(48{\cdot}5\times 0{\cdot}9659)$

$=45-46{\cdot}8=-1{\cdot}8$ ft/sec

$$\text{Work done/sec}=\frac{w \cdot Q}{g}(v_{u_1}-v_{u_2}) \cdot u_1 \qquad \text{...(See Eqn 1.25)}$$

$$=\frac{62{\cdot}4\times 28}{32{\cdot}2}\times(93{\cdot}5+1{\cdot}8)\times 45 \text{ ft lb/sec}$$

$$\therefore \quad P_t=\frac{\text{Work done/sec}}{550}=\frac{62{\cdot}4\times 28\times 95{\cdot}3\times 45}{32{\cdot}2\times 550}=\mathbf{423\ HP} \quad \textit{Answer}$$

$$\text{Efficiency } \eta_t=\frac{\text{Power developed}}{\text{Available power}}=\frac{423}{\frac{w \cdot Q \cdot H}{550}}=\frac{423\times 550}{62{\cdot}4\times 28\times 140}$$

$=0{\cdot}952$ or **95·2%** *Answer*

Problem 6.4 A single jet Pelton turbine is required to drive a generator to develop 10,000 KW. The available head at the nozzle is 2,500 ft (or 762 *m*). Assuming electric generator efficiency 95%, Pelton wheel efficiency 87%, co-efficient of velocity for nozzle 0·97, mean bucket velocity 0·46 of jet velocity, outlet angle of the buckets 15 degrees and the relative velocity of the water leaving the buckets 0·85 of that at inlet, find

a) the diameter of jet,

b) the flow in cusecs,

and *c*) the force exerted by the jet on the buckets.

If the ratio of the mean bucket circle diameter to the jet diameter is not to be less than 10, find the best synchronous speed for generation at 50 cycles per second and the corresponding mean diameter of the runner.

(AMIE Mech E—Oct 1954)

Solution

$KW=10{,}000$	$H=2{,}500$ ft (or 762 m)
$\eta_G=0{\cdot}95$	$\eta_t=0{\cdot}87$
$K_{v_1}=0{\cdot}97$	$K_{u_1}=0{\cdot}46\times 0{\cdot}97=0{\cdot}446$
$\beta_2=15°$	$w_2=0{\cdot}85\, w_1$
$m\geqq 10$	$f=50$ cycles/sec

(1 $KW=1{\cdot}36$ metric HP)

$$\text{Output of the turbine, } P_t=\frac{10{,}000}{0{\cdot}746}\times\frac{1}{0{\cdot}95}=14{,}120 \text{ BHP}$$

$$\left[\text{or } P_t=10{,}000\times 1{\cdot}36\times\frac{1}{0{\cdot}95}=14{,}320 \text{ metric HP}\right]$$

Available horsepower of the turbine $P_a = \frac{w \cdot Q \cdot H}{550}$ HP

$$\left(\text{or } = \frac{w \cdot Q \cdot H}{75} \text{ metric HP}\right)$$

Also $\quad P_a = \frac{P_t}{\eta_t} = \frac{14,120}{0\ 87} = 16,220$ HP

$$\left[\text{or} \quad P_a = \frac{14,320}{0{\cdot}87} = 16,460 \text{ metric HP}\right]$$

$\therefore \quad Q = \frac{16,220 \times 550}{62{\cdot}4 \times 2,500} = \mathbf{57{\cdot}2}$ **cusecs**

$$\left[\text{or} \quad \frac{16.460 \times 75}{1,000 \times 762} = \mathbf{1{\cdot}62\ m^3/sec}\right]$$

Also $\quad Q = \frac{\pi}{4} \cdot d_1^2 \cdot K_{v1} \cdot \sqrt{2gH}$

or $\quad 57{\cdot}2 = \frac{\pi}{4} \times d_1^2 \times 0{\cdot}97 \times \sqrt{2 \times 32\ 2 \times 2,500}$

$$\left[\text{or } 1{\cdot}62 = \frac{\pi}{4} \times d_1^2 \times 0\ 97 \times \sqrt{2 \times 9\ 81 \times 762}\right]$$

or $\quad d_1 = 0{\cdot}433$ ft $\quad$ or $\quad$ **5·2 in.**

[or $\quad d_1 = 0{\cdot}132\ m$ $\quad$ or $\quad$ **132 mm**]

Force exerted by jet $F_u = \frac{w \cdot Q}{g}(v_{u_1} - v_{u_2})$ ...(See Eqn 1.24)

Assume $\alpha_1 = 0$ and $\beta_1 = 180°$

and $\quad v_1 = K_{v_1} \cdot \sqrt{2gH} = 0\ 97 \times 8{\cdot}02 \times 50$

$= 390$ ft/sec

[or $\quad v_1 = 0\ 97 \times 4{\cdot}43 \times \sqrt{762} = 118{\cdot}5\ m/\text{sec}$]

$u_1 = K_{u_1} \cdot \sqrt{2gH} = 0{\cdot}446 \times 8{\cdot}02 \times 50$

$= 179$ ft/sec

[or $\quad u_1 = 0{\cdot}446 \times 4{\cdot}43 \times 27{\cdot}6 = 54{\cdot}5\ m/\text{sec}$]

From inlet velocity triangle (*see* Fig 6.22)

$w_1 = 390 - 179 = 211$ ft/sec

[or $\quad w_1 = 118{\cdot}5 - 54{\cdot}5 = 64\ m/\text{sec}$]

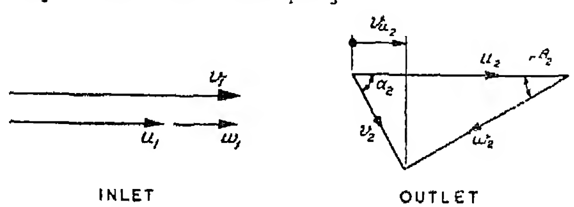


Fig 6.22

$w_2 = 0{\cdot}85 \times 211 = 179{\cdot}4$ ft/sec (or $w_2 = 0{\cdot}85 \times 64 = 54{\cdot}6$ m/sec)

$u_2 = u_1 = 179$ ft/sec (or $u_2 = 54\ 5$ m/sec)

$v_{u_1} = v_1 = 390$ ft/sec (or $v_{u_1} = v_1 = 118\ 5$ m/sec)

From outlet velocity triangle (*see* Fig 6.22)

$$v_{u_2} = u_2 - w_2 \cos \beta_2$$

$$= 179 - 179{\cdot}4 \times \cos 15^\circ \quad (\text{or} \quad = 54{\cdot}5 - 54{\cdot}6 \times \cos 15^\circ)$$

$$= 6 \text{ ft/sec} \quad (\text{or} \quad = 1{\cdot}83\ m/\text{sec})$$

$$\therefore\ F_u = \frac{w \,.\, Q}{g} (v_{u_1} - v_{u_2})$$

$$= \frac{62{\cdot}4 \times 57\ 2}{32{\cdot}2} \times (390 - 6) \quad \left[\text{or } = \frac{1{,}000 \times 1\ 62}{9{\cdot}81} \times (118\ 5 - 1{\cdot}83)\right]$$

$$= \mathbf{42{,}800\ lb} \quad [\text{or } = \mathbf{19{,}250\ kg}]$$

$$m = \frac{D_1}{d_1} = 10, \quad \therefore\ D_1 = 10 \times 5{\cdot}2 = 52 \text{ in.}$$

$$[\text{or } D_1 = 10 \times 132 = 1{,}320\ mm]$$

$$u_1 = \frac{\pi \,.\, D_1 \,.\, N}{60} \quad \text{or } N = \frac{60 \times 179 \times 12}{\pi \times 52} = 787 \text{ RPM}$$

$$\left[\text{or } N = \frac{60 \times 54\ 5}{\pi \times 0{\cdot}132} = 787 \text{ RPM}\right]$$

Frequency of generator $f = \frac{p \,.\, N}{60}$, where p = No. of pair of poles.

$\therefore$ If $p = 4$, $N_{syn} = \mathbf{750\ RPM}$ which is nearest to 787 RPM

$$\text{Now } D_1(_{revised}) = \frac{52 \times 787}{750} = 54\ 7 \text{ in.} = \mathbf{4\ ft - 6\tfrac{11}{16}\ in.}$$

$$\left[\text{or } = \frac{1{,}320 \times 787}{750} = \mathbf{1{,}386\ mm}\right]$$

Dia of jet	$d_1 = \mathbf{5{\cdot}2\ in.}$	(or **132 mm**)
Flow	$Q = \mathbf{57{\cdot}2\ cusecs}$	(or **$1{\cdot}62\ m^3$/sec**)
Force exerted by jet on buckets	$F_u = \mathbf{42{,}800\ lb}$	(or **19,250 kg**)
Best Synchronous Speed	$N_{syn} = \mathbf{750\ RPM}$	
Runner mean diameter	$D_1 = \mathbf{4\ ft - 6\tfrac{11}{16}\ in.}$	(or **1,386 mm**)

Answers

Problem 6.5 In Pelton wheel installations a small auxiliary nozzle is often fitted, from which a jet of high pressure water can be directed when required on to the reverse side of the buckets, thus acting as a brake and bringing the wheel quickly to rest. Estimate the time required to stop the wheel by the brake nozzle under the following conditions :

Diameter of brake nozzle=2·8 inches

Head at the nozzle=1,560 ft

Original velocity of buckets=146 ft/sec

Corresponding kinetic energy of wheel and all revolving parts=11,200,000 ft lb.

Assume that the dynamic pressure of the buckets is equivalent to that on flat radial blades.

Solution

Velocity of jet from brake nozzle $v_1=K_{v_1} \cdot \sqrt{2g\,H}$

Where K_{v_1} =velocity co-efficient

=0·985 (assumed)

$$\therefore \quad v_1=0{\cdot}985\times\sqrt{2\times32{\cdot}2\times1{,}560}$$
$$=312 \text{ ft/sec}$$

Cross-sectional area of brake nozzle, $a_1=\frac{\pi}{4}\cdot d_1^2$

$$=\frac{\pi}{4}\left(\frac{2{\cdot}8}{12}\right)^2=0{\cdot}043 \text{ sq ft}$$

$\therefore$ The quantity of water per sec flowing through the brake nozzle,

$$Q=a_1\,.\,v_1=0\,043\times312=13{\cdot}35 \text{ cfs}$$

Initial peripheral velocity of the buckets=146 ft/sec

Final peripheral velocity of the buckets=0 ft/sec

$\therefore$ Average peripheral velocity of the buckets=73 ft/sec

(assuming uniform retardation)

Then average force acting on buckets

$$F_u=\frac{w\,.\,Q}{g}\left\{v_1-(-u_1)\right\}$$

Where v_1=velocity of jet,

u_1=peripheral velocity of buckets acting in the opposite direction of v_1.

$$\therefore \quad F_u=\frac{62{\cdot}4\times13{\cdot}35}{32{\cdot}2}\times(312+73)=10{,}000 \text{ lb}$$

Now, let the mean radius of wheel be R_1, then

Torque or turning moment $M_t=F_u\,.\,R_1\doteq10{,}000\,.\,R_1$

Let I be the moment of inertia of all moving parts, then

$\frac{1}{2}I\,.\,\omega_1^2$=kinetic energy of all moving parts

=11,200,000 ft lb

where $\omega_1\doteq$angular velocity in radians/sec

$$=\frac{u_1}{R_1}=\frac{146}{R_1}$$

$$\therefore \qquad I=\frac{11,200,000}{\frac{1}{2}\,.\,\omega_1^2}=\frac{11,200,000\times 2\,.\,R_1^2}{146\times 146}$$

and torque or turning moment $M_t=I\,.\,\alpha$

where α=angular retardation in rad/sec²

Hence $F_u\,.\,R_1=I\,.\,\alpha$

or $$10,000\,.\,R_1=\frac{11,200,000\times 2\,.\,R_1^2}{146\times 146}\,.\,\alpha$$

or $$\alpha=\frac{10,000\times 146\times 146\,.\,R_1}{11,200,000\times 2\,.\,R_1^2}=\frac{9{\cdot}48}{R_1}$$

but $$\alpha=\frac{\omega}{t}=\frac{\frac{u_1}{R_1}}{t}$$

$$\therefore \qquad \frac{146}{R_1\,.\,t}=\frac{9{\cdot}48}{R_1}$$

or $$t=\frac{146}{9{\cdot}48}=\textbf{15·4 seconds} \qquad \textit{Answer}$$

UNSOLVED PROBLEMS

6.1 Write a short note how Pelton turbine was developed from Girard turbine.

6.2 Describe with the help of a simple sketch the main components of Pelton turbine.

6.3 Write short notes on—

By-pass nozzle, deflector, Seewar jet diffusor.

6.4 Why is the jet deflected by the buckets to 160° to 165° instead of 180° ?

6.5 How are the buckets of Pelton turbine secured to the rim of the wheel ?

6.6 Why a Pelton bucket has a notch like edge ? How does it effect the efficiency of the turbine ?

6.7 State the functions of casing of Pelton turbine. Why has it no hydraulic function to perform ?

6.8 What is the uses of hydraulic brake ? Describe the ratio of brake jet diameter to main jet diameter.

6.9 Sketch the layout of single jet, double jet and four-jet Pelton turbines.

6.10 Sketch the arrangements of single overhung and double overhung Pelton wheels.

6.11 What is meant by a single overhung and a double overhung Pelton turbine ? What are the advantages of the latter over the former ?

6.12 Under what conditions would you use more runners for a Pelton turbine ? *(Delhi University—1957)*

6.13 Under what conditions would you employ more than one nozzle for a Pelton turbine? What are the disadvantages of such an arrangement? *(AMIE—May 1955)*

6.14 On what factors does the number of nozzles depend in case of Pelton turbines?

6.15 What importance has the ratio

$$m=\frac{\text{mean diameter of Pelton wheel}}{\text{least diameter of jet}}$$

in designing the Pelton turbine. *(AMIE—Nov 1954)*

6.16 Evolve a formula for the specific speed of a Pelton wheel in the following form—

$$N_s=K\,.\,\sqrt{\eta}\,.\,\frac{d}{D}$$

where N_s=specific speed, η=overall efficiency,
d=diameter of jet, D=diameter of bucket circle,
K=constant.

Find out the value of K when co-efficient of discharge for nozzle is 0 96 and the bucket speed=0·47 of jet speed. *(AMIE—May 1954)*

6.17 On what factors the selection of speed of Pelton turbine depend?

6.18 How would you determine the minimum number of buckets for a Pelton turbine?

6.19 Sketch a Pelton wheel bucket and show how it is attached to the wheel. *(Madras University—1954)*

6.20 Write a short note on Turgo-Impulse turbine. How does it differ from Pelton turbine?

6.21 Derive an expression giving the relationship between the jet speed and bucket speed for maximum efficiency of a Pelton wheel. *(Madras University—1957)*

6.22 Prove that the horsepower of a Pelton turbine per unit head and per unit jet diameter is two-third.

6.23 Differentiate between volumetric, head, mechanical and overall efficiencies of a Pelton turbine.

6.24 A Pelton wheel is to be designed to develop 1,000 HP at 400 rpm. It is to be supplied with water from a reservoir whose level is 800 ft above the wheel through a pipe 3,000 ft long. The pipe line losses are to be 5% of gross head. The co-efficient of friction is 0 005. The bucket speed is to be 0·46 of jet speed and efficiency of wheel is 85%. Calculate the pipe line diameter, jet diameter and wheel diameter. (Pipe—1 ft 6 in., Jet—3·38 in., wheel - 4ft 10 in.) *(Punjab University—1955)*

6.25 The highest head so far used anywhere in the world upto date is for running a Pelton turbine, installed at REISSECK (Austria) which is rated at 31,000 BHP at 750 rpm under an effective head of 5,800 ft.

Determine—

a) Least diameter of jet,

b) Mean diameter of the Pelton runner,

c) Number of buckets of Pelton runner,

using emperical relation

$$Z=0\ 5\ m+15$$

where Z=no. of buckets

and $$m=\frac{\text{mean diameter of Pelton runner}}{\text{least diameter of jet}}$$

Assume suitable values of oveıall tuıbine efficiency (η_t), jet velocity co-efficient (K_{v_1}) and wheel peripheral velocity co-efficient or speed ratio (K_{u_1}) fıom the following table which gives all the required values against N_s, the specific speed.

N_s	7·5	5 4	2·5
η_t	0·82	0 89	0·9
K_{v_1}	0 98	0 985	0·99
K_{u_1}	0 44	0 45	0 46

(4 in. ; 7 ft 2 ın. ; 36) (*Jadavpur University—1957*)

6.26 Determine the dıscharge, least jet diameter, mean runner diameter, jet ratio and number of buckets of the following Pelton turbine :

Nainital Power House—

BHP=172, Head=950 ft, Speed=600 rpm

Assume the following constants :

$K_{v_1}=0·98$, $K_{u_1}=0·45$ $\eta_t=0·75$

(2·13 cfs ; 1·27 in. ; 3·54 ft ; 33·4 ; 32) (*Punjab University—1958 S*)

6.27 The buckets of a Pelton wheel deflects the jet impinging on them through 150°. Pressure at the nozzle is 600 lb/sq in. The centre line of the buckets is on a circle of dia 100 in. Find—

a) best speed of the wheel,

b) theoretical efficiency,

c) theoretical HP at a nozzle flow of 100 cusecs,

d) specific speed.

Prove the formula used for specific speed.

(310 rpm ; 90% ; 15,700 HP ; 4·36) (*Punjab University—1954*)

6.28 A Pelton wheel has a mean bucket speed of 40 ft/sec and is supplied with water at the rate of 9,000 gpm under a head of 100 ft. If the

buckets deflect the jet through an angle of 160°, find the HP and the efficiency of the turbine.

(256 HP ; 94%) (*Madras University—1957 & UPSC—Nov 1938*)

6.29 A Pelton wheel, which may be assumed to have semi-cylindrical buckets, is 2 ft diameter. The available pressure at the nozzle when it is closed is 200 lb per sq in. and the supply when the nozzle is open is 100 cu ft/min. If the revolutions are 600 per minute, calculate the horsepower of the Pelton wheel and its efficiency. (79·2 HP ; 91·2%) (*Madras University—1953*)

6.30 A Pelton turbine is required to develop 3,250 HP under a head of 900 ft. Its efficiency is 80%. Specific speed is 3·6 and the ratio of the rim speed to the jet speed is 0·47.
Assume $C=0.97$ for the nozzle. Calculate the jet diameter and the rpm of the wheel.

(5·58 in. ; 312 rpm) (*AICTE—1957*)

6.31 A jet of water impinges on a series of curved vanes at an angle of 30° to the direction of motion of the vanes while entering and leaves the vanes horizontally. The head under which the jet issues from the nozzle is 100 ft, the co-efficient of velocity for the nozzle is 0·9 and the diameter of the jet after leaving the nozzle is 2 inches. The speed of the vanes is 30 ft/sec and the relative velocity of the water at outlet is 0·8 times the relative velocity at inlet. Calculate

a) the angle of vane tips at inlet,

b) the horsepower developed by the jet,

and *c*) the efficiency of the system.

(48° ; 11·9 HP ; 66·7%) (*Rajputana University—1957*)

6.32 Show that in an impulse water turbine with equal bucket angles at inlet and outlet and with no frictional and other losses, the maximum efficiency is equal to $\cos^2\alpha$, where α is the jet angle with the direction of motion of the turbine.

A Pelton wheel working under a total head of 1,500 ft is supplied with 10 cfs/sec of water through a pipe 2¼ miles long, co-efficient of friction of the pipe material being 0·007. If the wheel develops 800 HP with a wheel efficiency of 85%, determine the necessary diameters of the nozzle and the pipe.

Neglect losses in nozzles.

(3·576 in. ; 1·046 ft) (*Roorkee University—1958*)

6.33 A Pelton turbine is required to develop 5,000 HP at 300 rpm under a head of 1,200 ft. Determine the least diameter of the jet impinging on the buckets of the turbine wheel if the co-efficient of velocity is 0·985. Assume turbine overall efficiency as 0.9. What would be the mean diameter of the wheel if speed ratio is 0·45. Calculate the size of the nozzle used if its diameter is 25% larger than the least diameter of jet. How many buckets will be used for the wheel for the best efficiency ? (5¼ in., 8 ft., 6$\frac{9}{16}$ in., 24)

6.34 Show that in a Pelton wheel, where the buckets deflect the water through an angle of $180°-\theta$, the hydraulic efficiency of the wheel is given by—

$$\eta_h = \frac{2v\,(V-v)\,(1+\cos\theta)}{V^2}$$

if V is the velocity of jet and v the velocity of the wheel at the pitch radius.

If the bucket speed is 100 ft per sec, $\theta=20°$, and the jet diameter is 5 in, find the jet velocity for maximum efficiency and the corresponding WHP of the turbine.
(200 ft/sec ; 1920 HP as Available Power) (*Bombay University* – 1957)

6.35 A Pelton wheel is supplied with water in the nozzle box at a pressure head of 500 ft. The velocity co-efficient for the nozzle is 0·96. The relative velocity of water while in contact with the buckets is turned through 150°, and is also reduced by 15% owing to friction loss. The required ratio of bucket to jet speed is 0·47 and 0 8 of the energy imparted to the wheel, as determined by the velocity diagrams, is available at the output shaft. The bucket circle diameter is ten times the jet diameter. Find the jet and wheel diameters, and the speed of rotation to develop 200 HP under these conditions. (2·426 in., 24 26 in., 765 rpm)

6.36 The water available for a Pelton wheel is 150 cu ft/sec and total head from reservoir to nozzle is 900 ft. The turbine has two runners with two jets per runner. All the four jets have same diameters. The pipe line is 10,000 ft long. The efficiency of the power transmission through pipe line and nozzle is 91% and efficiency of each runner is 90%. The velocity co-efficient of each nozzle is 0·975 and the co-efficient of friction for the pipe is 0·0045.

Determine—

i) the horse power developed by turbine
ii) the diameter of jet
iii) the diameter of pipe line.
(12,500 HP, 5·2 in , 4·5 ft) (*London University—1945*)

6.37 The following data were obtained from a test on a Pelton turbine :—

Area of jet = 12·0 sq in , Discharge = 6·35 cusecs
Head at nozzle = 100 ft ; BHP = 56·0
Horsepower absorbed in friction and windage = 3 0 HP

Determine the energy lost in the nozzle and also the energy absorbed due to losses in the wheel at discharge.
(7·1 HP ; 5·9 HP) (*Roorkee University—1957*)

6.38 In Mandi Hydro-Electric Scheme at Jogindar Nagar, the water from the high level reservoir at Brot is led through the tunnel (constructed in the mountains) which is connected to four penstocks, each of average diameter of 51 in. and each finally discharging through a nozzle of 8 in. diameter. The jet of each penstock impinges on the buckets of a Pelton turbine of horizontal type. Thus there are four turbines, each having its own penstock with a nozzle. Each of the penstock is one mile long. The total head available at the centre line of nozzle is 1,668 ft. Calculate the velocity of the water jet issuing out of each nozzle, assuming co-efficient of velocity as 0·98.

The mean diameter of the runner of Pelton wheel is 6 ft—6$\frac{3}{4}$ in. The speed ratio is 0·46. If the buckets turn the water to 165°, Calculate—

a) total water power available at the headworks assuming co-efficient of friction $f=0{\cdot}008$,

b) power of each jet,

c) hydraulic power of each turbine if 3% of the jet water drops to the tail race without doing any useful work,

d) BHP of the turbine is 2% of the power calculated under *c*) is lost in windage and bearing friction.

[321 ft/sec *a*) 86,800 HP *b*) 20,150 HP *c*) 19,200 HP *d*) 18,816 BHP]
(*Delhi University—1958*)

6.39 A Pelton turbine has a mean bucket speed of 130 ft/sec, and is supplied with water at the rate of 120 gallons/sec. The head of water behind the nozzle being 1,380 ft. If the jet is deflected by the buckets through 170°, find the HP developed and the efficiency of the wheel. Assume $\beta_1=0$. (2,840 HP, 94·4%) (*AMIE—1948*)

6.40 A Pelton wheel is to be designed to the following specifications :—

Power = 16,250 Metric HP ;	Head = 381 *m*
Speed = 750 RPM ;	Overall Efficiency = 86 per cent

Jet diameter not to exceed $\frac{1}{8}$ times the wheel diameter.

Determine—

a) the wheel diameter

b) number of jets required

and *c*) the diameter of the jet. (1 *m* ; 4 ; 118 *mm*)
(*Rajasthan University—1957* ; Converted to metric units)

6.41 A double overhung Pelton wheel unit is to operate a 30,000 KW generator under an effective head of 305 *m* at the base of the nozzle. Find the size of jet, mean diameter of runner, synchronous speed and specific speed of each wheel. Assume generator efficiency 93%, Pelton wheel efficiency 85%, co-efficient of nozzle velocity 0·97, speed ratio 0·46 and jet ratio 12.
(330 *mm* ; 3,975 *mm* ; 167·67 rpm ; 19·8 metric units)
(*Punjab University—1959A* ; Converted to metric units)

6.42 A twin jet Pelton wheel has a mean bucket circle diameter of 1,680 *mm* and runs at 500 rpm. When the jets are 152·5 *mm* diameter, the available head at the nozzle is 488 *m*. Assuming co-efficient of velocity for the nozzle as 0·98, outlet angle of bucket 15°, relative velocity of water leaving the bucket 0·88 of that of inlet ; and windage and mechanical losses 3% of the water horsepower supplied, find—

a) the water horsepower supplied and brake horsepower,

b) the force of one jet on the bucket,

c) the overall efficiency. (22,740 metric HP ; 17,100 kg ; 84·7%)
[*AMI Mech E* (*Lond.*)—*April 1955* ; Converted to metric units]

6.43 *a*) Obtain an expression for the theoretical efficiency of a Pelton wheel when the angle of the bucket at exit makes an angle of θ with the direction of jet. Show by a diagram how the efficiency of the wheel will vary as the relation of velocity of the jet to the velocity of bucket is varied.

b) A Pelton wheel has a mean speed of 12·2 *m*/sec and is supplied with water at the rate of 1,370 litres per second under a head of 30·5 *m*. If the buckets deflect the jet through an angle of 160°, find the horsepower and efficiency of the wheel.

(528 metric HP ; 95·4%)

(*Punjab University—1960 S* ; Converted to metric units)

CHAPTER 7

REACTION TURBINES

7.1 Francis Turbine—The modern Francis water turbine is an inward mixed flow reaction turbine *i.e.* the water, under pressure, enters the runner from the guide vanes towards the centre in radial direction and discharges out of the runner axially. The Francis turbine operates under medium heads and also requires medium quantity of water. It is employed in the medium head power plants. This type of turbine covers a wide range of heads. Water is brought down to the turbine and directed to a number of stationary orifices fixed all around the circumference of the runner (Fig 7.1). These stationary orifices are commonly termed as *guide vanes or wicket gates.*

A part of the head acting on the turbine is transformed into kinetic energy and the rest remains as pressure head. There is a difference of pressure between the guide vanes and the runner which is called the *reaction* pressure, and is responsible for the motion of the runner. That is why a Francis turbine is also known as a *reaction turbine.*

In this turbine the pressure at the inlet is more than that at the outlet. This means that the water in the turbine must flow in a closed conduit. Unlike the Pelton type, where the water strikes only a few of the runner buckets at a time, in the Francis turbine the runner is always full of water. The movement of the runner is affected by the change of both the potential and the kinetic energies of water. After doing its work the water is discharged to the tail race through a closed tube of gradually enlarging section. This tube is known as the *draft tube*. It does not allow water to fall freely to the tail race level as in the Pelton turbine. The free end of the draft tube is submerged deep in the tail water, making, thus, the entire water passage, right from the head race upto the tail race, totally enclosed.

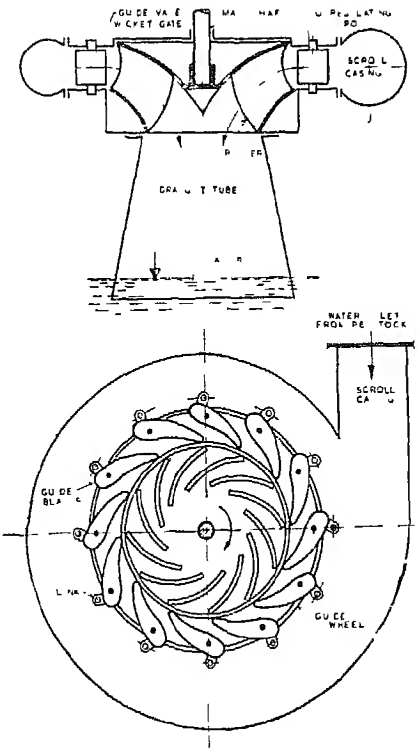


Fig 7.1 Outline of a Francis Turbine

7.2 Different Types of Francis Turbine—There are two main types of Francis turbine, namely—

i) Closed type, and

ii) Open flume type.

Closed Type Francis Turbine—In this type the water is led to the turbine through the penstock whose end is connected to the spiral casing of the turbine. This spiral casing directs the water evenly to the guide blades (Fig 7.1 and 7.2). The water then passes through the runner and finally goes to the tail race through the draft tube. The closed type Francis turbine may be of two types, horizontal or vertical. The horizontal type is used for medium and high heads and the vertical type for medium and low heads. The large units are, however, mostly of the vertical type.

Open Flume Type Francis Turbine—In the open flume type Francis turbine a concrete chamber replaces the spiral casing (*see* Fig 7.3). This type of turbine is used for heads upto 20 to 30 ft (or 6 to 9 *m*). In many power plants Kaplan turbine is fast replacing this type of turbine.

They are, however, still employed where the quantity of water is small and the fluctuation in head is more. The turbine is surrounded by water. The guide vanes are adjusted by means of a mechanism, also, submerged in water. As the heads are low, the mechanical stresses in the levers and links are also less. The modern tendency is towards

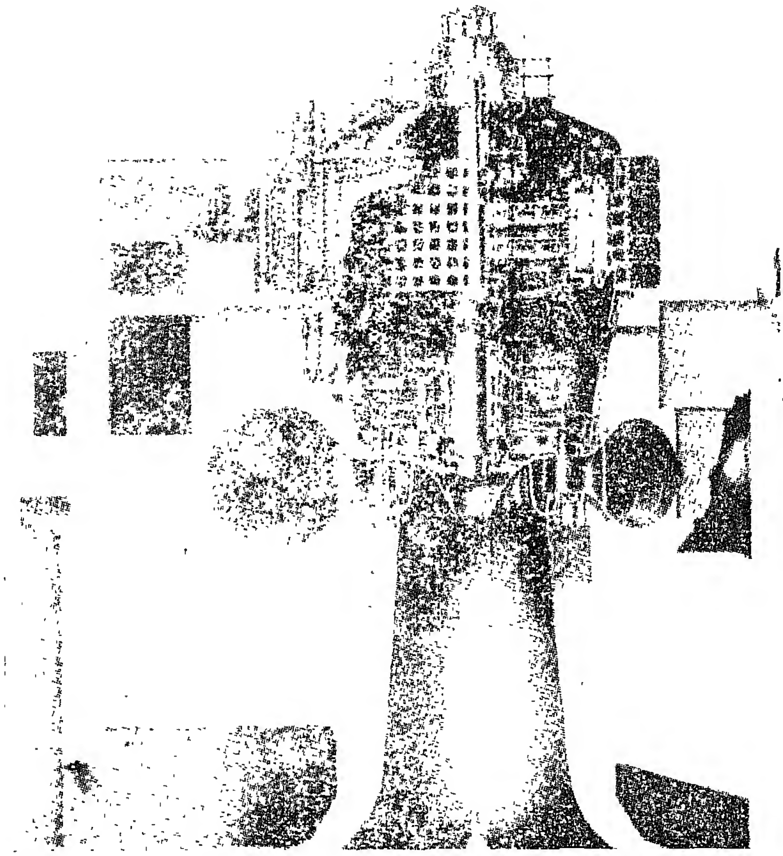


Fig 7.2 Medium Head, Vertical Francis Turbine for Niagara Falls Developing 70,000 HP at 213 ft Head (Manufactured by Allis-Chambers Mfg Co Ltd, USA)

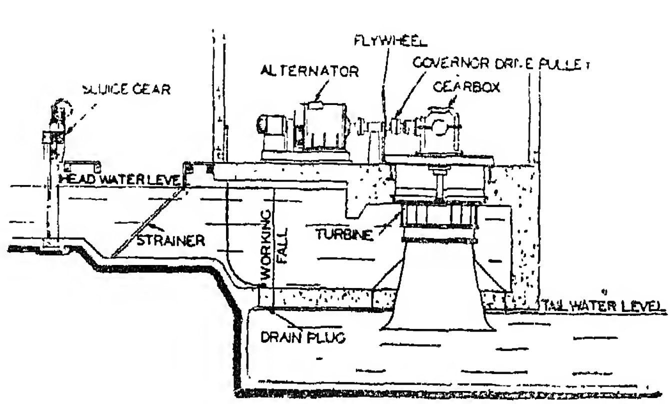


Fig 7.3 Open Flume, Vertical Francis Turbine Layout (Manufacturers : Gilbert Gilkes & Gordon Ltd, UK)

equipping the turbine with an external regulating mechanism. In case of a horizontal turbine that shaft is carried through a gland in the chamber wall. The guide vanes, the runner etc, which are submerged in water are on one side of the masonry wall. On the other side is the bearing and electric generator. As the bearing is out of water it is easily accessible. The vertical type open flume turbine is submerged in water through a vertical shaft, and the bearing which is fixed on a masonry structure carries the full turbine load. The generator is, generally, connected horizontally through gears, as it must run at a higher speed than the turbine.

Multiple Turbine—If the generator is to be directly coupled to the turbine, it must run at a higher speed. With smaller heads and more quantity of water (open flume type) the required speed is achieved by making a twin or triple turbine, *i.e.*, a turbine having two or three runners. These runners are keyed to a common shaft and discharge water in a common draft tube.

This arrangement reduces the cost of the generator but the cost of the turbine is increased. On the other hand the axial thrust on the bearing is balanced to a certain extent and the bearing can thus be made cheaper. Most of the multiple turbines are horizontal. In case of vertical multiple turbines less space is required, but on the other hand the installation becomes complicated and the lowest turbine runner lies much deeper in water.

Main Components of Francis Turbine

7.3 Spiral Casing or Scroll Casing (Fig 7.4)—To avoid loss of efficiency, the flow of water from the penstock to the runner should be such that it will not form eddies. In order to distribute the water around the guide ring evenly, the scroll casing is designed with a cross-sectional area reducing uniformly around the circumference, maximum at the entrance and nearly zero at the tip. This gives a spiral shape and hence the casing is named as spiral casing. In the case of big units, the inside circumference of casing has *stay vanes* each directing the water to the guide vanes. The position of the inlet to the spiral casing depends

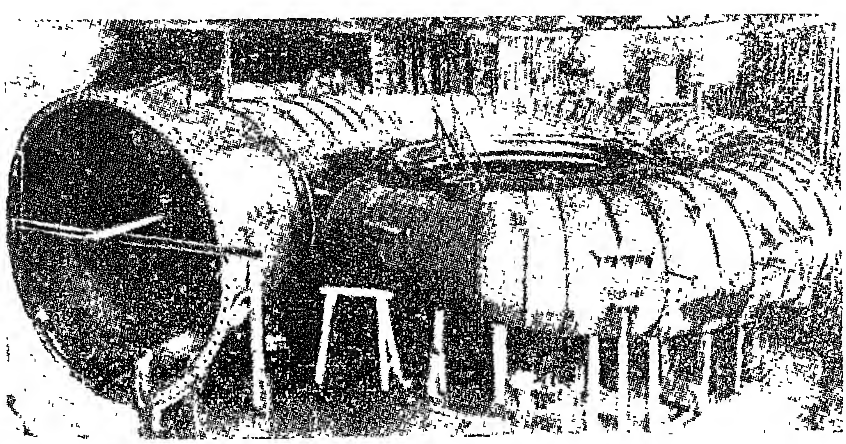


Fig 7.4 Spiral Casing Manufactured by Escher Wyss, has an Inlet Diameter of 15½ ft for Sungari Turbines (Manchuria), each developing 1,15,000 HP.

on the direction of water flowing out of the penstock which may vary according to the site. The spiral casings are provided with inspection holes and also with pressure gauge connections. If the spiral casing has to be quite large, it is made in parts for ease in transport. The spiral casings for vertical reaction turbines are partly or completely grounded in. In such cases the runner is not easily accessible and the dismantling is not so easy as in horizontal type turbine.

The material of scroll casing depends upon the heads which is as follows—

Concrete without steel plate lining	—upto about 100 ft (or 30 *m*)
Welded rolled steel plate	—upto about 300 ft (or 90 *m*)
Cast steel	—more than 300 ft (or 90 *m*)

7.4 Guide Mechanism (Fig 7.1)—The guide vanes or wicket gates, as they are sometimes called, are fixed between two rings in the form of a wheel, known as *guide wheel*. The guide vanes have a cross-section known as aerofoil section. This particular cross-section allows water to pass over them without forming eddies. Each guide vane can rotate about its pivot centre which is connected to the regulating ring by means of a link and a lever. The ring is connected to the regulating shaft by means of regulating rods, generally, two in number. By rotating the regulating shaft the guide vanes can be closed or opened thus allowing a variable quantity of water according to the needs. The regulating shaft is operated by means of a governor whose function is to keep the speed of the turbine constant at varying loads. With a decrease in load the speed of the turbine always tends to increase. To bring the speed back to the rated value, the governor is used to reduce the guide vane opening thereby allowing less water to strike the runner. The guide vanes are generally made of cast steel.

7.5 Runner and Turbine Main Shaft (See Fig 7.5)—The flow in the runner of a modern Francis turbine is not purely radial but a combination of radial and axial. The flow is inward, that is, from the periphery towards the centre. The height of the runner depends upon the specific speed. The high specific speed runner is higher than the one which has a low specific speed because the former has to work with a large amount of water. The runners may be classified as *i*) slow; *ii*) medium and *iii*) fast depending upon the specific speed (see Art 5.15). The runner may be cast in one piece or may be made of separate steel plates welded together. The

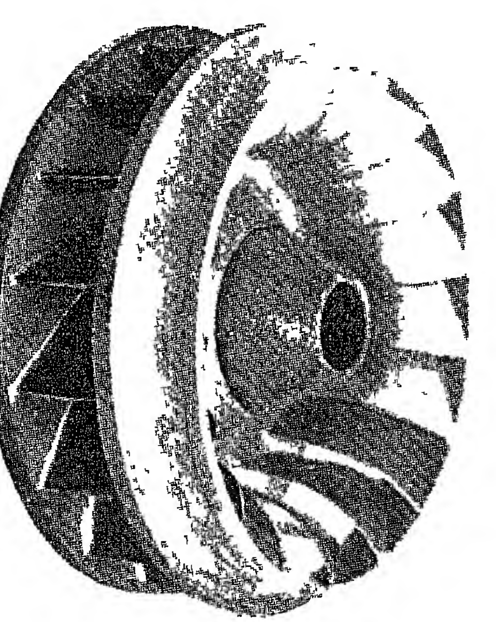


Fig 7.5 Model (1 : 7·56 of Francis Turbine Runner for Bhakra Dam Power House, Manufactured by Hitachi Ltd, Tokyo, Japan.

runners are made of cast iron for small output, cast steel for large output and stainless steel or a non-ferrous metal like broze, when the water is chemically impure and there is danger of corrosion. The runner blades should be carefully finished. Francis runners as large as 18 ft—10½ in. (or 5·75*m*) in diameter (for Drieprostrio—USSR) have been manufactured so far.

The runner is keyed to the shaft which may be vertical or horizontal. The turbine is accordingly specified as vertical or horizontal type.

The shaft is generally made of steel and is forged. It is provided with a collar for transmitting the axial thrust to the bearing.

The turbine is generally provided with one bearing (Fig 7.2). In vertical type the bearing carries full runner load and acts as thrust-cum-supporting bearing. Right selection of bearing is, therefore, extremely important. The lubrication of the thrust bearing also plays an important part in the running of the turbine. For this purpose an oil pump driven by the main turbine shaft is generally employed.

7.6 **Draft tube** (Fig 7.1 and 7.2)—The water after doing work on the runner passes on the tail race through a draft tube which is a riveted steel plate pipe or a concrete tunnel, its cross-section gradually increasing towards the outlet. Thus draft tube is a conduit which connects the runner exit to the tail race. The tube should be drowned—approximately 3 ft (or one metre) below the lowest tail race level. The functions of the draft tube are as follows :—

i) If the water is discharged freely from the runner, the turbine will work under a head equal to the height of the head race water-level above the runner outlet. If an airtight draft tube connects the runner to the tail race, workable head is increased by an amount equal to the height of the runner outlet above the tail race.

The draft tube will, thus, permit a negative (suction) head to be established at the runner outlet thus making it possible to install the turbine above the tail race without loss of head. This can be explained as follows—

The pressure in the draft tube at the tail race level is atmospheric. If the cross-section of draft tube is kept uniform, the pressure at the runner outlet is equal to the atmospheric pressure minus the height of runner outlet above the tail race level. The available head, measured from head race level to the discharge side of the turbine, is thus the same as if the turbine were erected at tail race level and discharged under atmospheric pressure.

ii) The water leaving the runner still possesses a high velocity and this kinetic energy would be lost if it is discharged freely as in a Pelton turbine. By employing a draft tube of increasing cross-section, the enclosed conduit is extended up to the outlet end of the tube and discharge takes place at a much reduced velocity thus resulting in a gain of kinetic head. This increases the negative pressure head at turbine runner exit, with which the net working head on the turbine increases. With the increase in net working head the turbine output will also increase, thus raising the efficiency of turbine.

7 7 Different Types of Draft Tubes—The draft tube is an integral part of a reaction turbine. The velocity energy of water at runner exit is 3 to 15% of the net working head in case of Francis turbines, depending upon the specific speed. As the specific speed increases, the value of velocity energy at the runner exit rises and it will be nearly 45% in case of a Kaplan turbines. Hence for high specific Francis turbine as well as for Kaplan turbine, the second function of draft tube, described in Art. 7 6, *i e.* recovery of kinetic energy at discharge, is more important. Therefore great attention is paid to the shape of draft tube especially meant for high specific speed turbines.

The following are some types of draft tubes, employed in the field :—

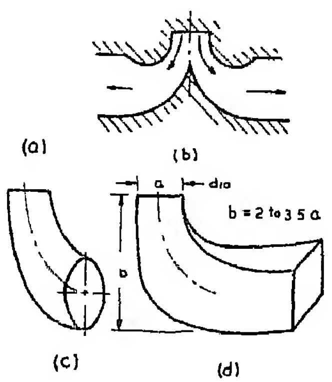


Fig 7 6 Different types of Draft Tubes

a) Straight Diverging Tube
b) Moody Spreading Tube or "Hydracone"
c) Simple Elbow Tube
d) Elbow Tube having Circular Cross-section at Inlet but Rectangular at Outlet

a) *Straight Divergent Tube* (Fig. 7.6*a*)—The shape of this tube is that of a frustrum of a cone. It is employed for low specific speed, vertical shaft Francis turbine. The maximum cone angle is 8° (or $\alpha=4°$). Experiments have shown that if this angle is greater than 8°, the water detaches away from the inner wall of the tube while flowing downwards, forming vortices and causing loss of head. The tube must discharge sufficiently low under tail water level. The maximum efficiency which this type of draft tube can yield is 90%, because the kinetic head to be recovered is less. This type of Draft tube improves speed regulation on falling load.

b) *Moody Spreading Tube or "Hydracone"* (Fig 7.6*b*)—It has a solid core in the entire bottom central portion of tube, which spreads the flow on the exit side. The central cone arrangement is made to reduce the whirl action of discharging water which may be developed due to the high velocity of water at runner exit and which is likely to cause eddy losses. The efficiency of such a draft tube is about 85%.

c) *Simple Elbow Tube* (Fig 7.6*c*)—In order to keep down the cost of excavation, particularly in rock, the vertical length of the draft tube should be minimum. Since the draft tube exit diameter should be as large as possible to recover the kinetic head and at the same time the maximum value of the cone angle is fixed, the draft tube must be bent to keep its definite length. Simple elbow type draft tube will serve such a purpose. Its efficiency is, however, low, about 60%.

d) *Elbow Type with a Circular Inlet and a Rectangular Outlet Section* (Fig 7.6*d*)—This type of draft tube has been designed to turn the water from the vertical to the horizontal direction with a minimum depth of excavation and at the same time having a high efficiency. The transition from a circular section in the vertical leg to a rectangular section in the horizontal leg takes place in the bend. The horizontal portion of the draft

tube is generally inclined upwards to lead the water gradually to the level of the tail race and to prevent entry of air from the exit end. The exit end of the tube must be totally immersed in water

In order to avoid any whirl component of velocity of water at runner exit, one or two piers are constructed in the bend of the draft tube. Such piers behave similar to the Central core of Moody's spreading tube, described above. Fig 7 7 (*a*) and (*b*) show the single and double pier elbow type draft tubes.

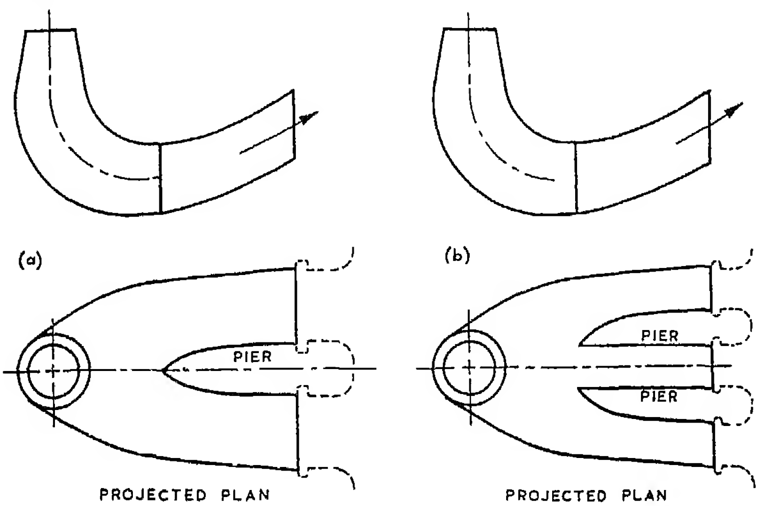


Fig 7.7 Elbow Type Draft Tube
a) Single Pier Type
b) Double Pier Type

7.8 Notable Francis Turbine Installations of the World— The most powerful Francis turbine in the world is being manufactured by NOHAB (Sweden) for Furnas Power House, Brazil, which will develop 225,000 HP at a net head of 98·7 *m* (323 ft—11 in.). The power house will be equipped with four such turbines. So far the most powerful Francis turbine also manufactured by NOHAB (Sweden), is installed at Stornorrfors Power Station, Sweden, developing 2,00,000 HP under a net head of 75 *m* (246 ft).

The most powerful Francis turbine in USA to date is installed at Grand Coulee, Washington (USA), which develops 165,000 HP at 330 ft head and 120 rpm. There are 18 such turbines in the power house. Still bigger turbine is installed at Shasta Dam (USA), which is rated at 187,000 HP at 475 ft head and 138·5 rpm, but it does not give its maximum output as it is coupled to a low-capacity generator developing 75,000 KW.

The Francis turbine installed at Bortla—Orgues (France), develops 158,000 BHP at 366 ft head and 187·5 rpm, and has been manufactured by Charmilles (Switzerland).

The powerful Francis turbine in Japan is at Suhio and Julu, manufactured by a Japanese Firm, generates 145,000 HP at 250 ft head.

7.9 SOME FRANCIS TURBINE

S. No.	Scheme	Site of Power House	Source of Water
1	Bhakra Dam Project (Punjab)	Bhakra (8 miles from Nangal Rly. Station)	Sutlej River (Bhakra Dam)
2	Ranbir Canal Scheme (Jammu & Kashmir)	Satwari (near Jammu where River Tawi & Ranbir Crosses)	Chenar River
3	Ginderbal Hydro-Station (Jammu & Kashmir)	Ginderbal (13 miles from Srinagar)	Sind River
4	Udhampur (Jammu & Kashmir)	Udhampur (42 miles from Jammu)	Tawi Canal
5	Hirakud Dam Project I (Orissa)	Hirakud (7 miles from Sambalpur)	Mahanadi (Hirakud Dam)
6	Damodar Valley Corp. (DVC), West Bengal & Bihar	Tilaya (District Hazaribagh, Bihar, nearest Railway Station Kodarma)	Barakar River (Tilaya Dam)
7	Mayurakshi Project (West Bengal & Bihar) (Seasonal Power Project linked with DVC grid)	Messanjar (Dist. Santhal Parganas) Bihar	Mayurakshi River
8	Gokak Hydro Electric Scheme (Bombay)	Gokak (Dist Belgaum)	Gokakprabha River
9	Chambal Hydro-Electric Scheme (Rajasthan)	Gandhisagar	Chambal River
10	Tungabhadra Hydro-Electric Scheme (Andhra)	*a*) Tungabhadra Dam *b*) Hampi	Tungabhadra Tungabhadra
11	Machkund Hydro-Electric Project (Andhra)	Machkund (125 miles from Vizagapatum)	Machkund River (Duduma Dam)
12	Cauvery Hydro-Electric Scheme (Mysore)	Sivasamudram (40 miles from Mysore)	Cauvery River
13	Cauvery-Mettur Project (Madras)	Mettur (District Salem) (Madras)	Cauvery River (Mettur Dam)
14	Periyar Hydro-Electric Scheme (Madras)	Periyar	Periyar
15	Kerala Govt	Peringal Kutha	Chalakudy River
16	Kerala Govt	Neriamangalam	Tailwaters of Sengulum & Penniar
17	Upper Sileru Hydro-Electric Project (Ist Stage)—Andhra	Upper Sileru	Sileru River at Guntavada Dam
18	Rihand Dam Project (U.P.)	(Mirpapur Rly. Stn.) Rihand	Rihand Dam

INSTALLATIONS IN INDIA

Manufacturers or Suppliers	No. of Turbines	Turbine Specifications: Power HP each	Head ft	Discharge cusecs each	Speed rpm	Vertical or Horizontal	
Hitachi (Japan)	5	1,50,000	400	3,610	166·7	Vertical	—
—	4	700	26	—	300	Horizontal	—
Escher Wyss	3	4,365	137 *m*	2760 lit/sec	1,000	Horizontal	—
Voith	2	490	190	27	1,000	Horizontal	—
Voith	2	32,000	75 (74 to 120)	4,500	—	Vertical	—
—	2	2,500	49 to 77	—	250	Vertical	$Z_1=20$ $Z_2=15$ $N_1=487$ rpm
—	2	2,500	—	—	—	—	—
a) —	3	900	200	—	—	—	—
b) —	1	1,500	—	—	—	—	—
Voith	3	34,000	150	—	—	—	—
a) Escher Wyss	1	680	88	—	—	—	—
b) Charmilles	2	12,500	110	1,100	214	Vertical	—
a) Morgan Smith	3	25,000	850	250	600	Vertical	—
b) Voith	3	36,000	840	—	—	Vertical	—
a) Boving	4	9,000	405	—	375	Horizontal	—
b) Boving	5	5,600	—	—	600	Horizontal	—
c) Escher Wyss	1	5,600	—	—	600	Horizontal	—
English Electric	4	4,000 to 16,000	60 to 160	—	250	Horizontal	$D_1=63{\cdot}5''$ $Z_1=20$ $A_1=12$ ft^2
Voith	3	50,000	1,225	400	750	Vertical	$N_s=21{\cdot}1$
Charmilles	4	13,500	595	—	600	Vertical	—
Charmilles	3	23,000	650	—	600	Vertical	—
Under construction	2	95,000	310	700	—	—	—
English Electric	5	77,000	225	—	150	Vertical	—

The hydel station at Sungari River (Manchuria) has eight Francis turbines, each developing 1,15,000 HP under a head of 170 to 220 ft.

The highest head used so far for a Francis turbine is 1,780 ft. This turbine, manufactured by Kvaerner Burg, Oslo, is in operation at Hemsil (Norway), developing 50,000 HP at 750 rpm. Next highest head turbine, manufactured by Escher Wyss, is installed at Fionnay (Switzerland) developing 63,200 HP under a head of 1490 ft at 750 rpm. For Sharavati Valley Project (Mysore) Francis turbine working under 1500 ft head are under consideration.

7.10 Power - Brake horse power and available power of Francis turbine are determined by methods similar to those adopted for Pelton turbine (See Art 6.10).

Problem 7.1 The source of a hydro-electric plant is a stream whose discharge varies with the season. The rainy season lasts for 3 months and gives an average discharge of 420 cusecs to the stream. For the rest of the year the discharge is 110 cusecs. A reservoir is built such that the whole of the discharge could be utilised at a uniform rate. The mean reservoir level is 650 ft above the turbine level. The power house has three turbines whose inlets are connected to the reservoir by means of three pipe lines, each 4,200 ft long. The head loss due to friction in each of these pipes is 3 per cent of the gross head. The overall efficiency of each of the turbines is 87%. Assume $f=0{\cdot}005$.

Determine (*a*) the capacity of the reservoir, (*b*) the diameter of the pipes and (*c*) the BHP output of the station.

Solution

a) Average discharge during rainy season for three months $=420$ cusecs

Average discharge during dry season for nine months $=110$ cusecs

$\therefore$ Total discharge for the year $=(420\times3)+(110\times9)=2{,}250$ cusecs

Average discharge for one month $=\dfrac{2{,}250}{12}=187{\cdot}5$ cusecs

The reservoir receives the water during the rainy season only and at the same time a discharge of 187·5 cusecs is utilised by the power station.

$\therefore$ Discharge going to the reservoir $=420-187{\cdot}5=232{\cdot}5$ cusecs

Hence the capacity of reservoir $=232{\cdot}5\times3\times30\tfrac{1}{3}\times24\times3{,}600$ cu ft

(Assuming average number of days of a month $=30\tfrac{1}{3}$)

$$=183\times10^7 \text{ cu ft}$$

$$=\frac{183\times10^7}{43{,}560} \text{ acres ft}$$

$$=\mathbf{42{,}000 \text{ acres ft}} \qquad \textit{Answer}$$

b) Gross head $=650$ ft

Head lost due to friction, $H_{L_f}=650\times0{\cdot}03=19{\cdot}5$ ft

but $H_{L_f}=\dfrac{4f\,.\,L}{d}\,.\,\dfrac{v^2}{2g}$ where $d=$dia of pipe
$L=$Length of pipe

$$=\frac{4f.L}{2g.d}.\left(\frac{Q}{\frac{\pi}{4}d^2}\right)^2=\frac{4f.L.Q^2}{2g\times\left(\frac{\pi}{4}\right)^2.d^5}$$

where Q=quantity of water flowing through each pipe

$$=\frac{187\cdot5}{3}=62\cdot5 \text{ cusecs}$$

$$\therefore \quad 19\cdot5=\frac{4\times0\cdot005\times4{,}200\times62\cdot5\times62\cdot5}{64\cdot4\times0\cdot785\times0\cdot785\times d^5}$$

or $d=\sqrt[5]{424}$

$=$**3·35 ft** *Answer*

c) The BHP output of the station—

$$\text{BHP}=\frac{w.Q.H}{550}.\eta_t$$

$$=\frac{62\cdot4\times187\cdot5\times(650-19\ 5)\times0\cdot87}{550}$$

$=$**11,700 BHP** *Answer*

Design of Component Parts of Francis Turbine

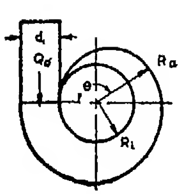


Fig 7.8 Spiral Casing

7.11 Spiral Casing - The outer curve of the spiral casing is of the form of an archimedean spiral (See Fig 7 8).

If the inlet diameter and velocity be d_i and v_i respectively, quantity of water flowing per second,

$$Q_o=v_i.\frac{\pi}{4}.d_i^2 \qquad \dots(7.1)$$

also $v_i \propto \sqrt{2g\,H}$

$=K_{v_i}.\sqrt{2g\,H}$

The maximum value of v_i is about 30 ft/sec (or 10 *m*/sec)

The inlet diameter d_i should be less than but may be equal to penstock diameter.

TABLE 7.1

Practical Data for K_{v_i}

K_{v_i}	0 2	...	...	...	0·15	
H	75	...	...	...	1,500	ft
	25	...	...	...	500	*m*
N_s	High	...	...	...	Low	

At any angle θ (See Fig 7 8)

Radius $R_\theta = R_1 + \frac{\theta}{2\pi} \cdot d_1$ ...(7.2)

Quantity of water flowing $Q = \frac{\theta}{2\pi} \cdot Q_o$ ...(7.3)

A full spiral is generally recommended for high head turbine and semi-spiral for low head installation.

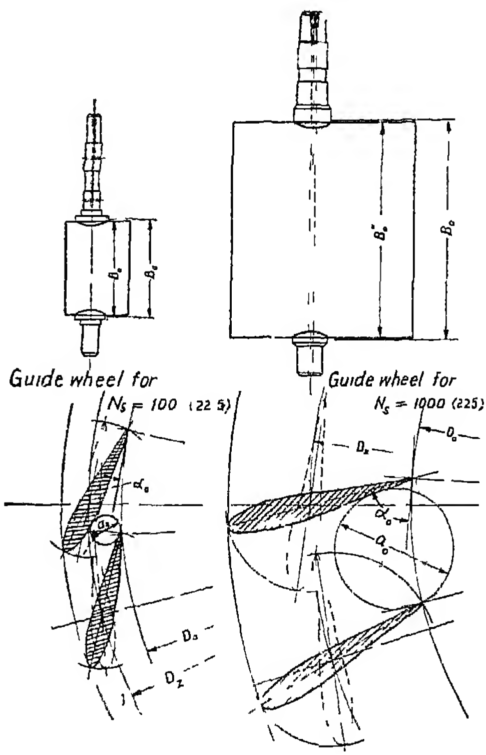


Fig 7.9 Guide Vanes and Guide Wheel

7 12 Guide Vanes—Guide Vanes have an aerofoil section to enable stream line flow (See Fig 7.9).

a) **Diameter**—Let D_o be the diameter, measured up to the trailing edge of the guide vane, then tangential velocity,

$$u_o = \frac{\pi \cdot D_o \cdot N}{60}$$

Here u_o is a hypothetical velocity assumed for the sake of convenience in the calculation of D_o. Actually guide vanes are not moving.

Since $u_o = K_{u_o} \cdot \sqrt{2g\,H}$

where $K_{u_o} = 0.7$ to 1.34 increasing with specific speed (See Fig 7.24)

also $u_a = \frac{\pi \cdot D_o \cdot N}{60}$

$$\therefore \quad D_o = \frac{60 \cdot K_{u_o} \sqrt{2g\,H}}{\pi \cdot N} \quad .. (7.4)$$

b) **Depth or Height**—Let the depth of a guide vane be B_o. Then, if the radial velocity at outlet be v_{m_o}, neglecting the thickness of trailing edge,

Area across flow $= \pi \cdot D_o \cdot B_o$

and discharge $Q = \pi \cdot D_o \cdot B_o \cdot v_{m_o}$

$$= \pi \cdot D_o \cdot B_o \cdot K_{v_{m_o}} \cdot \sqrt{2gH}$$

where $K_{v_{m_o}}$ increases from 0.15 to 0.35 with increase in specific speed (See Fig 7.24).

Finally, $B_o = \dfrac{Q}{\pi \cdot D_o \cdot K_{1_{m_o}} \cdot \sqrt{2gH}}$. (7.5)

c) **Length**—Length of the guide vane, $L_o \approx 0.3\ D_o$

d) **Number of Guide Vanes**—The number of guide vanes Z_o depends on the diameter D_o and is usually from 8 to 24. The guide vanes should be even in number so that the guide wheel can be divided evenly for ease in manufacture and transport when it is very large.

The following table will give the number of guide vanes for all types of reaction turbines.

TABLE 7.2

Practical Data for Z_o

Z_o	8	12	16	24	28	32	36	
D_o	10	→	40	→	→	→	280	inches
	Francis						Kaplan	
	250	→	1,000	→	→	→	7,000	*mm*

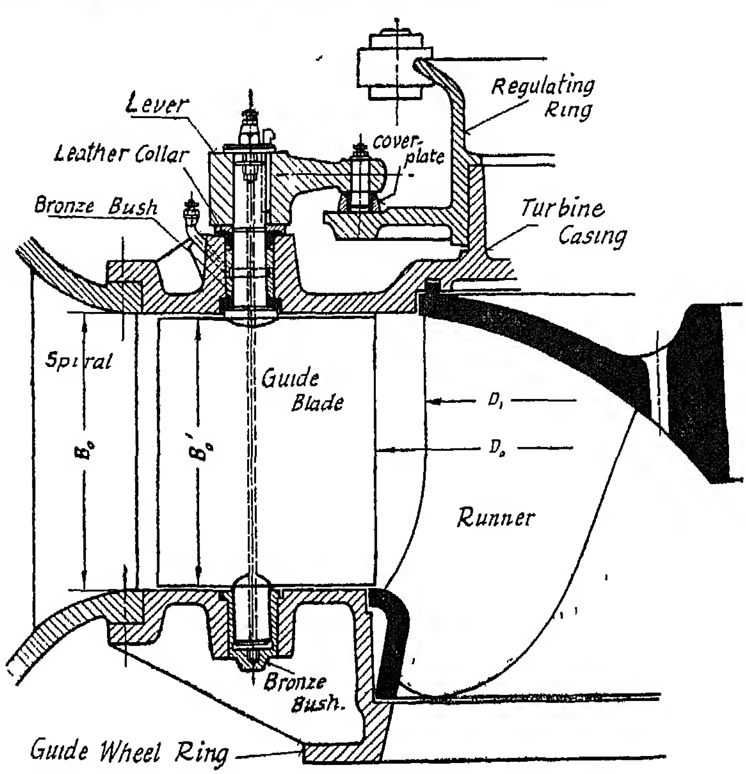


Fig 7 10 Francis Runner with Mechanism for Guide Blade Movement

7.13 Francis Runner—Inlet diameter D_1 is shown in Fig 7.10 up to the end of crown. Sometimes it is measured up to the centre of the entrance edge of runner blade.

In any case,

$$u_1 = \frac{\pi \cdot D_1 \cdot N}{60} \quad \text{and} \quad u_1 = K_{u_1} \cdot \sqrt{2gH}$$

where K_{u_1} varies from 0·62 to 0·82 increasing with specific speed (See Fig 7.24)

$$\therefore \quad D_1 = \frac{60 \cdot K_{u_1} \cdot \sqrt{2gH}}{\pi \cdot N} \qquad \text{...(7.16)}$$

Outlet diameter D_2 (See Fig 7.11) $\approx \frac{D_1}{2}$

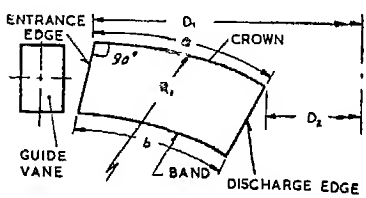


Fig 7.11 Construction of Francis Runner Outlines

Radius of curvature of crown

$$R_1 \approx \frac{D_1}{2}$$

When D_1, D_2 and R_1 are fixed, length of crown can be calculated and length b of band is 0·7 to 0·75 times a. The entrance edge is a potential line, and suitable curvature can be given to the discharge edge (See Fig 7.11).

Thickness of Runner Blades varies from $\frac{1}{4}$ inch to 1 inch depending on D_o.

Number of Runner Blades $z_2 = z_o \pm 1$ to avoid the synchronous effect.

7.14 Shape of Francis Runner and Evolution of Kaplan Runner—The exact shape of the Francis runner depends on its specific speed. It is obvious from the equation for specific speed (Eqn 3.39) that higher specific speed means lower head. This requires that the runner should admit a comparatively large quantity of water. This can be done by increasing either

i) Ratio $\frac{B_o}{D_1}$

or *ii*) non-dimensional factor $K_{v_{m_o}} = \frac{v_{m_o}}{\sqrt{2gH}}$

In the first case the height of guide vanes B_o and correspondingly the height of runner should increase while in the second case v_{m_o}, the radial velocity of flow through the guide vanes should increase.

Fig 7.12 shows in stages the change in the shape of Francis runner with variation of specific speed (*see* also Fig 5.6). The first three types have, in order, the slow, the normal and the fast runners. Flow of water through the runner is radial at entrance but more or less axial at exit. As the specific speed increases, discharge becomes more and more axial. Fourth type [See Fig 7.12 (*d*)] is the high specific speed Francis runner,

better known as Dubs runner. Here the water enters diagonally and the discharge is almost entirely axial. If the band of a Dubs runner is removed, it appears like a propeller type runner. In the strictly propeller type runner, number of blades is less and flow is throughout purely axial.

Fig 7.12 (e) shows such a runner. It is designated Kaplan runner after Kaplan who first designed this type of turbine runner.

By drawing the inlet velocity triangle for each of the five runners shown in Fig 7.12 it is seen that lessening of head and increase in specific speed are accompanied with a reduction of inlet velocity v_1 of water. But the radial velocity v_{m_1} increases letting in a large amount of water.

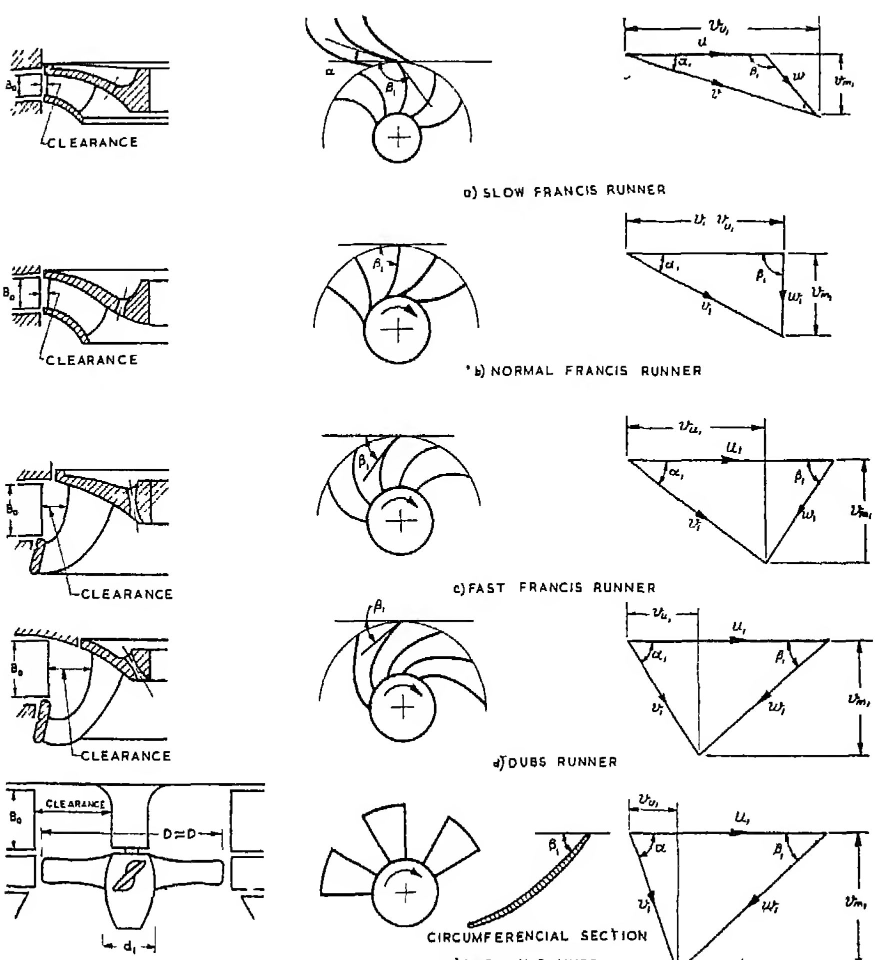


Fig 7.12 Change in the Shape of Francis Runner and its Inlet Velocity Triangles with the Variation of Specific Speed

The values of N_s, K_{u_1}, α_1 and β_1 for each of the runners are given below.

a) **Slow Runner** .

$N_s = 13{\cdot}5$ to 27 British Units (or 60 to 120 Metric Units)

$\alpha_1 = 15°$ to $25°$ $\quad\beta_1 = 90°$ to $120°$

$K_{u_1} = 0{\cdot}62$ to $0{\cdot}68$ $\quad\frac{B_o}{D_1} = \frac{1}{25}$ to $\frac{1}{30}$

Value of β_1 being more than 90°, course of water in the runner is sharply curved and this implies that blades have a similar curvature.

b) **Normal Runner** :

$N_s = 27$ to $40{\cdot}5$ British Units (or 120 to 180 Metric Units)

$\alpha_1 = 25°$ to $32\frac{1}{2}°$ $\quad\beta_1 = 90°$

$K_{u_1} = 0{\cdot}68$ to $0{\cdot}72$ $\quad\frac{B_o}{D_1} = \frac{1}{8}$ to $\frac{1}{4}$

c) **Fast Runner** :

$N_s = 40{\cdot}5$ to $67{\cdot}5$ British Units (or 180 to 300 Metric Units)

$\alpha_1 = 32\frac{1}{2}°$ to $37\frac{1}{2}°$ $\quad\beta_1 = 60°$ to $90°$

$K_{u_1} = 0{\cdot}72$ to $0{\cdot}76$ $\quad\frac{B_o}{D_1} = \frac{1}{4}$ to $\frac{1}{2}$

d) **Dubs Runner** :

$N_s = 63$ to 117 British Units (or 280 to 525 Metric Units)

$\alpha_1 = 37\frac{1}{2}°$ to $40°$ $\quad\beta_1 = 45°$ to $60°$

$K_{u_1} = 0{\cdot}46$ to $0{\cdot}76$ $\quad\frac{B_o}{D_1} = \frac{1}{2}$

e) **Kaplan Runner** :

$N_s = 67{\cdot}5$ to 225 British Units (or 300 to 1,000 Metric Units)

$\alpha_1 < 90°$ $\quad\beta_1 < 90°$

$K_{v_1} = 1{\cdot}2$ to $2{\cdot}4$ (*i.e.* more than one)

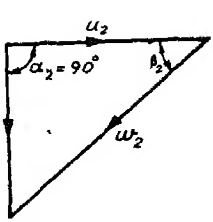


Fig 7.13 Outlet Velocity Triangle for all Reaction (Francis as well as Kaplan) Runners

Outlet Velocity Triangle for each of these five runners is more or less similar and is shown in Fig 7.13.

$\alpha_2 \approx 90°$ for Francis Runners

$\alpha_2 = 90°$ for Kaplan Runners

7.15 Draft Tube Theory—It has been observed earlier that the steadily increasing cross-sectional area of the draft tube causes a large portion of the kinetic head to be converted to pressure head as the water approaches the tail race.

Thus gain in head is

$$\frac{v_2^2}{2g} - \frac{v_3^2}{2g} = \left\{ 1 - \left(\frac{A_2}{A_3}\right)^2 \right\} \cdot \frac{v_2^2}{2g} \text{ (neglecting frictional loss)}$$

Where v_2, A_2 and v_3, A_3 are the velocities and areas at the points (2) and (3) respectively (See Fig 7.14).

Had there been no draft tube, the kinetic head $\frac{v_2^2}{2g}$ would have been entirely lost. Kinetic head thus saved by the draft tube adds to the effective head of the turbine. (See Eqn 7.7).

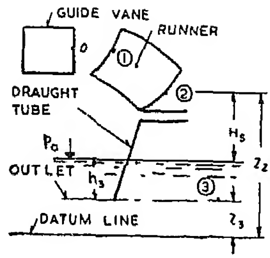


Fig 7 14 Draft Tube Theory

The other function of the draft tube is to permit a negative (suction) head to be established at the runner outlet (point 2 in Fig 7.14), which can be proved as follows :

a) Neglecting frictional loss in the draft tube and applying Bernoulli's Theorem between points (2) and (3)

$$\frac{p_3}{w}+\frac{v_3^2}{2g}+z_3=\frac{p_2}{w}+\frac{v_2^2}{2g}+z_2$$

$$\text{or}\quad \frac{p_2}{w}=\frac{p_3}{w}-(z_2-z_3)-\frac{v_2^2-v_3^2}{2g}$$

$$\text{But}\quad \frac{p_3}{w}=\frac{p_a}{w}+h_3$$

$$\therefore\quad \frac{p_2}{w}=\frac{p_a}{w}-(z_2-z_3-h_3)-\frac{v_2^2-v_3^2}{2g}$$

But $(z_2-z_3-h_3)=H_s$ (Height of runner outlet above tail race level)

$$\therefore\quad \frac{p_2}{w}=\frac{p_a}{w}-\left(H_s+\frac{v_2^2-v_3^2}{2g}\right) \qquad ...(7.7)$$

Where H_s=static suction head

and $\frac{v_2^2-v_3^2}{2g}$=dynamic suction head

Minimum value of $\frac{p_2}{w}$ is the vapour pressure at the working temperature. This can be obtained from the following table :

TABLE 7.3

Vapour Pressure Function of Temperature

Temperature	32	50	75	100	112	°F
Vapour Pressure H_v	0·2	0·4	1	2·1	3	ft of water

Temperature	0	10	20	30	40	100	°C
Vapour Pressure, H_v	0·06	0·13	0 024	0·43	0 75	0·91	*m* of water

Pressure p_2 should not be below vapour pressure, otherwise the vapours, will be formed causing cavitation. p_a and p_2 are thus fixed. If v_2 is more, H_s must be small. In order to keep the suction head under the cavitation limit, vertical distance between the runner outlet and tail race level must be reduced as the elevation above sea-level increases. H_s may be negative in which case the runner obviously would be placed below the tail race level.

Considering loss of head h_f due to friction in the draft tube and applying Bernoulli's Theorem between points (2) and (3),

$$\frac{p_3}{w}+\frac{v_3^2}{2g}+z_3=\frac{p_2}{w}+\frac{v_2^2}{2g}+z_2-h_f$$

or $$\frac{p_2}{w}=\frac{p_a}{w}-(z_2-z_3-h_3)-\frac{v_2^2-v_3^2}{2g}+h_f$$

Now $H_s=z_2-z_3-h_3$

and h_f is generally expressed as function of dynamic suction head.

or $$h_f=k\,.\,\frac{v_2^2-v_3^2}{2g}$$

$\therefore$ Substituting the values of H_s and h_f

$$\frac{p_2}{w}=\frac{p_a}{w}-\left\{H_s+(1-k)\,.\,\frac{v_2^2-v_3^2}{2g}\right\} \qquad .. (7.8)$$

Comparing Eqn 7.7 and 7.8, it is seen that with the loss of head h_f, which always occurs in practice, the value of pressure p_2 increases, thus reducing the danger of cavitation in case of a turbine, but increasing the danger of cavitation in a centrifugal pump.

Put $1-k=\eta_d$, the draft tube efficiency

$$\frac{p_2}{w}=\frac{p_a}{w}-\left(H_s+\eta_d\,.\,\frac{v_2^2-v_3^2}{2g}\right) \qquad .. (7.8a)$$

Problem 7 2 A Kaplan turbine is fitted with a vertical conical shaped draft tube. The top and bottom diameters are equal to 24 in. and 36 in. respectively. The tube is running full with water flowing downwards, and it has a vertical height of 20 ft out of which 5 ft is drowned in the tail race water. Assume friction loss of head between the top and the bottom points as 0·3 times the kinetic head at draft tube exit. The velocity at exit is 5 ft per sec. Determine—

a) the pressure head at the top point of the draft tube in ft of water and in lb per sq in.

b) the total head at the same point with reference to the tail race as a datum,

c) the total head at the bottom point with reference to the tail race as a datum,

d) the power in the water at the top of the tube,

e) the power in water at the bottom of the tube,

and f) the efficiency of the draft tube.

Solution

$D_2 = 24$ in. $= 2$ ft $\qquad D_3 = 36$ in. $= 3$ ft

$H_{L_{2-3}} = 0\,3 \cdot \dfrac{v_3^2}{2g} \qquad L_d = 20$ ft

$H_s = 20 - 5 = 15$ ft $\qquad v_3 = 5$ ft/sec

a) Applying Bernoulli's Theorem between points 2 and 3 (See Fig 7.14) :

$$\frac{p_3}{w} + \frac{v_3^2}{2g} + z_3 = \frac{p_2}{w} + \frac{v_2^2}{2g} + z_2 - H_{L_{2-3}}$$

$$\therefore \qquad \frac{p_2}{w} = \frac{p_3}{w} - (z_2 - z_3) - \left(\frac{v_2^2 - v_3^2}{2g}\right) + 0\,3 \cdot \frac{v_3^2}{2g}$$

$$= \frac{p_a}{w} - H_s - \left(\frac{v_2^2 - v_3^2}{2g}\right) + 0\,3 \cdot \frac{v_3^2}{2g}.$$

But $\qquad a_2 = \dfrac{\pi}{4} \times 2^2 = 3\,14$ sq ft

$$a_3 = \frac{\pi}{4} \times 3^2 = 7\,06 \text{ sq ft}$$

$$\therefore \qquad v_2 = \frac{a_3 \cdot v_3}{a_2} = \frac{7\,06 \times 5}{3\,14} = 11\,25 \text{ ft/sec}$$

Substituting the values—

$$\frac{p_2}{w} = 34 - 15 - \frac{11\,25^2 - 5^2}{64{\cdot}4} + 0\,3 \times \frac{5^2}{64\,4}$$

$$= 34 - 15 - 1{\cdot}575 + 0\,118$$

$$= 34 - 16\,457$$

$\therefore \qquad \dfrac{p_2}{w} =$ **17 543 ft of water absolute** *Answer*

or $\qquad \dfrac{p_2}{w} =$ **−16·457 ft of water**

or $\quad 17\,543 \times \dfrac{62\,4}{144} =$ **7·6 lb/sq in. absolute**

or $\; -16{\cdot}457 \times \dfrac{62\,4}{144} =$ **−7·12 lb/sq in.** *Answer*

b) Total heat at the top point—

$$H_2 = \frac{p_2}{w} + \frac{v_2^2}{2g} + z_2$$

$$= -16{\cdot}457 + \frac{11\,25^2}{64{\cdot}4} + 15$$

$$= -16{\cdot}457 + 1{\cdot}965 + 15$$

$= $ **0 508 ft of water** *Answer*

c) Total head at the bottom point—

$$H_3=\frac{p_3}{w}+\frac{v_3^2}{2g}+z_3=H_2-H_{L(2-3)}$$

$$=H_2-0\ 3\ .\ \frac{v_3^2}{2g}=0\ 508-0\ 118$$

$$=\mathbf{0{\cdot}39\ ft\ of\ water}\quad \textit{Answer}$$

d) Power in the water at the top of the tube—

$$P_2=\frac{w\ .\ Q\ .\ H_2}{550}=\frac{62\ 4\times(3{\cdot}14\times11{\cdot}25)\times0\ 508}{550}$$

$$=\mathbf{2\ 03\ HP}\quad \textit{Answer}$$

e) Power in water at the bottom of the tube—

$$P_3=\frac{w\ .\ Q\ .\ H_3}{550}=\frac{62\ 4\times(3\ 14\times11{\cdot}25)\times0{\cdot}39}{550}$$

$$=\mathbf{1{\cdot}56\ HP}\quad \textit{Answer}$$

f) Efficiency of draft tube—

$$\eta_d=\frac{\text{actual head converted}}{\text{theoretical head converted}}$$

$$=\frac{\dfrac{v_2^2-v_3^2}{2g}-0\ 3\times\dfrac{v_3^2}{2g}}{\dfrac{v_2^2-v_3^2}{2g}}=\frac{1{\cdot}457}{1{\cdot}574}=0{\cdot}925$$

or $$=\mathbf{92{\cdot}5\%}\quad \textit{Answer}$$

7.16 Cavitation—Whenever the pressure in any turbine part drops below evaporation pressure, vapour formation will result. This vapour is trapped in the liquid in the form of bubbles. It may also happen that a stream of water cuts short of its path giving rise to eddies and vortices which may contain voids or bubbles These bubbles mainly formed on account of low pressure, are carried by the stream to higher pressure zones where the vapours condense and the bubbles collapse. Since the collapsing pressure may be as high as 100 atmospheres, it causes *pitting* in the metallic surface of runner blades, draft tube etc.

The phenomenon which manifests itself in the pitting of the metallic surfaces of turbine parts is known as *Cavitation* as cavities will appear. Turbine parts should be properly designed in order to avoid cavitation.

Besides damaging the metallic surfaces, cavitation also lowers the efficiency of the turbine. The blade contours are designed for a certain streamline pattern and when the latter alters due to pitting, the resultant torque produced by the flow of water is reduced, thus decreasing the power developed.

From Eqn 7.8, it is seen that the cavitation depends upon—

a) *Vapour pressure* which is a function of temperature of flowing water (See Table 7.3)

b) *Absolute pressure* or barometric pressure due to the location of turbine above mean sea level (See Table 7.4).

c) *Suction pressure* H_s which is the height of runner outlet above tail race level.

d) *Effective dynamic suction head and absolute velocity of water at runner exit*

$$i.e., \quad \eta_d\left(\frac{v_2^2-v_3^2}{2g}\right)$$

and v_3 respectively.

Prof D. Thoma of Munich (Germany) suggested a cavitation factor σ (sigma) to determine the zone where turbine can work without being effected from cavitation. Its critical value is given by

$$\sigma_{crit}=\frac{H_b-H_s}{H}=\frac{(H_a-H_v)-H_s}{H} \quad (7.9)$$

where H_b=Barometric pressure in ft (or m) of water,
$=H_a-H_v$
H_a=Average atmospheric pressure in ft (or m) of water (See Table 7.4).
H_v=Vapour pressure in ft (or m) of water (See Table 7 3).
H_s=Suction pressure head in ft (or m) of water (height of runner outlet above tail race),
H=Working head of turbine in ft (or m) of water.

Table 7.4 gives the average atmospheric pressure which depends upon the altitude above sea level.

TABLE 7.4

Altitude vs Average Atmospheric Pressure

Altitude	Barometric Pressure	
ft	inches of mercury	ft of water
0	30	34
1,000	28 88	32 73
2,000	27·80	31 51
3,000	26·76	30 33
4,000	25·76	29·19
5,000	24·79	28·09
7,000	22·97	26 04
10,000	20·49	23·22
15,000	16 93	19 19

Altitude	Barometric Pressure	
metre	*mm* of mercury	metre of water
0	760	10·35
1,000	676	9 2
2,000	595	8·1
3,000	528	7·2
4,000	463	6·3

Now cavitation factor σ_{crit} depends upon the specific speed of the turbine. Table 7.5 gives these values for Francis turbine, depending on the experience.

TABLE 7.5

N_s vs σ_{crit} **(Francis Turbine)**

N_s	15	20	30	40	50	60	70	80	British Units
σ_{crit}	0 014	0 025	0·055	0·095	0 14	0·20	0·26	0 33	
N_s	50	100	150	200	250	300	350		Metric Units
σ_{crit}	0·04	0 05	0 07	0·10	0 14	0 2	0 27		

TABLE 7.6

N_s vs σ_{crit} **(Kaplan Turbine)**

N_s	300 to 450	450 to 500	550 to 600	650 to 700	700 to 800 Metric Units
N_s	67·5 to 101	101 to 112	123 to 135	146 to 157	157 to 180 British Units
σ_{crit}	0 35 to 0 4	0 4 to 0 45	0 6 to 0 65	0·85	1·05

According to Prof Thoma, cavitation can be avoided if the values of σ are not less than the critical value given above. Thus the deciding factors in the selection of reaction turbines are specific speed N_s and the cavitation factor σ. Prof F. H. Roger* (USA) suggested the following empirical relation for Francis turbines:

$$\sigma_{crit}=0{\cdot}625\,.\left(\frac{N_s}{100}\right)^2 \qquad \text{.. (7.10)}$$

The maximum permissible specific speed (British Units) can thus be calculated :

$$N_s= 100\,.\sqrt{\frac{\sigma_{crit}}{0{\cdot}625}} \qquad \text{...(7.11)}$$

$$=126{\cdot}5\,.\sqrt{\frac{H_b-H_s}{H}} \qquad \text{...(7.11}a\text{)}$$

A similar relation has been suggested for propeller turbines :

$$\sigma_{crit}=0{\cdot}3+0{\cdot}14\left(\frac{N_s}{100}\right)^{2{\cdot}73}, \text{ where } N_s \text{ is in British Units...(7.12)}$$

*See Proceedings of the American Society of Civil Engineers, Dec 1937.

For Kaplan turbine σ_{crit} is 10% higher than for a similar propeller turbine.

Problem 7.3 A 16,000 HP reaction water turbine has an effective head of 81 ft. The runner is 10 ft above the tail race. The turbine is installed at an elevation of 1,000 ft above sea level where the barometric head is 31·6 ft of water. The temperature of water is 80°F The following table gives values of the plant sigma σ (Thoma's Cavitation factor) against specific speed. The turbine is coupled to a 50 cycles alternator. Calculate the synchronous speed of the turbine.

σ	0·05	0·10	0 16	0 228
N_s	30	40	50	60

(*UPSC—Jan 1953*)

Solution

P_t=16,000 BHP $\qquad H$=81 ft

H_s=10 ft $\qquad H_b$=31·6 ft

(Assume the barometric pressure H_b at 1,000 ft altitude and at 80°F=31·6 ft of water)

$$\therefore \text{ Thoma's Cavitation factor } \sigma=\frac{H_b-H_s}{H}=\frac{31{\cdot}6-10}{81}=0{\cdot}267$$

The value of specific speed, N_s corresponding to above σ is found from the curve (See Fig 7.15) drawn with the help of the given table. Thus $N_s \leqq 65$. It is clear from the curve (See Fig 7.15) that if $N_s > 65$, then cavitation will appear.

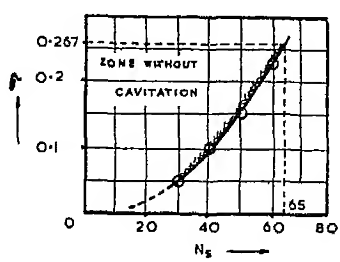


Fig 7.15 σ v_s N_s

$$\text{Now} \quad N_s=\frac{N \cdot \sqrt{P_t}}{H^{\frac{5}{4}}}$$

$$\text{or} \quad 65=\frac{N \cdot \sqrt{16{,}000}}{81 \times 3}$$

$$\text{or} \quad N=125 \text{ RPM}$$

Assuming p, the number of pair of poles for the alternator=24, the synchronous speed can be found with the help of equation given in Art 6.14.

$$\text{Thus} \quad f=\frac{p \cdot N}{60} \qquad \text{...(Eqn 6.9)}$$

$$\text{or} \quad 50 \times 60=p \cdot N$$

$$\text{or} \quad N=\frac{50 \times 60}{24}=\mathbf{125 \text{ RPM}} \quad \textit{Answer}$$

which is the same as found out with the help of specific speed.

7.17 Methods to Avoid Cavitation—

a) **Installation of Turbine Below Tail Race Level**—From Eqns 7.8 and 7.9 it is clear that if the value of H_s is less, the pressure p_2 and σ will increase, which means the turbine may work in a safe zone. In case of high specific speed turbines such as Kaplan turbine, runner exit velocity V_2 is very high which increases the value of expression $\frac{V_2^2 - V_3^2}{2g}$ in Eqn 7.8 and lowers the value of pressure p_2, thus bringing the cavitation zone nearer. In order to keep p_2 within cavitation limits, the value of H_s is made negative which means the runner is installed below the tail race level. For such installations the turbine remains always under water. Therefore when any repair or inspection of turbines is carried out, the water has to be pumped out by closing the passages. This is a disadvantage, because it is quite costly to pump the water out and also it requires considerable time. It is, therefore, advisable to avoid installation of the turbine below tail race level.

b) **Cavitation-Free Runner of Reaction Turbine**—The above method being very costly, the modern trend is to develop a cavitation-free runner It can be made after a thorough research.

The experiments are carried out on model turbine runner in a laboratory, because theoretical calculations merely do not help. A few suitable blade angles, blade lengths etc are tried for a particular runner and finally the best values are adopted. Photographs are taken by means of stroboscope while the experiments on turbine models are carried out. The parts of runner where cavitation appears, is clearly visible from the photographs taken. The most modern method of ascertaining cavitation phenomenon is by knowing the accurate pressure distribution along the runner blade section. This is also accomplished by carrying out tests on models driven by air in a wind tunnel, and running it exactly in the same working conditions as the actual-size machine. By this method the points of low pressure are found out. The velocity of these points should not be more than the critical values. If these values fall below the critical range, the velocities are to be altered to avoid danger of cavitation.

It has been found by the author while conducting research experiments on Kaplan turbine runner blades that if the blade profile is made thicker, in order to make it mechanically strong, the pressure at the thicker places, near the boss of runner, may fall below the evaporation pressure (critical pressure), thus bringing the possibility of cavitation. In such cases it has been felt to make the blades flatter. The cross-section of such blade is of air-foil shape and the pressure on both sides, especially on suction side is carefully tested.

c) **Use of Material**—*Cast iron* is less resistive against cavitation, because it contains more carbon contents. *Cast steel* is better than cast iron. In case cast steel parts are effected by cavitation, it is possible to repair them by welding. Welded places can resist the cavitation more than ordinary surfaces. *Stainless steel* is the best material to avoid cavitation. However it is expensive, therefore, the turbine runners are made of cast steel and then coated with stainless steel. Where water is chemically impure, *bronze* is preferred which also resists the cavitation better than cast steel. The pitting in case of bronze is filled by soldering.

d) **Use of Machining**—The turbine runner blades are highly polished, as it is seen in practice that with rough surfaces, the likelihood of cavitation occurance is more, due to interruption of smooth and stream line flow, causing vortices.

7.18 Selection of Speed—The empirical relations mentioned in Art 7.16 make it possible to predetermine N_s for given H_b, H_s and H. Also using equations 7.10 and 7.12, approximate values of specific speed N_s can be estimated. Further from N_s, the power output P_t of the turbine can be worked out from specific speed Eqn 3.39. Because the turbine is almost always directly coupled to an alternator, a synchronous speed (Eqn 6.9*a*) must be chosen.

7.19 Runaway Speed—The runaway speed of a turbine is a maximum speed attained by the runner under maximum head at full gate opening, when the external load (*i.e.* generator) is disconnected from the system and the governor ceases to function. All rotating parts must be designed to with stand the runaway speed which varies among the manufacturers with the design of turbine and generator. The range of runaway speed in terms of normal runaway speed for various types of turbine is generally as follows—

TABLE 7.7

Runaway Speed (N_r) in terms of Working Speed (N)

Turbine Type	Runaway Speed
Pelton turbine	1·8 to 1·9 N
Francis turbine	2 to 2·2 N
Kaplan turbine	2·5 to 3 N

The exact value of runaway speed of any turbine can be predicted from the model tests in the laboratory. R.E. Krueger in his work published by Bureau of Reclamation USA, has given to following equation to predict the runaway speed.

$$N_r = K_n \cdot N \cdot \left(\frac{H_{max}}{H}\right)^{\frac{1}{2}} \qquad \ldots(7.13)$$

where $K_n = 0{\cdot}003\, N_s + 1{\cdot}7$

N_s = specific speed (British units),

N_r = runaway speed,

H_{max} = maximum head not including water hummer in ft (or in m),

H = working head in ft (or in m).

Laboratory experiments have shown that with the appearance of cavitation the runaway speed of reaction turbine increases, probably because of reduced fluid friction. In case of Kaplan turbine this effect is significant. The runaway speed of reaction turbine may also vary with the gate opening.

KAPLAN TURBINE

7.20 Kaplan Turbine - The Kaplan like the Francis is a reaction turbine. It operates in an entirely closed conduit from inlet to tail race. As already explained, this turbine is used where comparatively low head and large quantity of water is available.

All parts such as spiral casing, guide mechanism and draft tube of the Kaplan turbine except the runner are similar (See Fig 7 16) to those

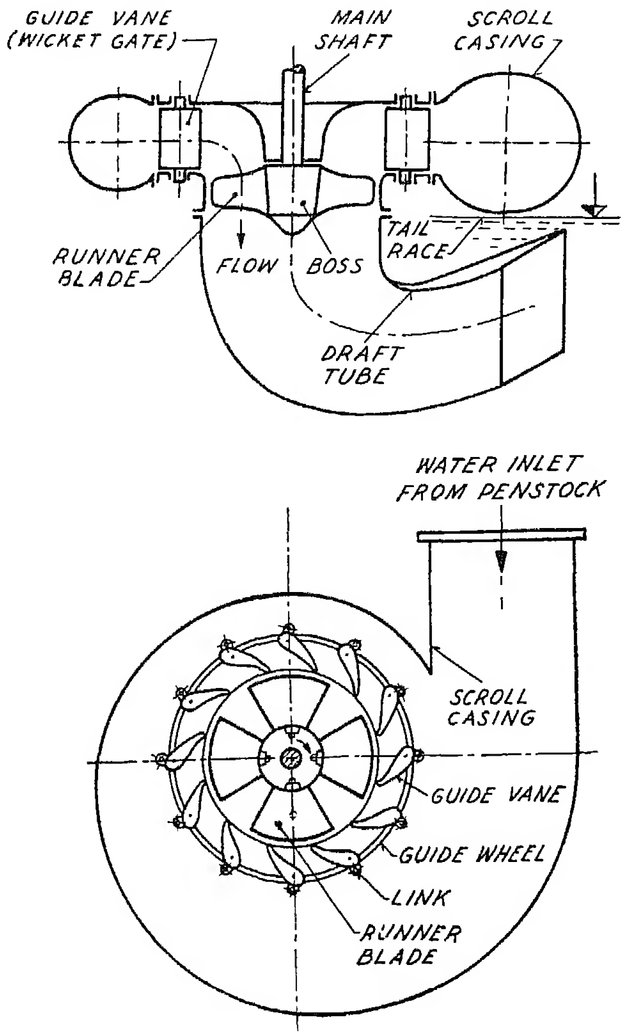


Fig 7.16 Outline of a Kaplan Turbine

of Francis type. Runner has two major differences. In Francis runner, the water enters radially while in Kaplan type, it strikes the blades axially. (See Fig 7.12). Number of blades in a Francis runner is 16 to 24 but in Kaplan it is only 3 to 6 or atmost 8 in exceptional cases. This reduces the contact surface with water and hence the frictional resistance. Kaplan blades are attached to the boss, dispensing with the band, thus eliminating the frictional losses likely to be caused by the latter.

7.22 Some Kaplan Turbine Installations in India

Sl No	Scheme	Turbine Details	Site of Power House	Source of Water
1	Nangal—Bhakra Project (Punjab)	Two vertical Kaplan Turbines each having $P_t=33{,}500$ HP, $H=98$ ft, $N=166{\cdot}7$ RPM, Run-away speed$=458$ RPM $Z_1=20$, $Z_2=6$	Two similar Power Houses at Ganguwal and at Kotla (7 and 14) miles from Nangal Railway Station respectively)	Nangal Hydel Channel
2	Ganga Canal Hydro-Electric Scheme (U P)	Ten stations having Kaplan Turbines with power varying from 400 to 4,000 HP (*Pathri*—3 Voith vertical turbines, each of 9,650 HP under 32·5 ft and *Mohammedpur*—3 English Electric turbines, each of 4,250 HP, 17·4 ft head at 125 rpm)	Ranipur Pathri, Bahadrabad, Salawa, Chitaura, Nirgajni, Mohammedpur, Sumera, Palra and Bhola	Ganga Canal
3	Sarda Hydel Project (U P)	Three Vertical Kaplan turbines each of 19,200 HP	Khatima	Sarda Canal
4	Hirakud Dam Project II (Orissa)	Four vertical Kaplan Turbines, each of 52,000 HP and two Voith vertical turbines, each of 36,700 HP under a head of 82 ft	Hirakud (7 miles from Sambalpur)	Mahanadi (Hirakud Dam)
5	Radhanagari Hydro-Electric Scheme (Bombay)	Four vertical Kaplan Turbines each having $P_t=1{,}720$ HP, $H=46$ to 119 ft (95 ft average), $N=600$ RPM	Radhanagari (Kolhapur)	Bhogavati River
6	Nizamsagar Project (Andhra)	Three vertical Kaplan Turbines each having 7,050 HP	Nizamsagar	Nanjira River
7	Tungabhadra Hydro-Electric Scheme (Madras)	Two vertical Kaplan Turbines from Hitachi (Japan) each having $P_t=13{,}800$ HP, $N=214$ rpm, $H=65$ ft	Tungabhadra Dam Power Station	Tunga-Bhadra Dam
8	Panchat Hill Power Station (DVC)	One vertical Kaplan turbine (Swedish make) $P_t=56{,}000$ HP at $H=81$ ft and 90,000 HP at 113 ft $D_1=17'-8''$, $N=125$ rpm $N_r=370$ rpm $Z_1=24$, $Z_2=5$ Runner weight$=$ 89 tons, Turbine shaft dia 35 5″ (bore 13·5″) Shaft [illegible]	Panchat Hill	Panchat Hill

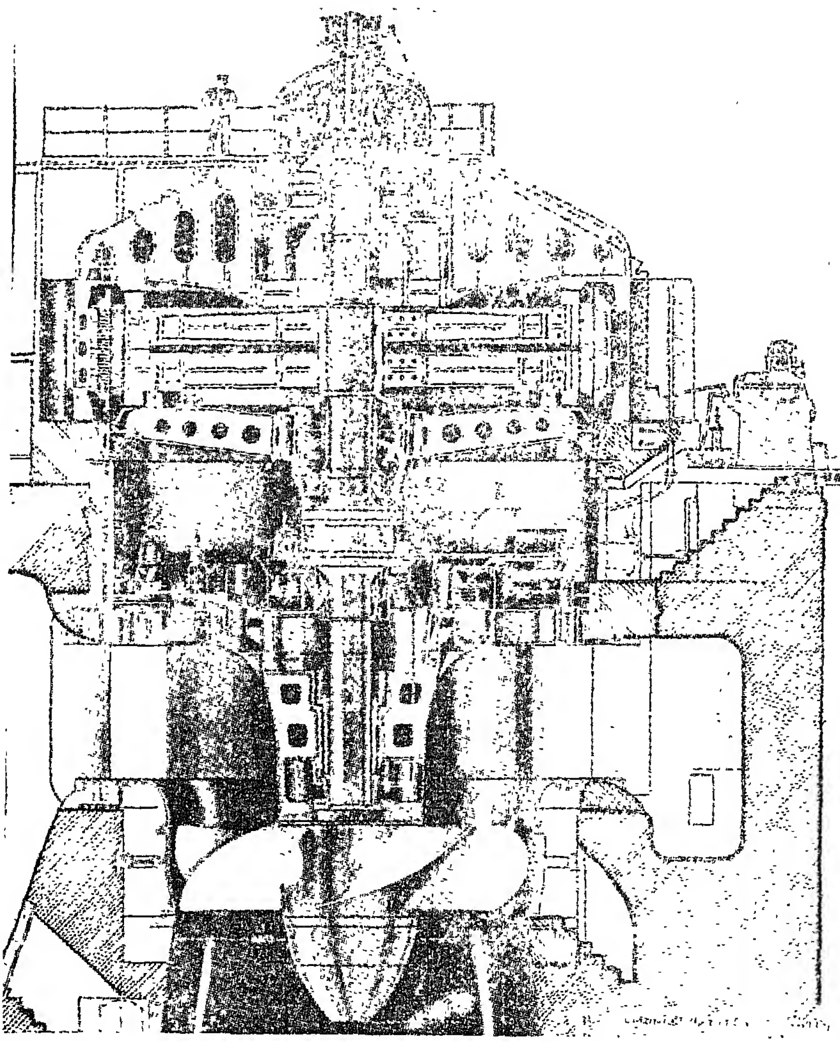


Fig 7.17 (a) One of the Largest Kaplan Turbine of Europe at Ryburg-Schwoerstadt (Swiss-German Border)

7.21 Notable Kaplan Turbines Installations of the World— The most powerful Kaplan turbine in the world to-date belongs to Dalls Power Station in USA. It develops 123,000 HP under a head of 81 ft. The largest Kaplan turbine of Europe is situated at Ligga (Sweden), developing 105,650 HP at 130 ft and 125 rpm. Fig 7.17 (*a*) and (*b*) show a Kaplan turbine on the Rhine River at Ryburg-Schwoerstadt (Swiss-German border). It has an outer runner diameter equal to $22\frac{1}{3}$ ft and develops 38,700 HP working under 38·5 ft head at a speed of 75 rpm. One could well appreciate the size of turbine on comparing it to the size of the man standing shown in Fig 7.17 (*a*) and (*b*).

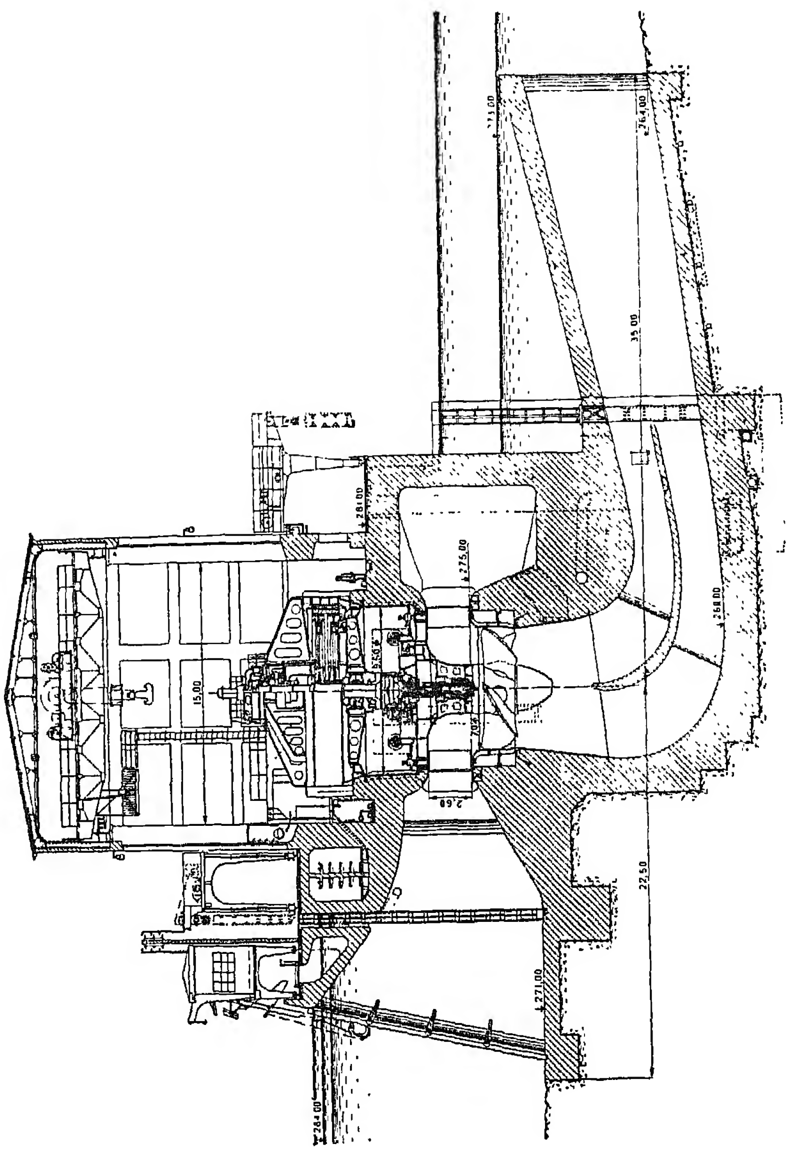


Fig 7.17 (b) Section Through Power House Having one of the Largest Kaplan Turbine of Europe at Ryburg Schwoerstadt P_t=38,700 HP ; H=38·5 ft (or 11·75 m) N=75 RPM, Manufactured by Escher Wyss & Co, Ltd, Zurich

In USA the English Electric Priest Rapids Kaplan turbines have a diameter of 23 ft 8 in., each rated at 131,000 HP under head of 84 ft and 85·7 rpm. The Rocky Reach and the Ice Harbour turbines which are scheduled to go into operation in 1961 have capacities of 140,000 and 113,000 HP respectively.

The largest Kaplan turbine in USA is at Bonneville, Oregan (USA) on Columbia River. It is rated at 66 000 HP under a head of 50 ft running at 75 rpm. The diameter of its adjustable blade-runner is 24¼ ft.

The largest Kaplan turbine of the world seems to be at Vargon (Sweden) on Vanern Lake. This turbine, manufactured by Karlstad, is rated at 15.200 HP at 14·1 ft and 46·9 rpm. Its runner diameter is 26 ft. This turbine has a low value of σ (sigma) which resulted in low excavation costs.

When completed the Kubyshev and Stalingrad Stations will have record size units, the former having 20 and later 22 Kaplan turbines of identical size each rated at 105 MW (or 140,000 HP) 68.2 rpm, 19 *m* (or 62.3 ft) head and runner diameter 9.3 *m* (or 30·5 ft).

The highest head Kaplan turbine so far installed are operating at Tres Marais Hydro Electric Power Station, Brazil. There are two Voith turbines, each developing 90,000 BHP under a head of 164 ft, having runner diameter of 15 ft and 8 blades.

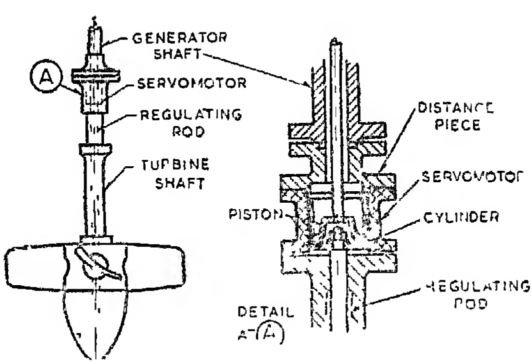


Fig 7.18 Mechanism for Controlling the Blade Movement of Adjustable Blade Runner

7.23 Adjustment of Kaplan Blades—Kaplan runner blades unlike the Francis can be adjusted to vary passage area between the two blades. This can be done while the turbine is in operation, by means of a servomotor mechanism operating inside the hollow coupling of turbine and generator shaft. Servomotor mechanism consists of a cylinder with a piston working under oil pressure on either side. The piston is connected to the upper end of a regulating rod, the up and down movement of which turns the blades. (See Fig 7.18). Regulating rod passes through the turbine mainshaft which is made hollow for this purpose. Movement of this rod is controlled by a governor (cf. Chapter 8). Motion of the regulating rod is transmitted to the blades through suitable link mechanism enclosed in the runner hub.

The Englesson's Patent, first used in 1922, to house the servomotor in the hub itself instead of in the coupling of turbine and generator shaft, has recently been modified by English Electric Co. The servomotor consists of a fixed piston surrounded by a mobile cylinder of small diameter and long stroke. The usual complicated connecting mechanism between the runner blade shafts and their operating levers is eliminated

by making each runner blade and operating lever as one solid casting. All operating levers are directly and simultaneously operated by the mobile servomotor cylinder. This arrangement offers easy accessibility to the servomotor without resorting to major dismantling.

7.24 Propeller Turbine—Kaplan turbine is just a propeller turbine with adjustable blades. The ordinary propeller turbine with fixed blades though economical can only work under full load (cf Chapter 9) for the best efficiency.

Largest fixed blade (propeller) turbine in the world is at George Plant in Washington (USA). It develops 70,500 HP at 120 RPM under 90 ft head. Still biggest propeller turbine installation in the world will be at St. Lawrance Power Plant. There will be 32 fixed blade turbines, 16 on the Canadian side and 16 on the American side. The Canadian side units are being supplied by English Electric Co. and the American side units by Allis-Chalmers and Baldwin-Lima Hamilton. The turbines are designed to develop 71,000 HP each under a head of 81 ft. At the maximum head of 87·5 ft they will develop 88,000 HP. The speed is 94·7 rpm and diameter of runner is 21 ft.

7.25 Outlines of Propeller or Kaplan Runner (Fig 7.19 and 7.20)—The flow of water at inlet and outlet is axial. The space between guide-vanes outlet and runner inlet is known as the "Whirl Chamber". The outer curve of this chamber is an ellipse (See Fig 7.20).

Semi-major axis of ellipse, $a \approx 0{\cdot}13\ D_1$

Semi-minor axis of ellipse, $b \approx 0{\cdot}16\ D_1$ to $0{\cdot}2\ D_1$

Diameter of boss $d_1 = 0{\cdot}35\ D_1$ to $0{\cdot}6\ D_1$

Section of the blades is an aerofoil. Open and closed positions are clearly shown in Section X-X, Fig 7.20).

Pitch $t = \dfrac{\pi \,.\, D_2}{z_2}$ where z_2=number of runner blades.

Ratio $\dfrac{l}{t_2} \approx 0{\cdot}9$ to $1{\cdot}05$

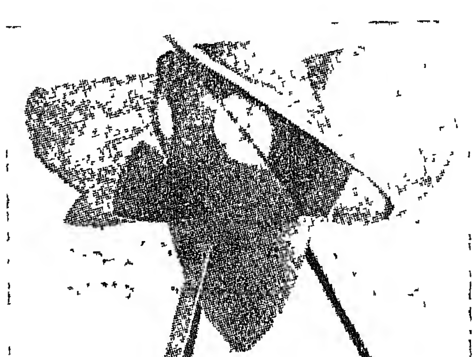


Fig 7.19 Kaplan Turbine Runner

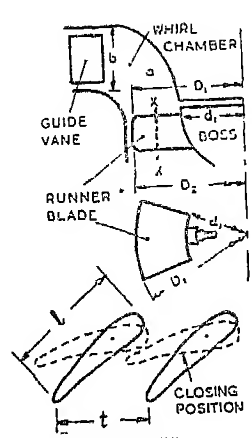


Fig 7.20 Outline of Kaplan or Propeller Runner

For velocity triangles at inlet and outlet refer to Art 7.14 and Fig 7.12 and 7.13.

TORQUE, POWER & EFFICIENCIES

7.26 Force and Torque—The resultant dynamic force exerted by the water on the runner vanes in the direction of rotation (*c f.* Eqn 1.22).

$$F_u = \frac{w \cdot Q}{g} (v_{u_1} - v_{u_2})$$

Force equivalent of motion at inlet

$$F_{u_1} = \frac{w \cdot Q}{g} \cdot v_{u_1}$$

Force equivalent of motion at outlet

$$F_{u_2} = \frac{w \cdot Q}{g} \cdot v_{u_2}$$

The action of the stream on the vanes of a radial flow runner can be determined by finding the total torque produced by all the elementary forces over the vanes. The runner is considered to be divided into a number of parts of equal area, each constituting what may be called a *fractional* turbine.

Let dM_H be the turning moment of a fractional turbine and dQ quantity of water flowing through it.

Equivalent turning movement of fluid motion at inlet

$$dM_{H_1} = dF_{u_1} \cdot R_1 = \frac{w \cdot dQ}{g} \cdot v_{u_1} \cdot R_1$$

Similarly equivalent turning moment at outlet

$$dM_{H_2} = dF_{u_2} \cdot R_2 = \frac{w \cdot dQ}{g} \cdot v_{u_2} \cdot R_2$$

Resultant torque

$$dM_H = dM_{H_1} - dM_{H_2} = \frac{w \cdot dQ}{g} \cdot (v_{u_1} \cdot R_1 - v_{u_2} \cdot R_2)$$

Hence, $$M_H = \int dM_H = \frac{w}{g} \int_0^Q (v_{u_1} \cdot R_1 - v_{u_2} \cdot R_2) \cdot dQ \quad \ldots(7.14)$$

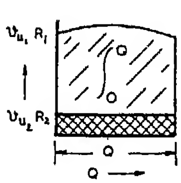


Fig 7 21 $v_u \cdot R$ vs Q

Exact value of M_H can be obtained by graphical integration (See Fig 7.21).

Roughly $$M_H = \frac{w \cdot Q}{g} (v_{u_2} \cdot R_1 - v_{u_1} : R_2) \quad \ldots(7.14a)$$

7.27 Power—Let P_H be the power developed by the turbine.

Then the power of a fractional turbine

$$dP_H = dM_H \cdot \omega$$

$$= \frac{w \cdot dQ}{g} \cdot (v_{u_1} \cdot R_1 \cdot \omega - v_{u_2} \cdot R_2 \cdot \omega)$$

$$= \frac{w \cdot dQ}{g} \cdot (v_{u_1} \cdot u_1 - v_{u_2} \cdot u_2)$$

$$\therefore \quad P_H = \int dP_H = \frac{w}{g} \int_0^Q (v_{u_1} \cdot u_1 - v_{u_2} \cdot u_2) \cdot dQ \qquad \ldots(7\ 15)$$

Thus the power P_H can also be determined be graphical integration (See Fig 7.22).

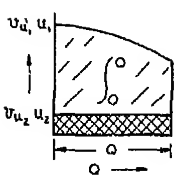


Fig 7.22 $v_u \cdot u$ vs Q

Approximately,

$$P = \frac{w \cdot Q}{g} (v_{u_1} \cdot u_1 - v_{u_2} \cdot u_2) \qquad \ldots(7.15a)$$

(See Art 1.11)

This is the expression for total hydraulic power developed by the turbine, considering the loss of head including that due to outlet velocity etc., but neglecting the volumetric losses.

7.28 Efficiencies—

a) **Head Efficiency**—Let the total head loss in turbine be $\triangle H$ the net operating head H.

Then efficiency $\eta_H = \dfrac{H - \triangle H}{H} = 1 - \dfrac{\triangle H}{H}$

This is known as the "*Head Efficiency*"

But $\quad P_H = P_a \cdot \eta_H$

$$= w \cdot Q \cdot H \cdot \eta_H$$

$$\therefore \quad \frac{w \cdot Q}{g} (v_{u_1} \cdot u_1 - v_u \quad \cdot u_2) = w \cdot Q \cdot H \cdot \eta_H$$

or

$$\eta_H = \frac{v_{u_1} \cdot u_1 - v_{u_2} \cdot u_2}{gH} \qquad \ldots(7.16)$$

$$= \frac{2\ (v_{u_1} \cdot u_1 - v_{u_2} \cdot u_2)}{2gH}$$

Substituting . $u_{u_1} = K_{v_{u_1}} \cdot \sqrt{2g\ H},\ v_{u_2} = K_{v_{u_2}} \cdot \sqrt{2g\ H}$

$u_1 = K_{v_1} \cdot \sqrt{2g\ H}$ and $u_2 = K_{u_2} \cdot \sqrt{2g\ H}$

$$\therefore \quad \eta_H = 2\ (K_{v_{u_1}} \cdot K_{u_1} - K_{v_{u_2}} \cdot K_{u_2})$$

But $\quad u_2 = \dfrac{R_2}{R_1} \cdot u_1 \qquad \left(\text{Since } \omega = \dfrac{u_1}{R_1} = \dfrac{u_2}{R_2}\right)$

or $$K_{u_2} = \frac{R_2}{R_1} \cdot K_v$$

$$\therefore \qquad \eta_H = 2K_{u_1} \left\{ v_{u_1} - K_{v_{u_2}} \cdot \frac{K_{u_2}}{K_{u_1}} \right\}$$

$$= 2K_{u_1} \left(K_{v_{u_1}} - K_{v_{u_2}} \cdot \frac{R_2}{R_1} \right) \qquad \text{.. (7.17)}$$

If the discharge is radial, *i.e.*, $\alpha_2 = \frac{\pi}{2}$

then $\cos \alpha_2 = 0$, and $K_{v_{u_2}} = 0$

$$\therefore \qquad \eta_H = 2K_{u_1} \cdot K_{v_{u_1}} \qquad \text{...(7.17}a\text{)}$$

b) **Volumetric Efficiency**—Let ΔQ be the amount of water that passes over to the tail race through some passage (See Fig 7.23) other than the runner and doing no useful work.

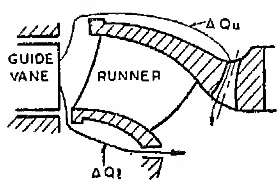


Fig 7.23 Water Quantity Loss Escaping Through the Clearance Between the Guide Wheel and the Runner.

Then $\Delta Q = \Delta Q_u + \Delta Q_l$

where ΔQ_u = upper clearance loss

ΔQ_l = lower clearance loss

(See Fig 7.23)

Volumetric efficiency,

$$\eta_Q = \frac{Q - \Delta Q}{Q} = 1 - \frac{\Delta Q}{Q} \qquad \text{...(7.18)}$$

TABLE 7.8

Practical Data for η_H and η_Q

K_s	5	96	980
N_s	16	69	225 (British)
	65	300	1,000 (Metric)
η_H	0·87	0·91	0·95
η_Q	0·955	0·965	0 985

c) **Hydraulic Efficiency**—Total hydraulic loss in a turbine is made up of total head loss and volumetric loss. The actual hydraulic power of the turbine is obtained by considering the total hydraulic loss.

Thus $P_h = w\,(Q-\triangle Q)(H-\triangle H)$

and $P_a = w\,.\,Q\,.\,H$

$\therefore$ Hydraulic efficiency

$$\eta_h = \frac{P_h}{P_a} = \frac{w\,.\,(Q-\triangle Q)(H-\triangle H)}{w\,.\,Q\,.\,H}$$

$$= \left(1-\frac{\triangle Q}{Q}\right)\left(1-\frac{\triangle H}{H}\right)$$

$$= \eta_Q\,.\,\eta_H \qquad \ldots(7.19)$$

d) **Mechanical Efficiency**—Brake horse power of a turbine is the hydraulic power minus the mechanical losses

$$P_t = P_h - \triangle P_{mech}$$

Mechanical losses may be due to bearing friction and windage.

Denoting the two by $\triangle P_{mech_1}$ and $\triangle P_{mech_2}$ respectively,

then $\triangle P_{mech_1} = K_1\,.\,G^{\frac{1}{2}}\,.\,N^{\frac{1}{2}}$

where G= total load on bearing,

N= speed in RPM,

K_1— a constant,

and $\triangle P_{mech_2} = K_2\,.\,N^3$

These formulae have been known from practical results. Now mechanical efficiency

$$\eta_{mech} = \frac{P_h - \triangle P_{mech}}{P_h} = 1 - \frac{\triangle P_{mech}}{P_h} \qquad \ldots(7.20)$$

$\therefore$ Brake horsepower,

$$P_t = P_h\,.\,\eta_{mech} \qquad \ldots(7.21)$$

e) **Overall Efficiency :** Let η_t be the overall efficiency of turbine.

Then $\eta_t = \dfrac{P_t}{P_a}$

$$= \frac{P_h\,.\,\eta_{mech}}{P_a}$$

$$= \frac{P_a\,.\,\eta_Q\,.\,\eta_H\,.\,\eta_{mech}}{P_a}$$

$$= \eta_Q\,.\,\eta_H\,.\,\eta_{mech} \qquad \ldots(7.22)$$

7.29 Rate of Flow in Reaction Turbines—Let quantity flowing per sec be Q

Then Q= Area across flow $\times$ velocity of flow

For a Francis runner,

area across radial flow at inlet $=(\pi\ D_1-z_2\ .\ t)\ .\ B_o$

where D_1 = inlet diameter of runner,

z_2 = number of blades in runner,

t = thickness of blades,

and B_o = height of runner = height of guide vanes.

Radial velocity of flow at inlet,

$$v_{m_1}=v_1\ .\ \sin\alpha_1$$

$$\therefore \quad Q=(\pi\ D_1-z_2\ .\ t)\ B_o\ .\ v_{m_1} \qquad \ldots(7.23)$$

Now the area occupied by blade edges is usually 5 to 10% of πD_1

In general, $(\pi\ D_1-z_2t)=K\ .\ \pi\ D_1$

where K = percentage of net flow area

i.e. 0 90 to 0·95

Also B_o is proportional to D_1

Let $\dfrac{B_o}{D_1}=K_B \quad \therefore \quad B_o=K_B\ .\ D_1$

and $v_{m_1}=K_{v_{m_1}}\ .\ \sqrt{2g\ H}$

$$\therefore \quad Q=K\ .\ \pi\ .\ K_B\ .\ K_{v_{m_1}}\ .\ \sqrt{2g}\ .\ D_1^2\ .\ \sqrt{H} \qquad \ldots(7.23a)$$

The factor $(K\ .\ \pi\ .\ K_B\ .\ K_{v_{m_1}}\ .\sqrt{2g})$ which is a constant for geometrically similar turbines is called **specific flow** and denoted by Q_{11}

Then $Q=Q_{11}\ .\ D_1^2\ .\ \sqrt{H}$ ...(See Eqn 3.35)

Q_{11} therefore is the quantity of water required by the turbine when working under unit head and with unit runner inlet diameter (D_1).

For a Kaplan runner,

Area across flow $=K\ .\ \dfrac{\pi}{4}\ .\ (D_1^2-d_1^2)$

Where K is percentage of net flow area obtained after deducting area occupied by the blades.

$D_1=D_2$ = external diameter of runner

d_1 = diameter of runner's boss.

Velocity of flow $v_{m_1}=K_{v_{m_1}}\ .\ \sqrt{2g\ H}$

$$\therefore \quad Q=\text{Area}\times\text{velocity}=K\ .\ \frac{\pi}{4}\ .\ K_{v_{m_1}}\ .\ \sqrt{2g}\ .\ (D_1^2-d_1^2)\ .\ \sqrt{H} \qquad \ldots(7.24)$$

$$=Q_{11}\ .\ (D_1^2-d_1^2)\ .\ \sqrt{H}$$

where $Q_{11}=K\ .\ \dfrac{\pi}{4}\ .\ K_{v_{m_1}}\ .\ \sqrt{2g}\ .$

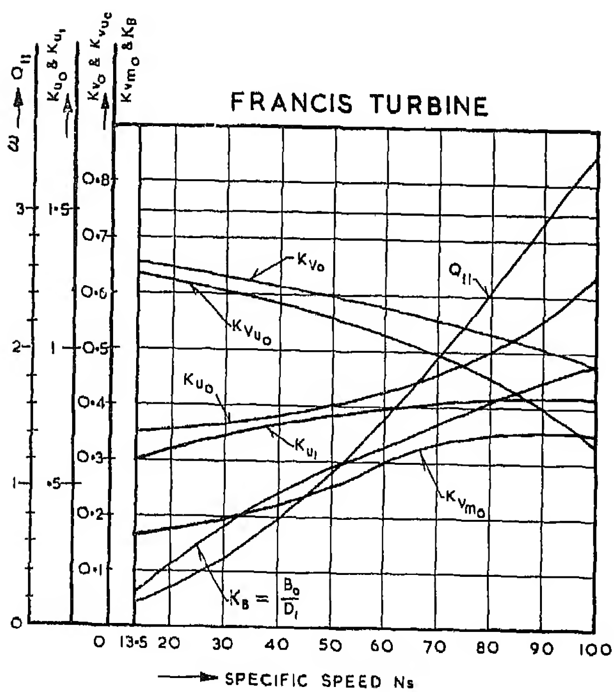


Fig 7.24 Francis Turbine Constants vs Specific Speed (British Units)
(N_s metric=4·45 N_s British)

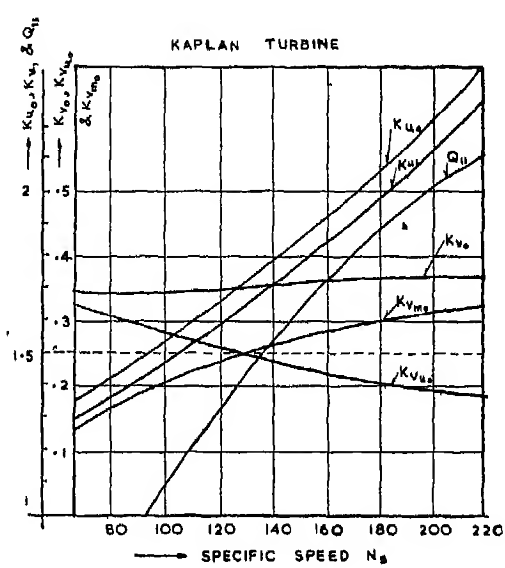


Fig 7.25 Kaplan Turbine Constants vs Specific Speed (British Units)
(N_s metric=4·45 N_s British)

For a Francis turbine Q_{11} varies from 0·09 to 2·7 while for a Kaplan turbine it is from 2·2 to 4·4.

Specific flow Q_{11} and non-dimensional factors K_{u_o}, K_{u_1}, K_{v_o}, $K_{v_{u_o}}$, $K_{v_{m_o}}$, K_B etc. can be plotted against specific speed N_s. Figs 7.24 and 7.25 show such curves for Francis and Kaplan turbines respectively.

Problem 7.4 An inward flow reaction turbine with a supply of 20 cusecs under a head of 50 ft develops 100 HP at 375 RPM. The inner and the outer diameters of the runner are 20 inches and 30 inches respectively. The velocity of water at exit is 10 ft/sec. Assuming that the discharge is radial and that the width of the wheel is constant, find the actual and the theoretical hydraulic efficiencies of the turbine and the inlet angles of the guide and wheel vanes.

(AMIE—April 1949)

Solution

$Q=20$ cusecs	$D_1=30$ inches
$H=50$ ft	$D_2=20$ inches
$P_t=100$ HP	$v_{m_2}=10$ ft/sec
$N=375$ RPM	$\alpha_2=90°$ (radial discharge)
	$B_1=B_2$

a) Peripheral velocity of wheel an inlet

$$u_1=\frac{\pi D_1 \cdot N}{60}=\frac{\pi\times30\times375}{12\times60}=49 \text{ ft|sec}$$

Velocity of flow at exit $v_{m_2}=10$ ft/sec

As $\alpha_2=90°$, $v_{m_2}=v_2$ (See Fig 1.22 *c*)

Work done/sec by the turbine per lb of water$=\dfrac{v_{u_1} \cdot u_1}{g}$

(See Eqn 1.37)

but this is equal to the head utilised by the turbine, *i e.*,

$$\frac{v_{u_1} \cdot u_1}{g}=H-\frac{v_2^2}{2g}$$

(Assuming there is no loss of pressure head at outlet)

or $$\frac{v_{u_1}\times49}{32{\cdot}2}=50-\frac{10^2}{64{\cdot}4}$$

$$=50-1{\cdot}55=48{\cdot}45$$

or $$v_{u_1}=\frac{48{\cdot}54\times32{\cdot}2}{49}=31{\cdot}8 \text{ ft/sec}$$

Work done/sec by the turbine wheel$=\dfrac{w \cdot Q}{g}(v_{u_1} \cdot u_1)$

...(See Eqn 1.37)

or Horsepower developed by the wheel$=\dfrac{w\,.\,Q}{g}\,.\,\dfrac{(v_{u_1}\,.\,u_1)}{550}$

$$=\frac{62{\cdot}4\times 20}{32{\cdot}2}\times\frac{31{\cdot}8\times 49}{550}$$

$$=108{\cdot}7 \text{ HP}$$

Available horsepower or water horsepower

$$=\frac{w\,.\,Q\,.\,H}{550}=\frac{62{\cdot}4\times 20\times 50}{550}=113{\cdot}5 \text{ HP}$$

Brake horsepower $=100$ HP

Overall turbine efficiency $\eta_t=\dfrac{100}{113{\cdot}5}\times 100=\mathbf{88{\cdot}15\ \%}$ *Answer*

(This is the actual hydraulic efficiency as required in this problem)

and hydraulic efficiency $\eta_h=\dfrac{108{\cdot}7}{113{\cdot}5}\times 100=\mathbf{95\ 8\ \%}$ *Answer*

(This is the theoretical hydraulic efficiency as required in this problem).

b) $Q=(\pi\,.\,D_1\,.\,B_1)\ v_{m_1}=\pi\,.\,D_2\,.\,B_2\,.\,v_{m_2}$ (See Eqn 2.11 and 7.23)

(neglecting blade thickness)

$\therefore\quad v_{m_1}=v_{m_2}\,.\,\dfrac{D_2}{D_1}$ ($\because$ width B is constant)

$$=10\times\frac{20}{30}=6{\cdot}67 \text{ ft/sec.}$$

Drawing inlet velocity triangle (See Fig 1.23) with the help of u_1, v_{m_1} and v_{u_1},

$$\tan\beta_1=\frac{v_{m_1}}{u_1-v_{u_1}}=\frac{6{\cdot}67}{49-31{\cdot}8}=\frac{6\ 67}{17{\cdot}2}=0{\cdot}388$$

or Inlet angle of wheel vane $\beta_1=\tan^{-1}0{\cdot}388$

$=\mathbf{21^\circ\ \text{-}10'}$ *Answer*

$$v_1=\sqrt{v_{u_1}{}^2+v_{m_1}{}^2}=\sqrt{31{\cdot}8^2+6{\cdot}67^2}=32{\cdot}4 \text{ ft/sec}$$

and $\quad v_{u_1}=v_1\cos\alpha_1$

$$\therefore\quad \cos\alpha_1=\frac{v_{u_1}}{v_1}=\frac{31{\cdot}8}{32{\cdot}4}=0{\cdot}982$$

$\therefore$ Inlet angle of guide vane $\alpha_1=\cos^{-1}0{\cdot}982=\mathbf{10^\circ-53'}$ *Answer*

Problem 7.5 An experimental inward flow reaction turbine rotates at 370 RPM. The wheel vanes are radial at inlet and the inner diameter of the wheel is half the outer diameter. The constant velocity of flow in the wheel is 6 ft/sec (or 1·83 *m*/sec). Water enters the wheel at an angle of 10°—4′ to the tangent to the wheel at inlet. The breadth of the wheel

at inlet is 3 in. (or 76·2 *mm*) and the area of flow blocked by the vanes is 5% of the gross area of flow at inlet. Find—

a) the net available head at the wheel,

b) the wheel vane angle at outlet,

c) the outer and inner diameters of the wheel,

and *d*) the theoretical water horsepower developed by the wheel.

(*AMIE—Nov* 1955)

Solution

$N=370$ RPM $\qquad D_1=2\ D_2$

$v_{m_1}=v_{m_2}=6$ ft/sec (or 1·83 *m*) $\qquad \alpha_1=10°-4'$

$B_1=3$ in. (or 76·2 *mm*) $\qquad k=1-0·05=0·95$

$\beta_1=90°$ ($\because$ vanes are radial at inlet)

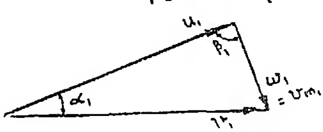


Fig 7.26 Inlet Velocity Triangle

c) $\quad v_{m_1}=v_1 \sin \alpha_1$

or $\quad v_1=\dfrac{v_{m_1}}{\sin 10°-4'}=\dfrac{6}{0·1748}$

$=34·3$ ft/sec

$\left[\text{or} \quad v_1=\dfrac{1·83}{0·1748}=10·47 \ m/\text{sec}\right]$

$\therefore$ Velocity of whirl at inlet

$v_{u_1}=v_1 \cos \alpha_1=34·3\times\cos 10°-4' \quad [\text{or } 10·47\times\cos 10°-4']$

$=33·8$ ft/sec $\quad$ [or 10·3 *m*/sec]

As $\quad \beta_1=90°,\ v_{u_1}=u_1$ (See Fig 7.26)

But $\quad u_1=\dfrac{\pi\,.\,D_1\,.\,N}{60}$

$\therefore \quad 33·8=\dfrac{\pi\times D_1\times 370}{60}\left[\text{or} \quad 10·3=\dfrac{\pi\times D_1\times 370}{60}\right]$

or $\quad D_1=\dfrac{33·8\times 60}{\pi\times 370}=1·746\approx$ **1 ft—9 in.** *Answer*

$\left[\text{or} \quad D_1=\dfrac{10·3\times 60}{\pi\times 370}=0·532\ m\right.$ or **532 mm** *Answer*$\Big]$

and $\quad D_2=\dfrac{D_1}{2}=\dfrac{1'\ \ 9''}{2}=$ **10·5 in.** *Answer*

$\left[\text{or} \quad D_2=\dfrac{532}{2}=\right.$ **266 mm** *Answer*$\Big]$

b) Net available head at the wheel

=Work done/sec by the wheel per lb of water

$=\dfrac{v_{u_1}\,.\,u_1-v_{u_2}\,.\,u_2}{g}$

$$= \frac{v_{u_1} \cdot u_1}{g} \quad \text{. (Assuming radial discharge at outlet)}$$

$$= \frac{33\ 8 \times 33\ 8}{32\ 2} \quad \left[\text{or} \quad \frac{10\ 3 \times 10\ 3}{9\ 81} \right]$$

$= \mathbf{33{\cdot}5\ ft}$ [or **10·8 m**] *Answer*

b) $$u_2 = \frac{\pi \cdot D_2 \cdot N}{60} = \frac{\pi \times 0\ 875 \times 370}{60} = 16{\cdot}95 \text{ ft/sec}$$

$$\left[\text{or } u_2 = \frac{\pi \times 0{\cdot}266 \times 370}{60} = 5{\cdot}16\ m/\text{sec} \right]$$

$v_{m_2} = 6$ ft/sec

$$\tan \beta_2 = \frac{v_{m_2}}{u_2} = \frac{6}{16{\cdot}95} = 0{\cdot}354 \left[\text{or } \frac{1\ 83}{5\ 16} = 0{\cdot}354 \right]$$

$\therefore$ β_2 the wheel vane angle at outlet $= \tan^{-1} 0\ 345 = \mathbf{19\tfrac{1}{2}^\circ}$ *Answer*

d) Quantity of water flowing through the wheel

= area of flow × velocity of flow

or $$Q = (k \cdot \pi \cdot D_1 \cdot B_1) \cdot v_{m_1} = 0{\cdot}95 \times \pi \times 1{\cdot}75 \times \tfrac{3}{12} \times 6$$

[or $$Q = 0{\cdot}95 \times \pi \times 0{\cdot}532 \times 0\ 0762 \times 1{\cdot}83]$$

$= 7{\cdot}825$ cu ft/sec [or $= 0{\cdot}222\ m^3$/sec]

Water Horsepower $$P_a = \frac{w \cdot Q \cdot H}{550} = \frac{62{\cdot}4 \times 7{\cdot}825 \times 35{\cdot}5}{550}$$

$$\left[\text{or} = \frac{1{,}000 \times 0{\cdot}222 \times 10{\cdot}8}{75} \right]$$

$= \mathbf{31{\cdot}5\ HP}$ [or **31·9 metric HP**] *Answer*

Problem 7.6 An inward flow Francis turbine is required to develop 5,000 BHP when operating under a net head of 100 ft and the specific speed is to be about 60 (in British Units).

Assuming guide vane angle at full gate opening 30°, hydraulic efficiency 90%, overall efficiency 87%, radial velocity of flow at inlet $0{\cdot}3\ \sqrt{2g\ H}$, blade thickness co-efficient 5%, draw the inlet velocity diagram and find—

a) the nearest synchronous speed to drive an alternator to give a frequency of 50 cycles per sec,

b) the diameter and the width of the runner at inlet,

and *c*) the theoretical inlet angle of the runner vanes.

[*AMIE Mech E (Lond)—April 1954*]

Solution

$P_t = 5{,}000$ BHP $\qquad \alpha_1 = 30^\circ$

$H = 100$ ft $\qquad v_{m_1} = 0{\cdot}3\ \sqrt{2g\ H}$

$N_s = 60 \qquad k = 0{\cdot}95$

$\eta_h = 0{\cdot}9$ $\qquad f = 50$ cycles/sec

$\eta_t = 0{\cdot}87$

a) Specific speed of the turbine $N_s = \dfrac{N \cdot \sqrt{P_t}}{H^{\frac{5}{4}}}$

or $\quad N = \dfrac{N_s \cdot H^{\frac{5}{4}}}{\sqrt{P_t}} = \dfrac{60 \times 100 \times 3{\cdot}16}{\sqrt{5{,}000}} = 268$ RPM

$\therefore$ Nearest synchronous speed$= \dfrac{3{,}000}{p} = \dfrac{3{,}000}{11} =$ **272·8 RPM** *Answer*

(Assuming p, the number of pair of poles$=11$)

b) Velocity of flow $v_{m_1} = 0{\cdot}3 \sqrt{2gH}$

$$= 0{\cdot}3 \times 8{\cdot}02 \times 10 = 24{\cdot}06 \text{ ft/sec}$$

But $\quad v_{m_1} = v_1 \cdot \sin \alpha_1$

or $\quad 24{\cdot}06 = v_1 \sin 30°$

or $\quad v_1 = \dfrac{24{\cdot}06}{0{\cdot}5} = 48{\cdot}12$ ft/sec

$$v_{u_1} = v_1 \cos 30° = 48{\cdot}12 \times 0{\cdot}866 = 41{\cdot}75 \text{ ft/sec}$$

Work done/sec by the turbine runner per lb of water

$$= \frac{v_{u_1} \cdot u_1}{g} = \eta_h \cdot H$$

$$= 0{\cdot}9 \times 100 = 90 \text{ ft lb/sec}$$

Peripheral velocity $u_1 = \dfrac{90 \times 32{\cdot}2}{41{\cdot}75} = 69{\cdot}4$ ft/sec

But $\quad u_1 = \dfrac{\pi \cdot D_1 \cdot N}{60}$

or $\quad D_1 = \dfrac{60 \cdot u_1}{\pi \cdot N} = \dfrac{60 \times 69{\cdot}4}{\pi \times 272{\cdot}8} = 4{\cdot}68$ ft$=$ **4 ft–8$\frac{1}{8}$ in.** *Answer*

Now $\quad P_t = \dfrac{w \cdot Q \cdot H}{550} \cdot \eta_t$

or $\quad 5{,}000 = \dfrac{62{\cdot}4 \times Q \times 100}{550} \times 0{\cdot}87$

or $\quad Q = 507$ cu ft/sec

Further quantity of water flowing through the runner

or $\quad Q = (k \cdot \pi \cdot D_1 \cdot B_1) \cdot v_{m_1}$

or $\quad B_1 = \dfrac{Q}{k \cdot \pi \cdot D_1 \cdot v_{m_1}}$

$\therefore \quad B_1 = \dfrac{507}{0{\cdot}95 \times \pi \times 4{\cdot}68 \times 24{\cdot}06} = 1{\cdot}505$ ft $\approx$ **1 ft 6 in.** *Answer*

c) Draw the inlet velocity triangle (See Fig 1.22)

$$\tan \beta_1 = \frac{v_{m_1}}{u_1 - v_{u_1}} = \frac{24{\cdot}06}{69{\cdot}4 - 41{\cdot}75} = \frac{24{\cdot}06}{27{\cdot}65} = 0{\cdot}87$$

or the inlet angle of the runner vanes

$$\beta_1 = \tan^{-1} 0{\cdot}87 = \mathbf{41^\circ - 1'} \quad \textit{Answer}$$

Problem 7.7 In an inward flow reaction turbine the peripheral velocity of the wheel at inlet is given by $u = \phi\ \sqrt{2gh}$ and the radial velocity of flow $v_f = \psi\ \sqrt{2gh}$

The breadth of the wheel at inlet is 'n' times the diameter of the wheel at inlet. If the turbine efficiency is 85 percent and the area taken up by vanes at inlet is 5 percent of the peripheral area at inlet, prove that the specific speed of the turbine is equal to $233\ \phi\ \sqrt{\psi}\ .\ n$

A runner of the above type having a specific speed of 50 RPM is required to develop 9,000 horsepower under a head of 278 ft. Taking ψ as 0·18 and $n = 0{\cdot}2$, calculate the RPM and the diameter of the runner.

(UPSC—Dec 1953)

Solution

$K_{u_1} = \phi$	$K_{v_{m_1}} = \psi$	$\therefore$	$K_{v_{m_1}} = 0{\cdot}18$
$v_{m_1} = v_f$	$B_1 = n\ D_1$	$\therefore$	$B_1 = 0{\cdot}2\ D_1$
$\eta_t = 0{\cdot}85$	$P_t = 9{,}000$ BHP		$u_1 = u$
$N_s = 50$	$H = 278$ ft		$k = 1 - 0{\cdot}5 = 0{\cdot}95$

a) The quantity of water flowing through the turbine = area of flow × velocity of turbine

$$\therefore \quad Q = 0{\cdot}95 \times \pi \times D_1 \times B_1 \times v_{m_1}$$

$$= 0{\cdot}95 \times \pi \times n \times D_1^2 \times \psi \times 8{\cdot}02 \times h^{\frac{1}{2}}$$

or $\quad Q_{11} = 0{\cdot}95 \times \pi \times 8{\cdot}02 \times n\ .\ \psi$

Now $\quad u_1 = \phi\ \sqrt{2gh}$

$$= \frac{\pi\ .\ D_1\ .\ N}{60}$$

$$\therefore \quad N = \frac{60\ .\ u_1}{\pi\ .\ D_1} = \frac{60\ \phi\ .\ \sqrt{2gh}}{\pi\ .\ D_1}$$

Now $\quad N_s = \dfrac{N\ .\ \sqrt{P_t}}{H^{\frac{5}{4}}}$

$$= \frac{\dfrac{60\ \phi\ .\ \sqrt{2gh}}{\pi\ .\ D_1} \times \sqrt{\dfrac{62{\cdot}4 \times 0{\cdot}95 \times \pi \times 8{\cdot}02 \times n \times \psi \times D_1^2\ .\ h^{\frac{1}{2}} \times h \times 0{\cdot}85}{550}}}{h^{\frac{5}{4}}}$$

$$\text{or}\quad N_s=\phi\times\frac{60\times 8.02}{\pi}\times\frac{\sqrt{h}}{D_1}\times\frac{\sqrt{\frac{62.4\times 0.95\times\pi\times 8.02\times 0.85}{550}\times D_1^2\times h^{\frac{3}{4}}\quad\sqrt{n\,.\,\psi}}}{h^{\frac{5}{4}}}$$

$$\text{or}\quad \mathbf{N_s=233\,.\,\phi\,.\,\sqrt{n\,.\,\psi}}\qquad \text{QED}$$

$$\text{or}\quad \mathbf{N_s=233\,.\,K_{u_1}\,.\,\sqrt{K_B\,.\,K_{v_{m_1}}}}$$

b) $$N_s=\frac{N\,.\,\sqrt{P_t}}{H^{\frac{5}{4}}}$$

$$\text{or}\quad 50=\frac{N\times\sqrt{9{,}000}}{278^{\frac{5}{4}}}=\frac{N\times 94.9}{278\times 4.1}$$

$$\text{or}\quad N=\frac{50\times 278\times 4.1}{94.9}=\mathbf{600\ RPM}\quad \textit{Answer}$$

$$Q_{11}=0.95\times\pi\times 8.02\times n\,.\,\psi$$
$$=0.95\times\pi\times 8.02\times 0.2\times 0.18$$
$$=0.862$$

$$\text{But}\quad Q=\frac{500\times P_t}{w\,.\,H\,.\,\eta_t}=\frac{550\times 9{,}000}{62.4\times 278\times 0.85}=335\text{ cu ft/sec}$$

$$\text{Now}\quad Q=Q_{11}\,.\,D_1^2\,.\,H^{\frac{1}{2}}$$

$$\therefore\quad D_1=\sqrt{\frac{Q}{Q_{11}\,.\,\sqrt{H}}}=\sqrt{\frac{335}{0.862\times\sqrt{278}}}$$

$$=\sqrt{23.3}=\mathbf{4.83\ ft\approx 4\ ft-10\ in.}\quad \textit{Answer}$$

Problem 7.8 A Kaplan turbine develops 8,000 HP (or 8,125 metric HP) under an effective head of 16 ft (or 4·88 m). Its speed ratio is 2 and flow ratio is 0.6, and the diameter of the boss=0·35 times the external diameter of the runner. Mechanical efficieny of the turbine is 90 percent. Calculate the diameter of the runner, speed of the runner and also its specific speed. *(Punjab University—1951)*

Solution

P_t=8,000 HP (or 8,125 metric HP) H=16 ft (or 4·88 m)

$K_{u_1}=2$ $K_{v_{m_1}}=0.6$

$\eta_t=90\%$ $d=0.35\,D_1$

(Here mechanical efficiency of the turbine means the overall turbine efficiency)

$$\eta_t=\frac{\text{Brake Horsepower}}{\text{Available Horsepower}}=\frac{P_t}{P_a}$$

$$\text{or}\quad 0.9=\frac{8{,}000}{\frac{w\,.\,Q\,.\,H}{550}}\quad\left[\text{or } 0.9=\frac{8{,}125}{\frac{w\,.\,Q\,.\,H}{75}}\right]$$

$$\therefore \qquad Q=\frac{8{,}000}{0{\cdot}9}\times\frac{550}{62{\cdot}4\times16}=4{,}900 \text{ cu ft/sec}$$

$$\left[\text{ or } \quad Q=\frac{8{,}125}{0\ 9}\times\frac{75}{1{,}000\times4{\cdot}88}=138\ 5\ m^3/\text{sec}\right]$$

Flow ratio $K_{v_{m_1}}=0{\cdot}6$

$$=\frac{v_{m_1}}{\sqrt{2gH}}=\frac{v_{m_1}}{8\ 02\times4}\quad\left[\text{ or } \quad \frac{v_{m_1}}{4{\cdot}43\times2{\cdot}21}\right]$$

or Velocity of flow, $v_{m_1}=0{\cdot}6\times8{\cdot}02\times4=19\ 25$ ft/sec

[or $v_{m_1}=0\ 6\times4\ 43\times2{\cdot}21=5\ 875\ m/\text{sec}$]

Discharge=velocity of flow × area of flow

$$\therefore \quad \text{Area of flow}=\frac{\text{Discharge}}{\text{velocity of flow}}=\frac{Q}{v_{m_1}}=\frac{4{,}900}{19{\cdot}25}=254{\cdot}5 \text{ sq ft}$$

$$\left[\text{or } \frac{138\ 5}{5{\cdot}875}=23{\cdot}6\ m^2\right]$$

But area of flow $=\frac{\pi}{4}D_1^2-\frac{\pi}{4}\ d^2$

$$\therefore \qquad 254{\cdot}5=\frac{\pi}{4}D_1^2\ (1-0\ 35^2)=0\ 689\ D_1^2$$

[or $23{\cdot}6=0{\cdot}689\ D_1^2$]

$$\text{or} \qquad D_1=\sqrt{\frac{254{\cdot}5}{0\ 689}}=\sqrt{370}=19{\cdot}25 \text{ ft}=\mathbf{19\frac{1}{4}\ ft} \quad \textit{Answer}$$

$$\left[\text{ or } \quad D_1=\sqrt{\frac{23{\cdot}6}{0\ 689}}=\sqrt{34{\cdot}3}=5{\cdot}86m \text{ or } \mathbf{5{,}860\ mm}\ \textit{Answer}\right]$$

$$\text{Speed ratio} \quad K_{u_1}=2=\frac{u_1}{\sqrt{2g\ H}}=\frac{\pi\ .\ D_1\ .\ N}{60\ .\ \sqrt{2g\ H}}$$

$$\therefore \quad N=\frac{K_{u_1}\ .\ \sqrt{2gH}\times60}{\pi\times D_1}=\frac{2\times8\ 02\times4\times60}{\pi\times19\ 25}=\mathbf{63\ 6\ RPM} \qquad \textit{Answer}$$

$$\left[\text{or } N=\frac{2\times4\ 43\times2\ 21\times60}{\pi\times5{\cdot}86}=\mathbf{63{\cdot}6\ RPM} \quad \textit{Answer}\right]$$

Best synchronous speed of the turbine would be **75 RPM**, when coupled to a 50 cycles/sec generator having 40 pair of poles.

$$\text{Specific speed of the turbine} \quad N_s=\frac{N\ .\ \sqrt{P_t}}{H^{\frac{5}{4}}}$$

$$=\frac{63\ 6\times\sqrt{8{,}000}}{16^{\frac{5}{4}}}=\mathbf{178} \quad \textit{Answer}$$

$$\left[\text{ or } \quad N_s=\frac{63{\cdot}6\times\sqrt{8{,}125}}{4\ 88^{\frac{5}{4}}}=\mathbf{791} \quad \textit{Answer}\right]$$

UNSOLVED PROBLEMS

7.1 Describe briefly an inward flow turbine. (*AMIE—May 1954*)

7.2 Show by the help of a diagram the layout of a medium head water power plant using a Francis turbine. The sketch of the turbine should also be drawn showing its different parts.

7 3 Where are open flume reaction water turbines employed ? Describe such a type of turbine briefly.

7.4 Write short notes on—

Scroll casing, Wicket gates, runners of reaction turbines.

7 5 What is the necessity of stay vanes in a spiral casing ?

7.6 What are the functions of a draft tube ?

7.7 Prove that a draft tube prevents the loss of head of a reaction turbine.

7.8 What are the uses of a draft tube ? Sketch and name the different types of draft tubes and state which one of them gives the maximum efficiency. Name the modern turbines which require the use of draft tube. Why does a Pelton turbine not possess any draft tube ? (*AMIE—Nov 1954*)

7.9 Describe different types of draft tubes.

7.10 What is Hydracone ? What are its advantages ?

7.11 How was the elbow type draft tube developed ?

7.12 What is the maximum head for which a Francis turbine has been installed ?

7.13 Why is it necessary to choose the number of Francis runner blades as odd and the number of guide vanes as even ?

7.14 Show by the help of diagrams and velocity triangles how the runner of a Francis turbine can be developed to that of a Kaplan type. (*AMIE—May 1955*)

7.15 Draw typical velocity triangles at inlet and at outlet for low, medium, and high specific speed Francis turbines as well as Kaplan turbines.

7.16 How do the losses in the draft tube affect the pressure at runner exit ?

7.17 What is meant by "cavitation" ? How and where does it occur in Water Power Plants ? Derive an equation for the lowest head required by a reaction turbine which is placed below the tail water level.

7.18 Why does it become necessary to install a water turbine below the tail race level ?

7.19 What is Thoma's factor of cavitation and what is its significance for water turbines ?

7.20 On what factors does the cavitation in water turbines depend ?

7.21 Describe some method to avoid cavitation in water turbines.

7.22 Write short notes on the selection of working speed and runaway speed of reaction turbines.

7.23 Describe with the help of sketch the special features of a Kaplan turbine. (*AMIE—Nov 1953*)

7.24 How are the runner blades of a Kaplan turbine rotated ?

7.25 Describe the latest development for the motion of Kaplan runner blades.

7.26 Describe the maximum diameter and maximum head of a Kaplan turbine till today.

7.27 What is the difference between a Propeller turbine and a Kaplan turbine ?

7.28 What are the hydraulic losses in a reaction water turbine ? Define hydraulic efficiency.

7.29 How will you determine the overall efficiency, if the volumetric, head and mechanical efficiencies are given ?

7.30 What do you understand by—

i) Setting of a turbine,

ii) Cavitation index,

iii) Speed ratio of a turbine. (*AMIE—May 1958*)

7.31 Talaiya Power House (DVC) is equipped with two water turbines each developing 2,800 HP when running at 250 rpm under a maximum head of 77 ft.

a) Calculate the specific speed of the turbine.

b) State the suitable type of the turbine and its runner.

c) Determine approximately the inlet diameter of the runner if the speed ratio is 0·8.

d) Draw the typical inlet and outlet triangles of velocities.

(58·2, Francis, fast runner, 4·3 ft) (*Jadavpur University—1955*)

7.32 Pathri Hydro Station on the Ganga Canal in U.P. is equipped with three VOITH water turbines, each having the following specifications—

Net working head = 32 ft,
Dischare = 2,950 cusecs,
Output = 9,500 BHP,
Speed = 125 rpm.

Determine

a) Overall efficiency of each turbine,

b) Specific speed and type of turbine,

c) Speed ratio, if the diameter of the runner is $12\frac{3}{4}$ ft.

Draw the outlines of the above turbine, showing the type of draft tube you would employ.

(88·7% ; 160 ; Kaplan ; 1·84 ; Elbow) (*Roorkee University—1958*)

7.33 A reaction turbine is to be selected from the following conditions—

Maximum net head = 42 ft
Average net head = 35 ft
Minimum net head = 29 ft

Full output at average head which is also design head is 18,000 BHP. What type of turbine would you suggest in this case ?

The full load total efficiency at average head may be taken as 89% and at maximum and minimum heads as 87% and 88% respectively. The speed ratio is 1·8 at average head and generation speed is 250 rpm.

Determine

a) Diameter of runner,

b) Designed specific speed,

c) Full load outputs at minimum and maximum heads.

[Kaplan (*a*) 6 ft—6½ in. (*b*) 394 (*c*) 23,100 HP, 13,420 HP]

(*Punjab University—1957A*)

7.34 An inward flow turbine, having an overall efficiency of 75% is required to give 175 HP. The head is 20 ft, velocity of the periphery of wheel is 0 8 $\sqrt{2g\,H}$, and the radial velocity of flow is 0·35 $\sqrt{2g\,H}$. The wheel is to make 250 rpm and the hydraulic losses in the turbine are 20% of the available energy. Determine

a) the angle of the guide blade at inlet,

b) the wheel vane angle at inlet,

c) the diameter of the wheel,

d) the width of the wheel at inlet.

Assume the turbine as reaction and radial discharge.

(35°—6′ ; 48°—8′ ; 2′—2¼″ ; 14·4 inches) (*Punjab University—1958A*)

7.35 An inward flow reaction turbine runner is required to operate under a head of 35 ft at a speed of 175 rpm and to develop 200 HP. Find the diameter of the runner at inlet and outlet, the discharge, the guide vane angle, and the runner vane angle at inlet and outlet, assuming the following data—

Outlet diameter $=0·66\times$ inlet diameter

Peripheral speed of inlet $=0·75\ \sqrt{2g\,H}$

Velocity of flow $=0·16\ \sqrt{2g\,H}$ $=$constant

Discharge radial

Hydraulic efficiency $=86\%$

Overall efficiency $=81\%$

(3·89 ft ; 2·57 ft ; 62 1 cusecs ; 42½° ; 17°—54′)

7.36 A Francis turbine gives the following performance :

$H=17·4$ ft, $Q=53·85$ cfs, BHP$=92·77$, rpm$=204·2$, Dia$=31$ in. Part gate opening$=0·873$.

a) Compute the efficiency and speed ratio.

b) Assuming the same wheel to operate at same speed and same gate opening under a head of 50 ft, what will be the new rpm, Q and HP. Check the results by recomputing from the efficiency and speed ratio,

[*a*) 87·2%, 0·823 ; *b*) 346 rpm, 91·2 cfs, 450 HP]

7.37 A Francis turbine has a wheel diameter of 4 ft at the entrance and 2 ft at the exit. The vane angle at the entrance is 90° and the guide vane angle is 15°. The water at the exit leaves the vanes

without any tangential velocity. The head is 100 ft and the radial component of flow is constant. What would be the speed of the wheel in rpm and vane angle at exit? State whether the speed calculated is synchronous one or not. If not what speed would you recommend to couple the turbine with an alternator of 50 cycles. (268 rpm ; 28° – 12′ ; 250 rpm) *(UPSC Jan—1952)*

7.38 An inward flow turbine wheel works under a head of 60 ft and makes 380 rpm. The diameter of the outer circumference of the wheel is 24 in., and of the inner circumference 12 in. The velocity of water entering the wheel is 44 ft/sec, and the angle it makes with the tangent to the wheel is 10°. Assuming the radial velocity of flow through the wheel to be constant, and that the water leaves the wheel in a radial direction, determine the direction of the tangent to the vane of the wheel at the inlet and outlet. Determine also the hydraulic efficiency of the turbine.
(114°—42′ ; 21° tangent to the periphery, 89%)

7.39 Find the leading dimensions of the runner of an inward-flow reaction water turbine to develop 850 HP at 1,000 rpm under a head of 340 ft, assuming a guide vane angle of 16°; axial length of blade at inlet 0·1 times the outer diameter, inner radius 0·6 times the outer radius; radial velocity of flow constant, final discharge radial; hydraulic efficiency 0·88; overall efficiency 0·86; allowance for blade thickness 5%.
($D_1 = 1{\cdot}633$ ft ; $D_2 = 0{\cdot}98$ ft ; $B_1 = 1{\cdot}95$ in. ; $\beta_1 = 130°$: $\beta_2 = 32° - 6'$)
(Jadavpur University—1958)

7.40 The following data refers to an inward flow reaction turbine :

Supply— 30 cu ft/sec at 80 ft head,

Wheel diameters—30 in. at outlet and 20 in. at inlet,

Radial exit velocity—8 ft/sec,

Inlet vane angle—35°.

Calculate the horsepower and rpm of the turbine. Assume the width of the wheel constant, and turbine efficiency 80%.
[217·5 HP; 320 rpm] *(AICTE—May 1958)*

7.41 An inward flow turbine works under a total head of 90 ft. The velocity of wheel periphery at inlet is 50 ft/sec. The outlet pipe of the turbine is one foot in diameter and the turbine is supplied with 50 gallons of water per second. The radial velocity of flow through the wheel is the same as the velocity in the outlet pipe. Neglecting friction, determine—

a) the vane angle at inlet,

b) the guide blade angle,

c) HP of the turbine.

(Madras University—1957)

7.42 Discuss the general theory of inward flow reaction turbine, and deduce an expression for the maximum hydraulic efficiency. State the reasons that lead to the adoption of comparatively smaller inlet angles for runner blades for higher heads and larger ones for lower heads. Show also that when the inlet angle is 90° for

the medium heads, the velocity of flow being constant, the hydraulic efficiency is expressed by $\frac{2}{2+\tan^2\alpha}$, where α is the guide blade angle. (*Bombay University—1957*)

7.43 A Kaplan turbine has a vertical conical draft tube. The diameter of the tube on the upper side connected to the outlet of the runner is 2 ft and that of the tube outlet is 3 ft. The tube is running full with water flowing downwards, and it is 20 ft long with 5 ft of its bottom length in tail water. The frictional loss between the top and the bottom point is 0·15 times the velocity head at the top point where the water has a velocity of 20 ft/sec. Find the water pressure in the top point of the draft tube in lb/sq in. and ft of water. (—8 25 lb/sq in.,—19·05 ft) (*Jadavpur University—1954*)

7.44 A reaction water turbine is equipped with a straight flaring draft tube having top and bottom diameters of 20 in. and 30 in. respectively. The water velocity at the top is 10 ft/sec, where the elevation is 15 ft above the level of the tail water. Assuming a loss in the draft tube equal to half the velocity head at exit, compute

a) the pressure head at the top,

b) the total head at the top with reference to the tail race as a datum,

c) the total head at the exit,

d) the power in water at the top and at exit,

and *e*) the power loss in the tube due to friction.
(—16·1 ft, 0·46 ft, 0·31 ft, 1·14 HP, 0 77 HP, 0·37 HP)

7.45 A turbine runner has an exit velocity of 32 ft/sec. The loss of head due to friction and other causes in the draft tube should not exceed 5 ft. What maximum height of settings will you recommend for the turbine if the cavitation is to be avoided ?

Assume

i) the velocity of water at the outlet of draft tube as 8 ft/sec,

and *ii*) the cavitation commences when the pressure is 8 ft of water absolute. (16·1 ft) (*Delhi University—1957*)

7.46 A plant is located at an elevation of 300 MSL. The average water temperature is 60°*F*. The turbine develops 50,000 HP at a rated speed of 112·5 rpm and under a head of 135 ft. Calculate the maximum allowable suction head for the draft tube. Given—

σ	1	0·5	0·4	0·2	0·1	0·05	0·03	0 016
N_s	200	166	90	60	41	29	22	16

and at 300 ft MSL.

°F	165	155	135	115	85	60
H_b	21	24	28	30	32	33

H_b=Height of barometric column in ft. (10 ft)

7.47 A Francis turbine develops 550 BHP at an overall efficiency of 82% when working under a static head of 5 metres, the draft tube being cylindrical and of diameter 2·3 metres. What increase in power and efficiency of turbine would you expect, head, speed and discharge remaining the same; if a tapered draft tube having an outlet diameter of 3·2 metres and efficiency of conversion of 90% were substituted for the cylindrical one?

(50 HP ; 4%) (*Madras University—1955*)

7.48 The Thoma-factor (σ) of cavitation can approximately be written as—

$$\sigma=\eta_s \,.\, K_m^2+\lambda \,.\, K_u^2$$

where $K_m=\dfrac{v_m}{\sqrt{2g\,H}}$ and $K_u=\dfrac{u}{\sqrt{2g\,H}}$

v_m and u being the axial and tangential velocities. η_s is the draft tube efficiency and

$$\lambda=\frac{\dfrac{p}{w}-\dfrac{\overline{p}}{w}}{\dfrac{u^2}{2g}}$$

is the dimensionless parameter describing the pressure difference between a point on the runner vane and at the exit. $\dfrac{p}{w}$ is the absolute pressure at a point on the vane and $\dfrac{\overline{p}}{w}$ at the exit.

A 250 rpm Kaplan turbine using 900 cusecs at full load under a net head of 60 ft has a runner diameter of 8 ft and hub diameter of 4 ft. The difference $\dfrac{p}{w}-\dfrac{\overline{p}}{w}$ as indicated by the model test can be taken equal to 6 ft of water. Neglect the vapour pressure and determine the maximum permissible suction head for the above machine taking barometric pressure as 28 ft of water and the draft tube efficiency of 85%.

(14·44 ft) (*Punjab University—1957A*)

7.49 The following data were obtained from an efficiency test of a Kaplan turbine—

Diameter of boss of runner=0·35 times the external diameter,

Speed ratio	=2,	Flow ratio=0·6	
Speed	=75 rpm,	Head	=25 ft
Brake horsepower	=17,500		

Find the efficiency of the turbine.

(89·8%) (*AMIE—Nov 1958*)

7 50 Kaplan turbine installed at Radhanagri (Bombay State) Power House develops 1,720 HP at an average head of 95 ft. The speed and flow ratios are 2·1 and 0·62 respectively. The diameter of the boss = 0 34 times the external diameter of runner. Overall efficiency is 0·89. Calculate the diameter and speed of the runner.

(2·31 ft ; 1,360 rpm)

7.51 As an element of water passes through the runner of an inward-flow turbine, its total energy and its pressure energy undergo considerable changes. Show this by sketching graphs roughly to scale, between "radial distance of element from the axis" (abscissae) and "pressure energy" and "total energy" of element (ordinates) for an idealised vertical shaft turbine as follows—

Outer diameter = 1·25 *m* ; inner diameter = 0·7625 *m* ; total head above wheel = 186 *m* ; radial component of flow velocity of water = 6·23 *m*/sec ; inlet runner blade angle = 90°. Assume that 5% of the total head is lost in friction and eddying in the guide apparatus, before the water reaches the wheel, and that 5% is lost in the runner itself. The water escapes from the runner into the tail-race at atmospheric pressure. Make any other reasonable assumptions. What would be the pressure head of the water in the clearance space between guide blades and runner blades ?

[*AMI Mech E (Lond)—Oct 1959* ; Converted to metric units]

7 52 233 litres of water per second are supplied to an inward flow reaction turbine. The head available is 11 *m*. The wheel vanes are radial at inlet and the inlet diameter is twice the outlet diameter. The velocity of flow is constant and equal to 1·83 *m*/sec. The wheel makes 370 rpm. Find—

a) guide vane angle ;
b) wheel vane angle at inlet ;
c) inlet and outlet diameters of the wheel ; and
d) the width of the wheel at inlet and exit. Neglect the thickness of the vanes. Assume that the discharge is radial and that there are no losses in the wheel.

(10°—4′ ; 90° ; 534 *mm* ; 267 *mm* ; 76·25 *mm* ; 152·5 *mm*)
(*Osmania University—1952* ; Converted to metric units)

7.53 Two inward flow reaction turbines working under the same head and same hydraulic efficiency have also runners of the same diameters *viz*, 550 *mm* and the velocity of flow for both is 5·5 *m*/sec. One of the runner A has an inlet blade angle of 65° and a speed of 520 rpm, while the other B has an inlet angle of 115° What is the speed of the runner B ?

(432 rpm) (*Bombay University—1956* ; Converted to metric units)

7.54 The largest Kaplan turbine installed anywhere in the world is at Vargon (Sweden). It is rated at 15,400 metric HP when working under an average head of 4 3 *m*. The diameter of the boss is 0·3 times the external diameter of runner. Overall efficiency of turbine is 0 91. Find the speed and diameter of turbine runner. Take the values of speed ratio and flow ratio as 2 and 0·65 respectively.

(42·2 rpm ; 8·33 *m*) (*BHU—1960* ; Converted to metric units)

CHAPTER 8

GOVERNING OF WATER TURBINES

8.1 Introduction 8.2 Functions of a Water Turbine Governor 8.3 Types of Water Turbine Governors 8.4 Qualities of a Governor (Sensitiveness, Rapidity, Stability, Isochronism, Hunting, Capacity, Speed Regulation and Regulating Time) 8.5 Elements of Oil Pressure Governor (Speed-Responsive Element, Power Element and Stabilising or Compensating Element) 8.6 Outlines of Oil Pressure Governor 8.7 Working of Oil Pressure Governor 8.8 Modern Types of Oil Pressure Governors (Gate Shaft Type and Actuator Type) 8.9 Different Types of Actuators (Speed-Responsive Actuator, Speed cum Acceleration-Responsive Actuator and Electro-Hydraulic or Electronic Actuator) 8.10 Governing of Impulse Turbines 8.11 Governing of Reaction Turbines 8.12 Relief Valve or Pressure Regulator 8.13 Governing of Kaplan Turbines by Double Regulation 8.14 Safety Devices for Penstocks 8.15 Surge Tanks 8.16 Forebay.

8.1 Introduction—Governing means regulation of speed. The main function of a governor is to maintain a constant speed when load on the turbine fluctuates. The governor is, therefore, the point of co-ordination between the turbine and external controls. This is accomplished by controlling the rate of flow of liquid leading to the runner, in proportion to the load. In case of Pelton turbines the governor closes or opens the needle valve, in case of Francis turbines, it closes or opens the wicket gates and in case of Kaplan turbines it moves the runner blades in addition to closing and opening of wicket gates.

Almost all modern hydraulic turbines are directly coupled to alternators which must run constantly at the designed speeds. The speed of an alternator is fixed by the number of pair of poles and the required frequency (cf Art 6.13). For various reasons the frequency cannot be allowed to vary. The speed must, therefore, remain constant when the load changes, hence the necessity of speed control. Governors are also used for other primemovers such as diesel engines, steam turbines and steam engines. In the former it operates by regulating the fuel supply and in the latter by controlling the steam pressure. The water turbine governor controls the rate of flow of water.

The governor of a diesel engine has to perform much less work than that of a hydraulic turbine. While a small centrifugal governor will suffice to control the supply of a fuel like oil, it needs a very strong governing mechanism to exert forces necessary to divert or to stop the flow of water into a turbine runner.

The turbine governor is called upon to satisfy conflicting conditions. It should be quick acting, *i.e.*, it should restore the speed to normal as soon as possible after it has been disturbed due to a variation of load. On the other hand it should not close the pipeline so quickly as to cause a serious hammer blow. Obviously, the governor should be so designed as to keep both speed rise of runner and pressure rise in pipe

within limits. In order to save the pipeline from water hammer effects, the turbines are equipped with a safety device operated by the governor. A deflector or diffusor in Pelton turbines and relief valve (*i.e.* pressure regulator) in reaction turbines will serve this purpose.

8.2 Functions of a Water Turbine Governor—

a) It controls the speed of turbine—set and match it to the hydro-electric system for synchronising.

b) It controls the speed and frequency of turbine unit or units to maintain a desired frequency and voltage.

c) It sets the amount of load a turbine unit has to carry.

d) It helps in starting and shutting down the turbo-unit by opening and closing the nozzle of Pelton wheel and wicket gates of reaction turbine.

8.3 Types of Water Turbine Governors—The governing mechanism may operate mechanically, electrically or by fluid pressure. Mechanical governors such as ball and spring governors as well as electric brake governors are used for small turbines. Speed control in large water turbines requires a mechanism capable of relatively heavy duty and therefore oil pressure governor is the only suitable type.

8.4 Qualities of a Governor—The following are the fundamental qualities or performance characteristics of a governor if it is to regulate satisfactorily.

a) Sensitiveness
b) Rapidity of Action
c) Stability
d) Isochronism
e) Hunting or Racing
f) Capacity
g) Speed Regulation
h) Regulating Time or Governor's Time.

a) **Sensitiveness**—The ratio of the mean speed of action to the difference between the maximum and minimum speeds is a measure of sensitiveness of a governor.

i.e. $$\text{Sensitiveness} = \frac{\omega_2+\omega_1}{2} \cdot \frac{1}{\omega_2-\omega_1} \qquad \ldots(8.1)$$

or in other words sensitiveness of governor means the alteration of position of governor which is caused by the smallest increase or decrease in speed at any turbine gate opening.

b) **Rapidity of Action**—The action of governor must be rapid. If the load increases, the governor's action should be quick to open the gate. However, if the load falls, the quick closing of gates checks the flow of water in the pipeline and may cause water hammer. If the pipeline is long relative to head (*i.e.* pipe length is more than 5 to 6 times the static head), the problem of pressure rise becomes serious. In order to avoid such a pressure rise, the closing time of governor should be increased, but the delay in shutting off the water allows a greater time for the turbine to speed up and hence the speed rise becomes considerable unless a very heavy flywheel is fitted.

c) **Stability**—If a governor is stable and its balls are slightly displaced from their equilibrium position, the speed remaining constant, they will have a tendency to return to the normal configuration corresponding to speed. Thus a stable governor helps the gates to come to their proper position with the least possible oscillation.

d) **Isochronism**—A governor is said to be isochronous if, neglecting friction, the equilibrium speed is same for all radii of balls. This is possible if the governor is *infinitely* sensitive and the governor always flies to one or the other extreme position. The condition is representated by $\omega_1=\omega_2$ in Eqn 8.1.

e) **Hunting or Racing**—A governor is said to hunt or race if there is periodic speed fluctuations with steady load. This is caused if the governor is too sensitive.

f) **Capacity**—The capacity of governor which controls the flow of oil under pressure to the turbine servomotor, is determined from the force required to move the turbine wicket gates. The capacity of governor is the sum of the capacities of gate servomotor and blade servomotor (in case of Kaplan turbine only).

The capacity of a servomotor in ft lb or kg m is the total force in lb or kg exerted by oil under pressure on the piston or pistons multiplied by the stroke in ft or metres.

Capacity of gate servomotor $G_g=G_1D_1^2\,H_{max}$ ft lb (8.2)

where $G_1=\text{Constant}=\dfrac{N_s+10}{10}$ ...(N_s in FPS units)

D_2=Runner discharge diameter

H_{max}=Maximum head including water hammer

N_s=Specific speed

Blade servomotor capacity $G_b=\dfrac{20\,P_t\,.\,N_s^{\frac{1}{4}}}{H^{\frac{1}{2}}}$ ft lb .. (8.3)

where P_t=rated horsepower

H=rated head.

g) **Speed Regulation**—The speed variations which are caused by sudden fall or increase of full or partial load, are to be regulated by the governor in order that the normal speed is returned immediately.

h) **Regulating Time or Governor's Time**—This is the time in seconds required by the governor to move the turbine gates from the closed to the open position or vice versa. The governor system is designed to close the wicket gates in 3 seconds for a short penstock and upto 5 seconds for a long penstock.

8.5 Principal Elements of Oil Pressure Governor—The principal elements of a water turbine governor which is generally of oil pressure type, are as follow—

a) Speed-responsive element, *b*) power element,

c) Stablilising or compensating element,

a) **Speed-Responsive Element** which starts its action with the change of speed, consists of—

i) *Pendulum or governor's head* which functions by centrifugal action.

ii) *Drive of Pendulum or of Governor's head* from the turbine main shaft.

b) **Power Element** which supplies power to open or to shut the turbine gates according to load, consists of—

i) *Servomotor*—A piston operating inside a cylinder by oil pressure, is made to transmit power to turbine gates through suitable mechanism.

ii) *Distributor of Relay Valve* supplies oil to either side of the piston of servomotor for its (piston's) to and fro motion.

iii) *Pressure Oil Supply Unit* supplies oil to distributor or relay valve. It has a pumping unit with an air vessel and a sump tank.

c) **Stabilising or Compensating Element** which prevents hunting or racing of governor, consists of—

i) *Dash pot*—The action of the pendulum is transmitted through a system of floating levers to the dash pot. The action of dash pot is *asymptotic*.

ii) *Pilot Valve*—Dash pot transmits the action of pendulum to the pilot valve which causes the oil pressure to be transmitted to the distributor or relay valve (*ii* of *b* above).

In addition to above, the stabilising or compensating element consists of the following—

iii) *Starting & Stopping Gears*—Starting of turbine is accomplished by a lever, operated by hand, which brings the distributing valve gradually into opening position. As soon as the turbine reaches normal speed, the oil pressure is admitted to the emergency cylinder by turning the bypass cock into "to run" position. To shut down, the bypass cock is turned into "to close" position whereby the emergency or safety oil pressure cylinder is exhausted and the turbine shuts.

iv) *Safety Devices* against oil pressure and drive failures.

v) *Load Limiting Device* with which the normal load can be limited before and/or during operation.

vi) *Speed Changer or Speed Setting* can be done by hand or by an electromotor.

vii) *Hydraulic Return Motion Gear* is fitted over the dash pot, which helps in transmitting the motion is one direction only and thus no *lost* motion can develop with such a gear.

8.6 **Outlines of Oil Pressure Governor**—It consists of the following main components (*See* Fig 8.1) :

i) **Servomotor or relay cylinder**—A piston operating inside a cylinder is made to transmit power to the turbine gates through suitable mechanisms.

ii) **Relay valve or distributing valve**—It controls the flow of oil under pressure to either direction of the servomotor cylinder.

iii) **Actuator or pendulum**—It functions by centrifugal action and it is driven from the turbine main shaft.

iv) **Dash Pot and Pilot Valve** which supplies the motion of pendulum, in stablised form, to the distributing valve.

v) **Pressure oil supply** including pumping units, supply oil to distributing valve.

vi) **Casing**—It encloses all above components and also serves as an oil pressure tank.

vii) **Governor driving mechanism**

The function of each part is explained below—

i) **Servomotor**—It consists of a cylinder inside which a piston moves under the action of oil pressure. For large units, two cylinders may be employed.

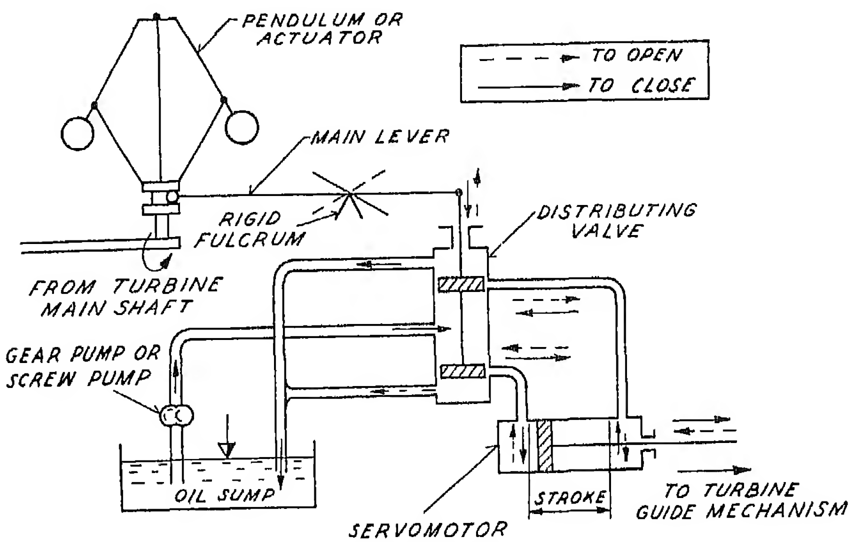


Fig 8 1 Oil Pressure Governor with Fixed Fulcrum in Neutral Position—Simple Line Diagram

The motion of the piston is transmitted to the spear rod in Pelton turbines and the guide vanes in reaction turbines. An auxiliary governing device is the hand wheel driving the piston. This is used either when starting the unit or when automatic governing is not required.

ii) **Relay Valve**—This is generally a slide valve of piston type. Distribution of oil supply to the chambers of the servomotor cylinder is regulated by the relay valve operated by a pilot valve which itself is controlled by some speed-responsive elements such as flyballs. The relay valve allows the oil under pressure to flow to either side of the servomotor piston to push it for moving the guide vanes to opening or closing positions.

iii) **Actuator or Pendulum**—It is usually a flyballs mechanism which responds quickly to speed variation. The centrifugal governor and the pumps are driven by tapping power from the turbine mainshaft through belts, gears or an electric motor. The centrifugal governor also operates the runner blade adjusting mechanism in Kaplan turbines.

iv) **Dash Pot and Pilot Valve** are the stabilising and the compensatory elements. The motion of pendulum's flyballs is transmitted to the relay valve through dash pot and pilot valve. The dash pot prevents sudden or too large opening out or closing in of flyballs, thus acting as a damping device. The pilot valve receives the damped motion from the pendulum and transmits it to the relay valve.

v) **Pressure Oil Supply**—Oil under pressure, ranging from 150 to 300 lb/sq in., is supplied to the distributing valve by a pumping unit. It consists of a rotary (gear or screw type) plunger pump, an oil sump tank, and a pressure tank with suitable valves and pipings. In some cases there are two gear pumps of different sizes each running continuously and connected to a common shaft. The smaller unit supplies oil to counteract gradual load fluctuation while the larger operates in case of sudden load drops or increments. No pressure tank is then required.

vi) **Casing**—The mechanisms enumerated above are enclosed in the casing which protects them against dust and damage. It also serves as an oil sump.

vii) **Governor Driving Mechanism**—The centrifugal governor and the rotary gear pump are driven by the turbine main shaft through belts or gears. In modern units, the governor incorporates an electric motor which is energised by a small alternator coupled to the main shaft. The motor is designed to respond to any fluctutation of frequency caused by a variation of speed of the main unit.

In very modern installations the alternator and motor have been replaced by electrical devices responsive to frequency variation. Precision control can be effected by employing electric apparatus extremely sensitive to variation of frequency.

8.7 Working of Oil Pressure Governor (Fig 8.1)—When the load on the generator increases, the speed of turbine which is directly coupled to generator falls. Now the pendulum or the actuator is driven

1 Plate-sprig Flyballs,
2 Flyball Drive,
3 Flyball Collar with Pilot Sleeve,
4 Pilot Oil Circuit,
5 Relais Valve,
6 Servomotor,
7 Pilot Valve Retardation,
8 Pilot Valve,
9 Retardation Cam,
10 Retardation Gear,
11 Dashpot Cylinder,
12, 13 Coil Springs,
14 Dashpot Piston,
15 Connecting Rod,
16 Fulcrum of Lever 7

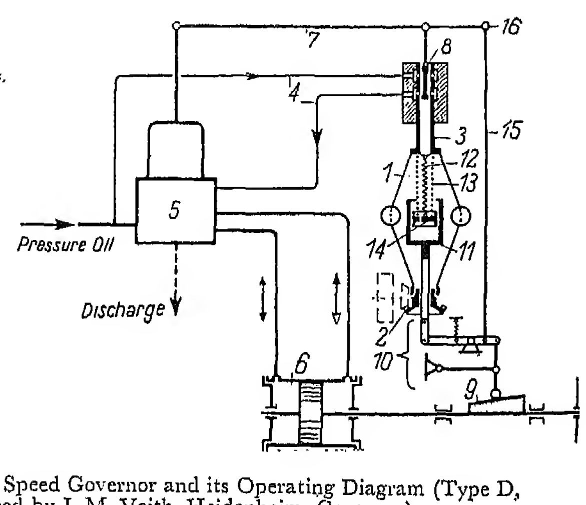


Fig 8.2 Water Turbine Speed Governor and its Operating Diagram (Type D, Manufactured by J. M. Voith, Heidenheim, Germany)

by the main shaft of the turbine. Therefore on reduction of speed of the turbine, the pendulum balls occupy the bottom piston with which the lever at right hand end, is raised. This raises the pistons of the distributing valve with which the passage for pipeline 1 opens for the high pressure oil from the gear pump to pass and to strike to the right hand side of the servomotor piston, allowing the servomotor piston rod to move towards left. This movement can be transmitted to open the nozzle of an impulse turbine or guide vanes of a reaction turbine. If the load on the turbine is decreased, the operation explained above will be reversed.

Sectional view with a simplified operating diagram of Turbine Governor manufactured by J. M. Voith Heidenheim (Germany) and by

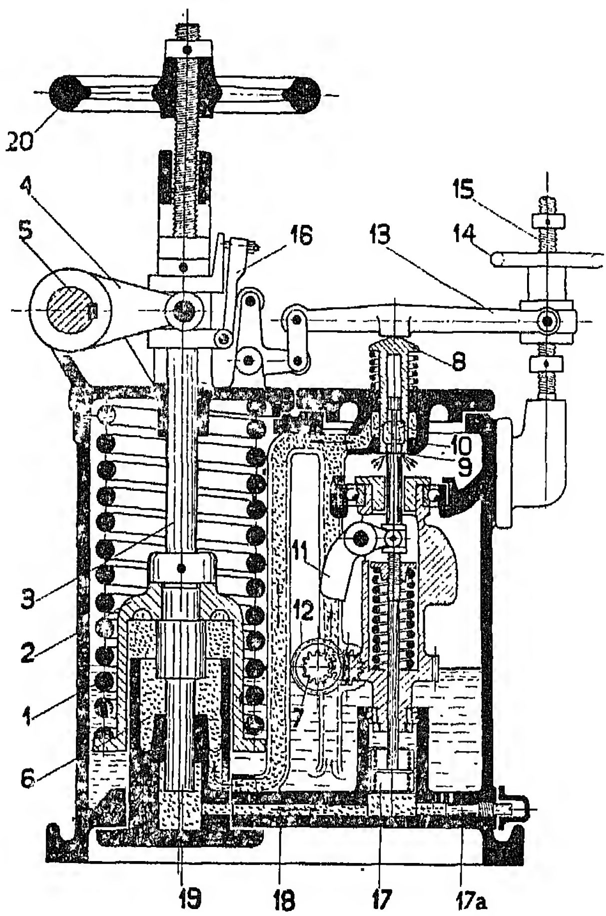


Fig 8.3 Water Turbine Speed Governor, manufactured by Escher Wyss & Co Ltd, Zurich

1. Spring 2. Piston 3. Piston rod 4. Crank 5. Shaft 6. Opening Cylinder Space 7. Gear Pump 8. Small Cylinder 9. Oil Outlet 10. Valve 11. Pendulum 12. Gear Pump Drive 13. Lever 14. Handwheel (small) 15. Screw 16. Cam 17. Piston with Spring 17(a). Drain Plug 18. Connecting Pipe 19. Plunger 20. Hand Wheel.

Escher Wyss and Co; Zurich (Switzerland), are shown in Fig 8.2 anb 8.3 respectively.

8.8 Modern Types of Oil Pressure Governors—The oil pressure governor may be of two types—

a) Gate shaft type *b*) Actuator type.

a) **Gate Shaft Type** (Fig 8.2 and 8.3)—In this type all the components of the governor *viz*, pendulm, dash pot, pilot valve, relay valve, servomotor and oil pressure supply are enclosed in a casing and form a single integrated unit. This is used for smaller units and the governor's capacity is upto about 50,000 ft lb (or about 7,000 kg m).

b) **Actuator Type** (Fig 8.4 and 8.5)—In this type the servomotor or servomotors being of large size are separated from the other components. In some cases the governor unit is divided in three parts namely oil pressure unit, servomotor unit and actuator, and they are normally located at different places in the power station. The governor's capacity in this case is not limited.

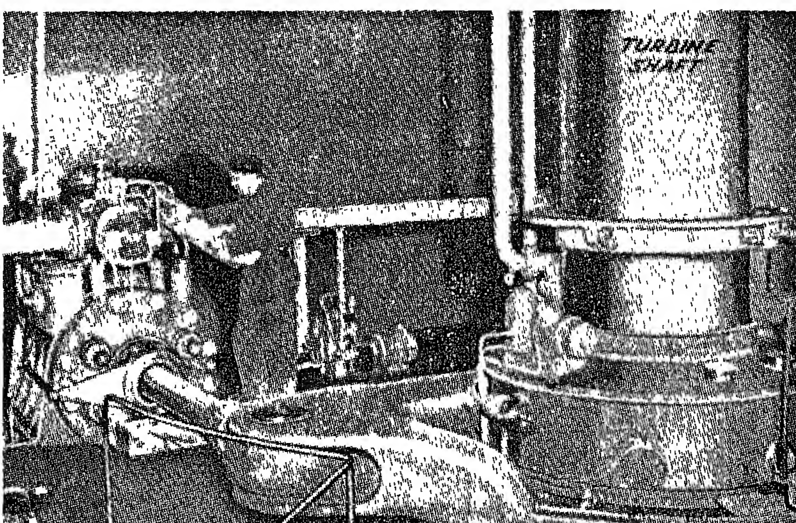


Fig 8.4 Turbine Shaft and one of the Gate Servomotors of Actuator Type Oil Pressure Governor, for Haspranget Power Plant

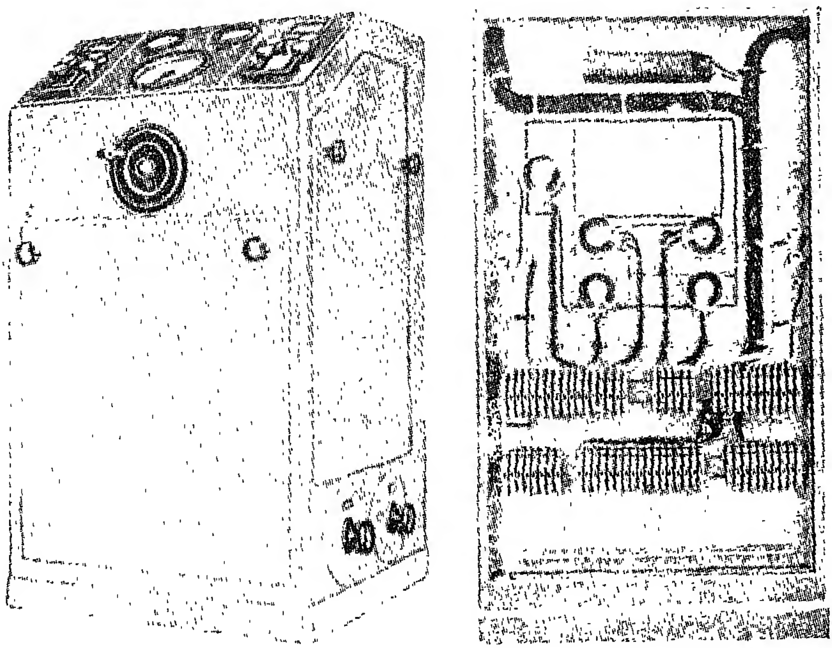
8.9 Different Types of Actuators—The type of governor's actuator depends upon the principle of pendulum operating mechanism. The different types are as follows—

a) Speed-responsive actuator,

b) Speed cum acceleration-responsive actuator,

c) Electro-hydraulic or electronic actuator.

a) **Speed Responsive Actuator** (Fig 8.2 and 8.3)—Mostly the governors are equipped with speed responsive element *i.e.* flyballs. The dash pot acts as a stabilizing device.



a) Front b) Back Cover Removed, Showing Electric Connection and Wiring

Fig 8.5 Electro-Hydraulic Actuator for Harspranget Power Plant

b) **Speed Cum Acceleration-Responsive Actuator**—The pendulum in this case is sensitive to both speed and acceleration. The sensitivity of the governor is said to increase as the accelerometer connected

Read the para c) on page 233 as under :—

c) **Electro-Hydraulic or Electronic Actuator** (Fig 8.5)—Recently ASEA (Sweden) have developed an electro-hydraulic governor in which the pendulum is replaced by an electric circuit which is sensitive to frequency. The electrical part is built by ASEA while the electro-mechanical part, the actuator cabinet, is at present built by NOHAB, KMW and Finshyttan, all of Sweden.

factors influencing the regulation are transformed into electric impulses, mainly alternating voltages, which are analysed and mixed in an electron tube governor and so combined that its outgoing impulses give positive indications on how the gate mechanism shall be altered with regard to direction and speed, so that the desired regulation will be ideal. In the so called electro-hydraulic operating device the impulses of the electron tube governor are transmitted by an electro-magnetic system to the hydraulic part of the turbine governor.

The voltage, which is fed into the electron tube governor is taken from a source, which has a frequency proportional to the speed of the turbine, for instance a pendulum generator.

8.10 Governing of Impulse Turbines—The quantity of water ejected from the turbine nozzle and striking the buckets may be regulated in one of the following ways :

i) Spear regulation,

ii) Deflector regulation,

and *iii*) Combined spear and deflector control.

The spear and deflector in all cases are operated by the servomotor mechanism.

i) *Spear Regulation*—To and fro movement of the spear inside the nozzle alters the cross-sectional area of stream, thus, making it possible to regulate the rate of flow according to the load. Spear regulation is satisfactory when a relatively large penstock feeds a small turbine and the fluctuation of load is small. With the sudden fall in load, the turbine nozzle has to be closed suddenly which may create water hammer in the penstock.

ii) *Deflector Regulation*—The deflector is generally a plate connected to the oil pressure governor by means of levers when it is required to deflect the jet, the plate can be brought in between the nozzle and buckets, thereby diverting the water away from the runner and directing into the tail race.

Deflector control is adopted when supply of water is constant but the load fluctuates. The spear position can be adjusted by hand. As the nozzle has always a constant opening, it involves considerable wastage of water and can be used only when supply of water is not scarce.

iii) *Combined Spear-Deflector Regulation*— As both of the above methods have some disadvantages, the modern turbines are provided with double regulation which is the combined spear and deflector control. Double regulations means regulation of speed and pressure. The speed is regulated by spear and the pressure is regulated by deflector arrangement.

The jet deflector controlled directly by governor, deflects jet from runner within a very short period so that no further energy is imparted to the latter. The deflector engages until the spear has been adjusted to a new position of equilibrium. The closing of the spear can thus be retarded to avoid undue pressure rise, whilst only the short time which the deflector requires to act need be considered in determining speed rise and flywheel momentum. The flywheel momentum should, however, be sufficient to deal with a sudden increase in load.

English Electric Co. (U.K) turbines are equipped with diffusor (Seewer patent) instead of deflector.

Working (Fig 8 6)—As the load on the turbine is decreased, the pendulum balls occupy the top position, thus moving the bell-crank lever towards bottom with which the deflector is brought in front of the nozzle, diverting whole or part of the water jet direct to the tail race.

With the movement of bell-crank lever, the roller on the cam is also raised. The distributing valve also functions simultaneously thus moving

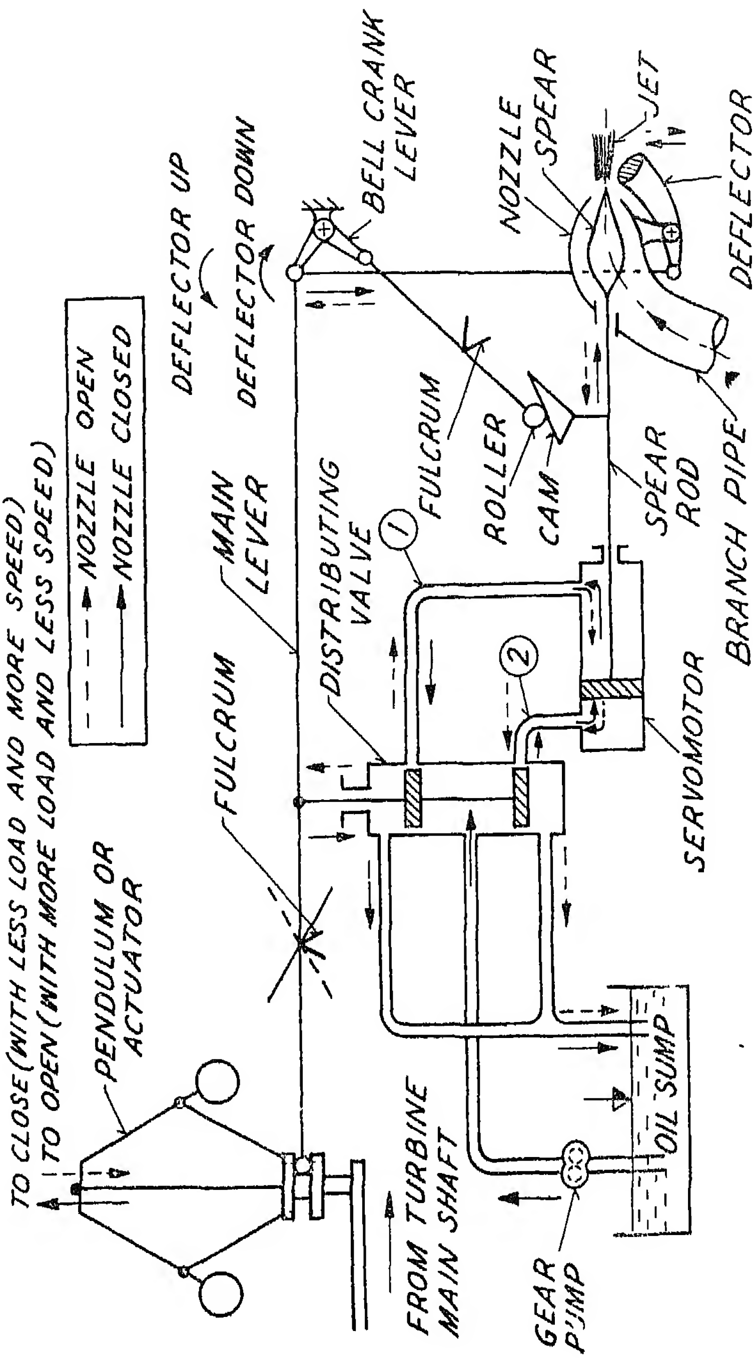


Fig 8.6 Governing of Impulse Turbines by Double Regulation System in Neutral Position—Simple Line Diagram

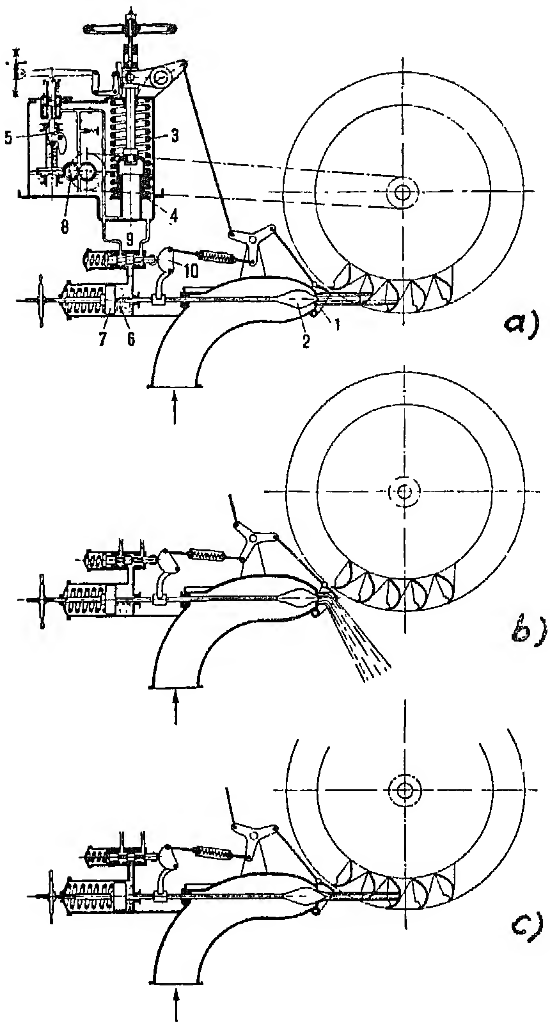


Fig 8.7 Double Regulation of a Pelton Turbine (Escher Wyss & Co Ltd., Zurich)

1. Deflector 2. Spear 3. Spring 4. Servomotor for Governor 5. Centrifugal Pendulum 6. Spear Servomotor 7. Piston 8. Gear Pump 9. Control Valve 10. Cam.

the servomotor piston rod towards right hand side (as explained in Art 8.7). This reduces the nozzle outlet by the movement of spear which occupics the position so that the constant turbine speed is maintained. With the movement of spear rod the cam also moves towards right, with which the roller slips and finally with the movement of bell-crank lever, the deflector comes to its original position. The whole of the above operation taken place in about 50 to 80 seconds.

Fig 8.7 shows the action of Escher Wyss governor to accomplish double regulation. The governor moves the deflector 1 whilst the spear 2 is controlled by the regulating rods of the deflector. The governor is subject to a closing force exerted by spring 3, for the purpose of safety, so that in event of any failure in the supply of oil under pressure the deflector 1 is still capable of deflecting the jet away from the runner wheel. The Governor servomotor 4 is controlled by the centrifugal pendulum 5 (See Fig 8.3). The spear servomotor 6 also has its safety closing spring, whilst for the opening movement oil under pressure from pump 8 is passed through valve 9 in the piston 7. The profile of the cam 10 is such that each position of the deflector corresponds to a given spear opening. When extra load is put on the turbine, the deflector 1 is raised and simultaneously the nozzle 2 is opened. When the load is thrown off, the deflector diverts the jet from the runner in less than a second.

8.11 Governing of Reaction Turbines—The guide blades of a reaction turbine (Fig 8.8) are pivoted and connected by levers and links to the regulating ring. To the regulating ring are attached two long regulating rods connected to a regulating lever. The regulating

lever is keyed to a regulating shaft which is turned by a servomotor piston of the oil pressure governor.

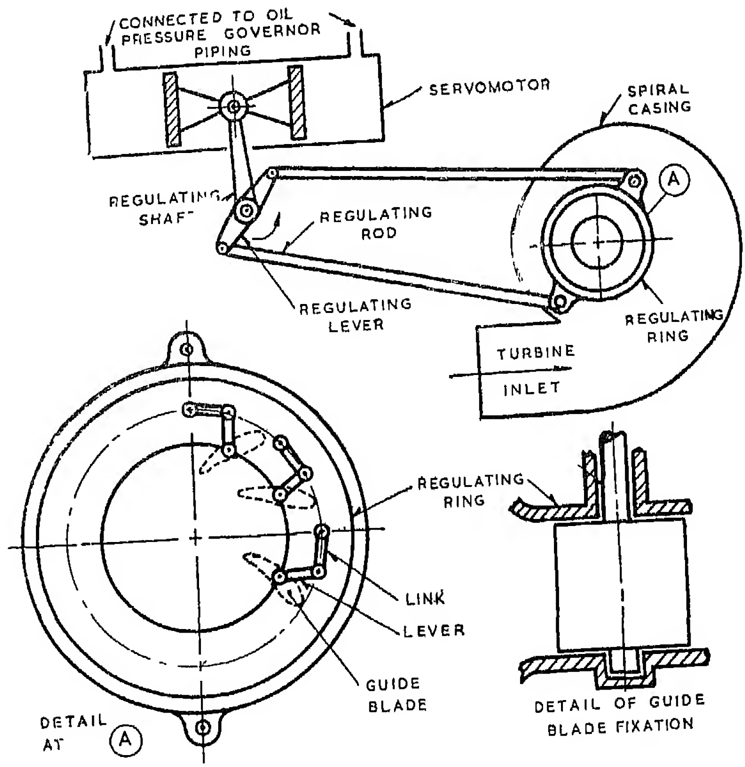


Fig 8.8 Governing of Reaction Turbines—Simple Line Diagram

The penstock feeding the turbine inlet has a relief valve better known as '*Pressure Regulator*'. When the guide vanes have to be suddenly closed, the relief valve opens and diverts the water direct to the tail race. Its function is, therefore, similar to that of the deflector in Pelton turbines. Thus the double regulation, which is the simultaneous operation of two elements is accomplished by moving the guide vanes and relief valve in Francis turbines, by the governor.

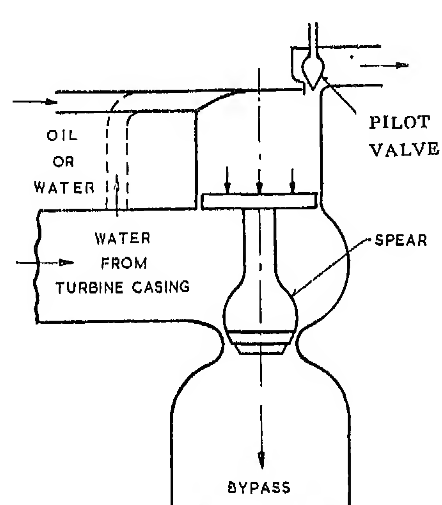


Fig 8.9 Relief Valve—Simple Line Diagram

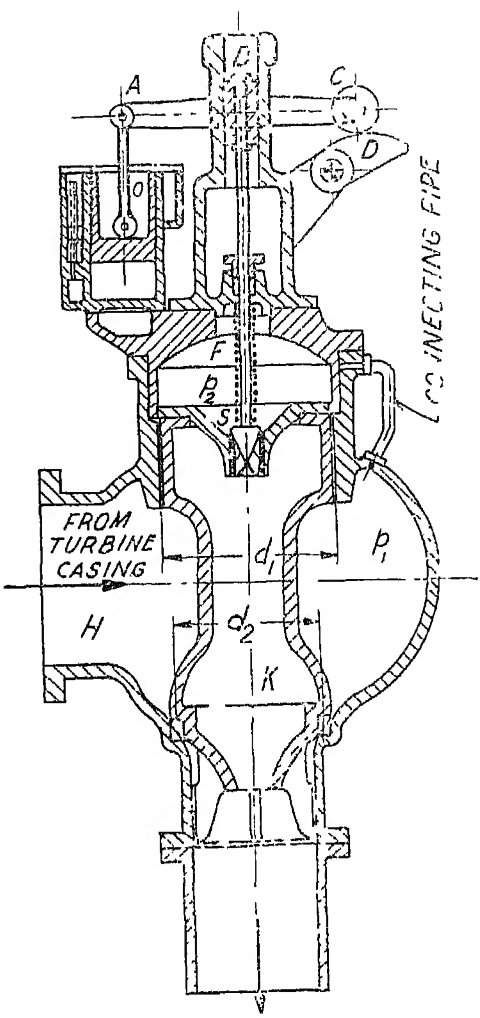
8.12 Relief Valve or Pressure Regulator—It is used as a part of the double regulating gear for speed control in Francis turbines. The object of the pressure

regulator is to bypass a portion of turbine discharge when load suddenly drops and in consequence the admission of water to the turbine is suddenly reduced in such a manner that no great increase of pressure can take place in the pipeline and that the waste of water is a minimum.

Fig 8.9 shows a simple line diagram of relief valve. Having the same principle as that of a Pelton nozzle, a relief valve consists of a spear which opens the by-pass by a servomotor piston, by oil or water pressure. The release of pressure on the upper side of piston, by opening the pilot valve, causes the spear to move upwards thus closing the by-pass.

Fig 8 10 shows a pressure regulator manufactured by Bell & Co Ltd, Kriens (Switzerland). The essential parts are the casing H, the piston valve K, the regulating valve S the dash pot O, the pawl D and the lever ABC.

When the load on the turbine suddenly drops, the automatic governor immediately closes the turbine gates. The pawl D quickly raises the end C of the lever. The other end A cannot follow quickly because of the brake action of the dash pot; and it may be assumed that during this period the lever turns around A lifting the valve S with which it is connected at B. Some water escapes through valves S and K, the pressure p_2 falls below p_1 because of a filter in the small connecting pipe. This difference of pressure raises the piston valve K and water is discharged through the pressure regulator, without passing through the runner.



WATER OUTLET

Fig 8.10 Relief Valve of Pressure Regulator (Theoder Bell & Co Ltd, Kriens, Lucerne, Switzerland)
H-Casing, K-Piston Valve, S-Regulating Valve O-Dashpot, D-Pawl, ABC—Lever, F-Spring

Force exerted by the spring F on S pulls the lever down at B, and while the end C of the lever rests on the pawl, piston of the dash pot slowly descends. Valves S and K slowly follow this motion and pressure regulator closes gradually.

If only a small part of the load is taken off, the regulating mechanism of turbine moves slowly, the piston of the dash pot can follow this motion and as the lever turns about B, the pressure regulator does not act.

If the load rises suddenly the dash pot exerts no brake action because a non-return valve permits the piston to move freely in one direction. Again B is at rest and pressure regulator does not operate.

Fig 8.11 shows a Francis turbine equipped with a relief valve, manufactured by Escher Wyss & Co Ltd, Zurich.

8.13 Governing of Kaplan Turbines by Double Regulation—Double regulation system for Kaplan turbines comprises the movement of guide vanes as well as runner vanes. It is employed to ensure the most satisfactory relationship between the relative positions of guide wheel and runner blades. The single weight pendulum 1 (See Fig 8.12) operates through valve 2, the guide wheel servomotor 3 and from the regulating rods of the guide mechanism, the runner blade 6 are controlled through valve 4 and servomotor 5. Each position of the runner blades is brought into an exact relationship with the guide wheel by means of cam 7, arranged before the runner distributing valve 4. When the load increases the guide wheel and the runner come into play simultaneously whereas on load being thrown off, the guide wheel goes ahead and is slowly followed by the runner. The larger sized turbines are equipped with emergency pressure oil pump 8. As long as the service oil pressure supply 9 is in order, the emergency pump 8 supplies oil without pressure, however, if any breakdown occurs, the valve 10 is automatically switched over and the emergency pump 8 delivers oil under high pressure immediately to the closing side of runner servomotor. The servomotor piston 11 is displaced upwards and turns the runner blades 6 into the closing position quite independently from the prevailing position either of the guide wheel or of runner control gear. This valve 10 is, however, not only controlled by the pressure oil supply, but also by the speed of the turbine. If this speed exceeds a given limit, the safety governor 12 closes a circuit and with the aid of magnet 13 switches the change-over valve 10 to its closing position.

8.14 Safety Devices for Penstocks—As the load on the generating units varies, the openings of the turbine gates are to be changed, depending upon the load, adjusting the rate of flow of water entering the runner. Whenever the turbine gates are suddenly closed, the water flowing in the long penstock which is on its way to the turbine has to be retarded. This leads to a phenomenon known as *water hammer* which causes excessive pressure in the pipe line with which the pipe may burst. It is necessary, therefore, to provide the plant with a safety device. Such a device may be *relief valve* or *pressure regulator* in case of Francis turbine (See Art 8.11 and 8.12). However to relieve the penstock of its excessive pressure the following devices are also employed—

a) **Surge tank** in case of high and medium head power plants, where the penstock is very long.

b) **Forebay** in case of medium and low head power plants, where the length of penstock is short.

Both of them will be described in the following articles.

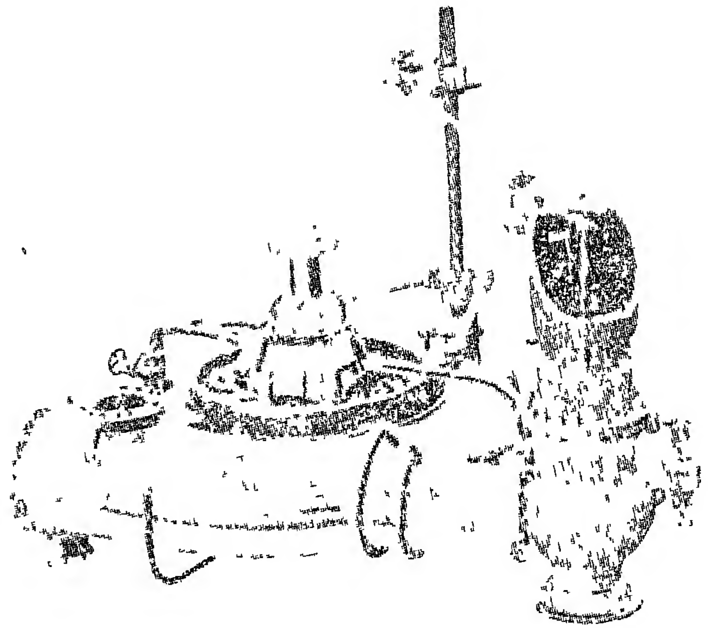


Fig 8.11 Relief Valve Connected to a Francis Turbine (Manufactured by Escher Wyss & Co Ltd, Zurich)

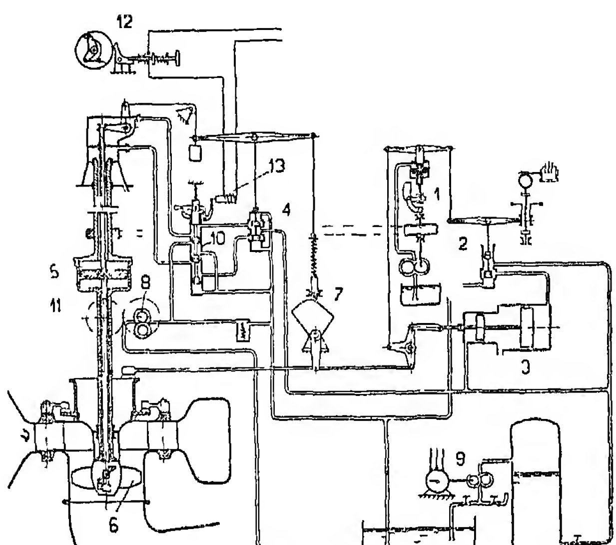


Fig 8.12 Double Regulation of a Kaplan Turbine (Escher Wyss & Co Ltd Zurich)

1 Single Weight Pendulum, 2 Valve, 3 Guide Wheel Servomotor, 4 Valve, 5 Runner Blades Servomotor, 6 Runner Blades, 7 Cam, 8 Emergency Pressure Oil Pump, 9 Service Pressure Oil Supply, 10 Valve 11 Servomotor Piston, 12. Safety Governor, 13 Magnet.

8.15 Surge Tank (See Fig 8.13)—A surge tank is a storage reservoir fitted at some opening made on a long penstock to receive the rejected flow when the penstock is suddenly closed by a valve fitted at its steep end. Surge tank, therefore, relieves the pipe line of excessive pressure produced due to closing of penstock, thus eliminating positive water hammer effect by admitting in it a large mass of water which otherwise would have flown out of the pipe line. It is necessary for medium and high head water power plants, especially when the water has to travel a long way from the intake to the power house. It is also used in a large pumping plant to control the pressure variations resulting from rapid changes in the flow.

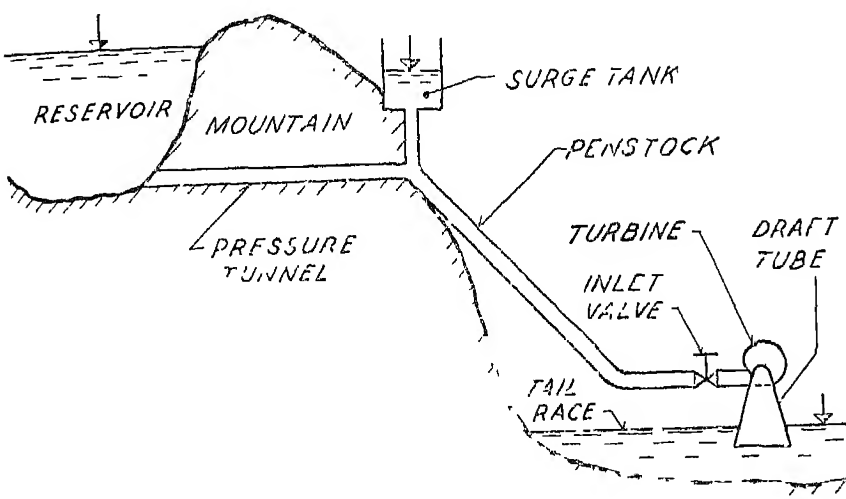


Fig 8.13 Surge Tank Installation

Surge tank also serves the purpose of supplying initial water for an increasing load on the turbines while the water in the pipe line is being accelerated.

In case of water power plant, when there is sudden reduction of load on the turbine, it becomes necessary for the governor to close the turbine gates for adjusting the flow of water in order to keep the speed of the turbine constant. However, the water is already on its way to the turbine. When the turbines gates are closed, the moving water has to go back. A surge tank would then act as a receptacle to store the rejected water and thus avoid water hammer. In other words as the demand of water by the turbine is less, the pipe line velocity must be decreased according to the requirement. A surge tank would then build up a retarding head which, in turn, reduces the pipe line velocity until the flow therein no longer exceeds that required by the turbine. On the other hand when there is an immediate demand on the turbine for more power, the governor re-opens the gates in proportion to increased load, thus making it necessary to supply more water. For a long pipe it takes a considerable time before the entire mass of water can be accelerated. Surge tank which is generally located near the turbine, will meet the sudden increased demand for water till such time the velocity in the upper portion of the line assumes a new value.

For a large pumping plant with a long delivery pipe, a surge tank can also be employed to control the pressure variations on the delivery side, which result due to sudden shut down or starting up of pump. When the pump is started, most of the initial flow from the pump enters the surge tank thus reducing the water hammer effects in the delivery pipe. On the other hand when the pump is shut down suddenly, the surge tank provides extra space to accommodate water which would come back, thus relieving water hammer pressure.

Functions of Surge Tank can be summed up as follows—

a) Control of pressure variations resulting from rapid changes of flow in penstock, relieving the line of excessive pressures, thus eliminating water hammer effects.

b) Regulation of flow in power and pumping plants by providing necessary accelerating or retarding head. The more effectively the accelerating or retarding head is applied the shorter will be the duration of surge, less amount of water will then have to be stored or given up by the tank, thereby smaller will be size of the tank required.

c) Regulation of turbine speed with the help of (*b*).

Location of Surge Tank—Theoretically a surge tank should be located as close to a power or pumping plant as possible. The ideal place in case of power plant is at the turbine inlet, but it is seldom possible in case of medium and high head plants because it will have to be made very high. In order to reduce its height, it is generally located at the junction of pressure tunnel and penstock (See Fig 8.13) or on the side of mountain.

Surge Tank Types—There are three main types: Simple, restricted orifice and differential.

a) **Simple Surge Tank** is a reservoir directly connected to a pipe line (penstock) as shown in Fig 8.14. In this type of surge tank the accelerating and retarding heads induced by a change of water surface accumulate slowly, therefore its action is sluggish. It is liable to set up considerable oscillations unless it is of such a large size as to render its cost prohibitive. It is seldom used in modern practice.

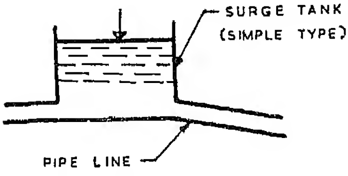


Fig 8.14 Simple Surge Tank

b) **Restricted Orifice Type Surge Tank**—It consists of a restricted orifice (See Fig 8.15) installed between the pipe line and the simple tank. With the help of orifice the water storage function of surge tank can be separated from accelerating and retarding functions. When the turbine gates are closed suddenly, a retarding head develops by the orifice more quickly than with a simple surge tank. Similarly the orifice will create a rapid accelerating head when additional flow has to be supplied by the surge tank. With the development of sudden

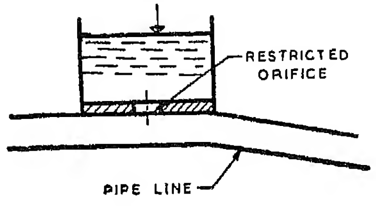


Fig 8.15 Restricted Orifice Type Surge Tank

retarding or accelerating heads, the head on the turbine is liable to fluctuate thus making the governor mechanism more cumbersome and costly.

c) **Differential Surge Tank**—It is a combination of simple and restricted orifice types, with an addition of an internal riser (See Fig 8.16).

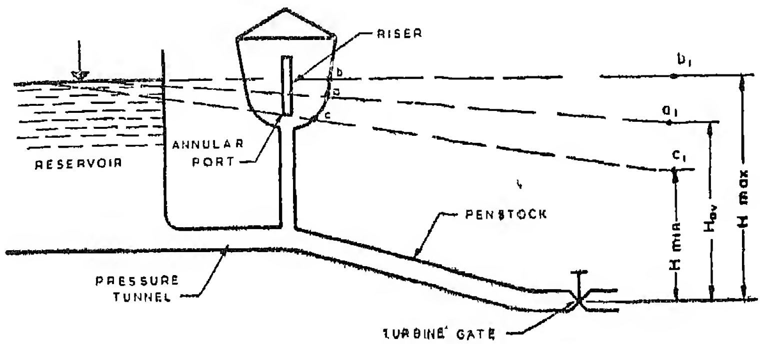


Fig 8.16 Differential Surge Tank

The riser is of smaller diameter than the connection to the pipe line. At the base of the riser there is an annular port, communicating with the tank, the area of which is proportional to suit the conditions under which the tank is to operate. The water enters the tank through the annular port, therefore the function of the tank depends upon the area of this port.

When there is a sudden drop in load on the turbine, the water from the pipe line rises immediately in the riser and spills over from its top into the tank. This establishes an immediate retarding head on the pipe line, as well as a differential head on the port. The differential head is responsible for forcing the water by the turbine, through the port into the tank. Due to restriction of flow through the riser there will be frictional loss which dissipates a part of increased pressure.

When there is a sudden demand of water by the turbine, the water level in the riser falls rapidly, thus developing an immediate considerable accelerating head on the pipe line. The additional water required is supplied by the tank slowly through the ports at the base of the riser.

Referring to Fig 8.16—

For *steady* turbine load, the velocity variations in the penstock do not exist, then the pressure gradient is normal *i.e.* Oaa_1 for *increasing* turbine load, the water is being drawn and therefore the gradient takes the shape of Occ_1.

For *falling* turbine load, extra water is pushed to the tank and then the gradient is shown by Obb_1.

Largest Differential Tank of the World till today is at Wallenpanpack (Pennsylvania, USA), made of steel, 55 ft diameter, 135 ft high, having a storage capacity of 2·4 million gallons.

Differences Between Simple and Differential Surge Tanks—

1) The capacity of the differential surge tank is less than the simple surge tank for the same stabilising effect, because a considerable retarding head is available in the former, while in case of latter the head builds up gradually as the tank fills.

2) In the case of simpe surge tank the heads which builds up is the accelerating head, but in case of differential tank the head in the surrounding tank is independent of the accelerating head as well as of the head acting on the turbines. This is due to the restricted flow of water through the riser.

***8.16* Forebay**—Like *surge tank*, a *forebay is a storage reservoir* at the beginning of penstock. When a hydro-power plant is located at the end of a canal (See Fig 5.4), a forebay is made by enlarging the canal at the intake to the penstocks. The functions of the forebay will thus be to store the rejected water when the load on the turbine is reduced and to supply the initial water for an increasing load, while the water in the penstock is being accelerated.

UNSOLVED PROBLEMS

8.1 What is governing and how is it accomplished in different types of water turbines ?

8.2 State the difference between the governing of different prime movers.

8.3 State the different types of water turbine governors.

8.4 What is the function of a governor in a water power plant ?
(*Jadavpur University—1955*)

8.5 What should be the qualities of a governor ?

8.6 Describe the following qualities of a governor—

a) Sensitiveness, *e*) Hunting or Racing,
b) Rapidity of Action, *f*) Capacity,
c) Stability, *g*) Speed Regulation,
d) Isochronism, *h*) Governor's Time.

8.7 State the principal elements of a water turbine governor. State each element together with its components.

8.8 What is hydraulic return motion gear in a water turbine governor ?

8.9 What is the function of the following in water turbine governor— Relay valve, Dashpot, Pilot valve ?

8.10 What is an oil pressure governor ? Draw a line sketch and describe its principal components. (*AMIE—May 1955*)

8.11 Why is the oil pressure governor employed to control the speed of water turbine ? Show by help of a line sketch the different parts of the oil pressure governor and describe them briefly.

8.12 Sketch and describe a modern method of regulation to maintain a constant speed for either

a) a Pelton wheel or *b*) a Francis turbine.
(*AMI Mech E—Oct 1954*)

8.13 Show with the help of a line sketch how the speed of a reaction water turbine is governed by servomotor. (*AMIE—Nov 1954*)

8.14 What are the different types of oil pressure governors ?

8.15 What are different types of actuators ? Describe them briefly.

8.16 Describe how does the electro-hydraulic or electronic actuator work.

8.17 What is the necessity of double control regulation and how it is effected in different types of water turbines ?

8.18 What is the difference between a deflector and a pressure regulator in water turbine ?

8.19 What is the function of a pressure regulator ? (*AMIE—May 1955*)

8.20 Describe briefly how the governing of a Kaplan turbine is carried out.

8.21 What is surge tank and a Forebay and what are their function ?

8.22 Describe differential type surge tank. How will you differentiate it with that of simple type ?

CHAPTER 9

MODELS & SELECTION OF TURBINES

A. Turbine Models

9.1 Turbine Models and their Testing 9.2 Conditions of Geometrical Similarity 9.3 Relation between the Characteristic Data of a Turbine and that of its Model—(Speed, Rate of flow Available Power, Brake Horse Power and Efficiency).

B. Selection of Turbines

9.4 Preliminary Data (Selection of Site and Field Information) 9.5 Selection of Type of Hydraulic Turbine 9.6 Specific Speed 9.7 Rotational Speed 9.8 Efficiency 9.9 Part-Load Operation 9.10 Overall Cost 9.11 Cavitation 9.12 Disposition of Turbine Shaft 9.13 Number of Units 9.14 Head (Very High Head, High Head, Medium Head and Low Head)

A. Turbine Models

9.1 Turbine Models and Their Testing—Water turbines are usually large-sized units. They are manufactured for specific conditions, therefore it is not possible to produce them on mass scale. If the turbines are tested after the manufacture is complete, then

a) the extensive tests on prototype machines are very costly and take a long time to conduct them,

b) the test results on prototype machines may not tally with the design data, supplied to the manufacturers. To change the dimensions and size of the turbine at this stage will be very costly.

c) There are hydraulic limitations to the test performed on the prototype—

i.e., *i*) it is not possible to vary the head

ii) the speed of the unit is constant

iii) the load available may not be as steady as is desired for accurate observations.

Hence due to these reasons it is essential to make a turbine model, which is geometrically similar for carrying out the tests in order to predict the behaviour and working conditions of the full sized turbine. The actual turbine and the model are identical in shape and all parts *viz* spiral casing, guide mechanism, runner and draft tube of the one are geometrically similar to the corresponding parts of the other.

Model should be of a suitable size so that it can be conveniently tested in the laboratory. Too small a model may be difficult to manufacture accurately and also it may not give accurate results. It is

therefore desirable to make a turbine model having an output of not less than 5 BHP and not more than 50 BHP.

Hence model tests as a substitute for tests in the field are justified on the ground of accuracy, comprehensiveness and convenience.

9 2 **Conditions of Geometrical Similarity**--Geometrically similar turbines must satisfy the following conditions—*

i) Theoretical specific speeds must be equal.

Thus $$N_{s_m}=N_{s_a} \qquad ...(9.1)$$

i) The specific speed depends upon brake horsepower (BHP) which is obtained by multiplying the available power by the turbine overall efficiency (η_t). Now η_t of model and that of prototype are not exactly the same, as proved later on. Therefore strictly speaking the specific speeds of model and prototype should not be equal. Theoretically it is assumed that the overall efficiencies of model and prototype are equal, then it can be said that theoretical specific speeds must be equal.

Due to the reason explained above, it is better to use non-dimensional factor K_s which will be equal for the model and the actual-sized turbine as it does not contain the turbine overall efficiency (See Eqn 5 2).

$$K_{s_m}=K_{s_a} \qquad ...(9.1a)$$

ii) Velocity triangles for the model and the turbine should be similar Thus, velocity constants–

$$(K_{u_1})_m=(K_{u_1})_a \qquad ..(9\ 2a)$$

$$(K_{v_1})_m=(K_{v_1})_a \qquad ...(9\ 2b)$$

$$(K_{v_{u_1}})_m=(K_{v_{u_1}})_a \qquad ...(9.2c)$$

$$(K_{v_{m_1}})_m=(K_{v_{m_1}})_a \text{ etc} \qquad ...(9.2d)$$

iii) The specific flows or the rates of flow per unit head and unit inlet diameter of runners should be the same for similar turbines.

Thus, $$(Q_{11})_m=(Q_{11})_a \qquad ...(9\ 3)$$

Where $$Q_{11}=\frac{Q}{D_1^2 \cdot \sqrt{H}} \qquad ..[\text{See Art } 3.14(i)]$$

iv) Runner exit and extrance diameters should be proportionate.

Thus, $$\left(\frac{D_2}{D_1}\right)_m=\left(\frac{D_2}{D_1}\right)_a \qquad ...(9.4)$$

Also, since $$\frac{D_2}{D_1}=\frac{K_{u_2}}{K_{u_1}},$$

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*(1) Symbols have their usual meanings. Suffixes *m* and *a* are used for the model and actual turbines respectively.
(2) See Chapter 3.

$$\left(\frac{K_{u_2}}{K_{u_1}}\right)_m=\left(\frac{K_{u_2}}{K_{u_1}}\right)_a \quad \ldots(9.5)$$

v) The efficiencies η_H and η_Q for the two turbines should theoretically be equal. In practice, however, it can never be realized. Since the frictional factor f of the turbine is different from that of its model; Reynolds' Number R_e is also different. Usually $f_m > f_a$,

Now, Relative roughness

$$\delta_m > \delta_a \quad \ldots(\text{Art } 3.5)$$

and $R_{e_m} < R_{e_a}$

$$\therefore \quad \eta_{H_m} \leq \eta_{H_a} \quad \ldots(9.6)$$

Since the leakage loss in the model is relatively more than in the actual turbine,

$$\eta_{Q_m} \leq \eta_{Q_a} \quad \ldots(9.7)$$

Similarly the mechanical losses of the model are relatively more than those of the full size unit.

$$\therefore \quad \eta_{mech_m} \leq \eta_{mech_a} \quad \ldots(9.8)$$

Hence $\eta_{t_m} \leq \eta_{t_a}$ ...(9.9)

9 3 Relation Between the Characteristics Data of a Turbine and those of its Model—Let the scale of the model be n. Then the full size turbine is n times bigger than the model.

And $n=\dfrac{D_{1_a}}{D_{1_m}}$. Also sometimes $n=\dfrac{H_a}{H_m}$

The following quantities are measured during the model tests :

H_m, Q_m, N_m, P_{a_m}, P_{t_m} and η_{t_m}.

From these, the corresponding values for actual turbine can be obtained as hereunder :

i) **Speed of Turbine :**

$$(K_{u_1})_a=(K_{u_1})_m$$

$$\therefore \quad \frac{\pi \cdot D_{1_a} \cdot N_a}{60 \cdot \sqrt{2g} \cdot \sqrt{H_a}}=\frac{\pi \cdot D_{1_m} \cdot N_m}{60 \cdot \sqrt{2g} \cdot \sqrt{H_m}}$$

or
$$N_a=\frac{D_{1_m}}{D_{1_a}} \cdot \sqrt{\frac{H_a}{H_m}} \cdot N_m \quad \ldots(9.10)$$

$$=\frac{1}{n}\sqrt{n} \cdot N_m$$

$$\therefore \quad N_a=\frac{N_m}{\sqrt{n}} \quad \ldots(9.10a)$$

ii) **Rate of Flow :**

For similar turbines

$$(Q_{11})a=(Q_{11})m$$

or $$\frac{Q_a}{D_{1_a}^2 \cdot \sqrt{H_a}}=\frac{Q_m}{D_{1_m}^2 \cdot \sqrt{H_m}}$$

or $$Q_a=\frac{D_{1_a}^2}{D_{1_m}^2} \cdot \sqrt{\frac{H_a}{H_m}} \quad Q_m \qquad \text{.. (9 11)}$$

$$=n^2 \cdot \sqrt{n} \cdot Q_m$$

$$\therefore \quad Q_a=n^{\frac{5}{2}} \cdot Q_m \qquad \text{...(9.11}a\text{)}$$

iii) **Available Power :**

$$(P_a)_a=w_a \cdot Q_a \cdot H_a/550$$

and $$(P_a)_m=w_m \cdot Q_m \cdot H_m/550$$

$$\therefore \quad \frac{(P_a)_a}{(P_a)_m}=\frac{w_a}{w_m} \cdot \frac{Q_a}{Q_m} \cdot \frac{H_a}{H_m}$$

If same fluid is used, $w_a=w_m$

and, therefore $$\frac{(P_a)_a}{(P_a)_m}=\frac{Q_a}{Q_m} \cdot \frac{H_a}{H_m}=\frac{D_{1_a}^2}{D_{1_m}^2} \cdot \left(\frac{H_a}{H_m}\right) \qquad \text{...(9.12)}$$

$$\therefore \quad (P_a)_a=n^{\frac{7}{2}} \cdot (P_a)_m \qquad \text{...(9.12}a\text{)}$$

iv) **Brake Horse Power :**

In general, $P_t=P_a \cdot \eta_t=P_a \cdot \eta_H \cdot \eta_Q \cdot \eta_{mech}$

$$\therefore \quad \frac{(P_t)_a}{(P_t)_m}=\frac{P_{a_a}}{P_{a_m}} \cdot \left(\frac{\eta_{H_a}}{\eta_{H_m}}\right) \cdot \left(\frac{\eta_{Q_a}}{\eta_{Q_m}}\right) \cdot \left(\frac{\eta_{mech_a}}{\eta_{mech_m}}\right)$$

$$=\frac{D_{1_a}^2}{D_{1_m}^2} \cdot \left(\frac{H_a}{H_m}\right)^{\frac{3}{2}} \cdot \left(\frac{\eta_{H_a}}{\eta_{H_m}}\right) \cdot \left(\frac{\eta_{Q_a}}{\eta_{Q_m}}\right) \cdot \left(\frac{\eta_{mech_a}}{\eta_{mech_m}}\right) \qquad \text{(9 13)}$$

$$\therefore \quad P_{t_a}=n^{\frac{7}{2}}\left(\frac{\eta_{H_a}}{\eta_{H_m}}\right) \cdot \left(\frac{\eta_{Q_a}}{\eta_{Q_m}}\right) \cdot \left(\frac{\eta_{mech_a}}{\eta_{mech_m}}\right) \cdot P_{t_m} \qquad \text{.. (9.13}a\text{)}$$

v) **Efficiency :** Efficiency of the turbine is usually calculated from the model efficiency by means of empirical relations. The following relations are in vogue to-day :

a) Dr. J. Ackeret (Switzerland), 1930, formula—

$$\eta_{H_a}=1-(-\eta_{H_m})\left\{0{\cdot}5-0{\cdot}5\left(\frac{R_{e_m}}{R_{e_a}}\right)^{\frac{1}{5}}\right\} \qquad \text{.. (9.14)}$$

b) Prof Moody (USA), 1942, modified formula—

$$\eta_{H_a}=1-\left(1-\eta_{H_m}\right)\left(\frac{D_{1_m}}{D_{1_a}}\right)^{\frac{1}{5}} \qquad ...(9.15)$$

c) Dr R Gregorig (Switzerland), 1933, formula—

$$\eta_{H_a}=1-\left(1-\eta_{H_m}\right)\left(\frac{R_{e_m}}{R_{e_a}}\right)^{\frac{1}{4}}-K_{v_{m_3}}\left\{1-\left(\frac{R_{e_m}}{R_{e_a}}\right)^{\frac{1}{4}}\right\} \qquad ...(9.16)$$

d) Dr S. P. Hutton* (England), 1954, formula—

$$\eta_{H_a}=1-\left(1-\eta_{H_m}\right)\left\{0{\cdot}3-0{\cdot}7\left(\frac{R_{e_m}}{R_{e_a}}\right)^{\frac{1}{5}}\right\} \qquad ...(9.17)$$

Ackeret and Hutton formulae would be more useful for Kaplan turbines. In the above formulae—

$$R_e=\frac{\sqrt{2gH}\,.\,D_1}{\nu} \quad \text{where, } \nu=\text{Kinematic Viscosity}$$

and $K_{v_{m_3}}=\dfrac{v_{m_3}}{\sqrt{2gH}}$ where v_{m_3} = Velocity of flow of water at the outlet of draft tube.

Problem 9.1 A turbine model having the following specifications was made to perdict the behaviour of the actual turbine :

$P_t=15{\cdot}8$ HP $\qquad H=12$ ft $\qquad N=474$ rpm

Model scale $\quad n=10$

Determine the speed of the actual turbine runner and its power developed if the overall efficiencies of the turbine and its model are equal. Find the type of turbine of which the model was made.

Solution

$$\text{Model scale } n=\frac{D_{1_a}}{D_{1_m}}=\frac{H_a}{H_m}=10$$

where '*a*' and '*m*' denote actual sized turbine and model turbine respectively.

$\therefore \quad H_a=10\times H_m=10\times 12=\mathbf{120\ ft}$ *Answer*

a) As the overall efficiencies of the turbine and its model are same, their specific powers must be equal,

$$(P_{t_{11}})_a=(P_{t_{11}})_m$$

or $$\frac{P_{t_a}}{D_{1_a}^2\,.\,H_a^{\frac{3}{2}}}=\frac{P_{t_m}}{D_{1_m}^2\,.\,H_m^{\frac{3}{2}}}$$

$$\therefore \quad P_{t_a}=P_{t_m}\left(\frac{D_{1_a}}{D_{1_m}}\right)^2\left(\frac{H_a}{H_m}\right)^{\frac{3}{2}}$$

$$=P_{t_m} \cdot n^2 \cdot n^{\frac{3}{2}} = P_{t_m} \cdot n^{\frac{7}{2}}$$

$$=15{\cdot}8 \times 10^{\frac{7}{2}} = 15{\cdot}8 \times 1{,}000 \times 3{\cdot}16$$

$$=\mathbf{50{,}000\ HP} \quad \textit{Answer}$$

b) For similar turbines the speed ratios are equal.

$$\therefore \qquad (K_{u_1})_a = (K_{u_1})_m \qquad \text{...(See Eqn 9.2}a\text{)}$$

or
$$\frac{\pi \cdot D_{1_a} \cdot N_a}{60 \cdot \sqrt{2g} \cdot \sqrt{H_a}} = \frac{\pi \cdot D_{1_m} \cdot N_m}{60 \cdot \sqrt{2g} \cdot \sqrt{H_m}}$$

or
$$N_a = N_m \cdot \frac{D_{1_m}}{D_{1_a}} \times \sqrt{\frac{H_a}{H_m}} = N_m \cdot \frac{\sqrt{n}}{n}$$

...(See Eqn 9.10)

$$= \frac{N_m}{\sqrt{n}} = \frac{474}{\sqrt{10}} = \frac{474}{3{\cdot}16} = \mathbf{150\ rpm} \quad \textit{Answer}$$

c) Specific speed of any of the turbines :

$$N_{s_m} = N_{s_a} = \frac{150 \times \sqrt{50{,}000}}{120^{\frac{5}{4}}} = \frac{150 \times 223\ 5}{120 \times 3{\cdot}31} = \mathbf{84{\cdot}5}$$

∴ the turbine is of **Kaplan** type. *Answer*

Problem 9.2 The runner of a Pelton turbine installed at Tata Hydro-Electric Co Ltd, Khapoli, was built according to the following specifications :

$P_t = 17{,}100$ HP $\qquad H = 1{,}685$ ft

$Q = 104$ cusecs $\qquad N = 300$ rpm

Find the specifications of a one-ninth scale model of this turbine, assuming the efficiencies of the two turbines to be the same.

Solution

Model scale $\quad n = 9$

a) Head under which the model turbine would work :

$$\frac{H_a}{H_m} = \frac{1{,}685}{H_m} = 9$$

$$\therefore \qquad H_m = \frac{1{,}685}{9} = \mathbf{187{\cdot}2\ ft} \quad \textit{Answer}$$

b) Speed of the model turbine :

$$(K_{u_1})_a = (K_{u_1})_m \qquad \text{...(See Eqn 9.2}a\text{)}$$

$$\therefore \qquad N_m = \sqrt{n} \cdot N_a = \sqrt{9} \times 300$$

$$= \mathbf{900\ rpm} \quad \textit{Answer}$$

c) Rate of flow for the model turbine :

$$(Q_{11})_a = (Q_{11})_m \qquad \text{...(See Eqn 9.3)}$$

$$\therefore \quad Q_m = Q_a \cdot \frac{d_{1m}^2}{d_{1a}^2} \cdot \frac{H_m^{\frac{1}{2}}}{H_a^{\frac{1}{2}}} = Q_a \cdot \left(\frac{1}{n}\right)^2 \cdot \left(\frac{1}{n}\right)^{\frac{1}{2}}$$

$$= \frac{Q_a}{n^{\frac{5}{2}}} = \frac{104}{9^{\frac{5}{2}}} = \frac{104}{81 \times 3} = \frac{104}{243}$$

$= \mathbf{0\ 428\ cusecs}$ *Answer*

d) Power developed by the model turbine :

$$(P_{t_{11}})_a = (P_{t_{11}})_m \quad \text{or} \quad \frac{P_{t_a}}{d_{1a}^2 \cdot H_a^{\frac{3}{2}}} = \frac{P_{t_m}}{d_{1m}^2 \cdot H_m^{\frac{3}{2}}}$$

$$\therefore \quad P_{t_m} = P_{t_a}\left(\frac{d_{1m}}{d_{1a}}\right)^2 \cdot \left(\frac{H_m}{H_a}\right)^{\frac{3}{2}} = P_{t_a} \cdot \frac{1}{n^{\frac{7}{2}}}$$

$$= \frac{17{,}100}{9^{\frac{7}{2}}} = \frac{17{,}100}{81 \times 27} = \mathbf{7{\cdot}82\ HP} \quad \textit{Answer}$$

$\therefore$ **Specifications of the turbine model—**

$\mathbf{P_t = 7{\cdot}82}$ **HP,**	**H=187·2 ft**	*Answer*
Q=0·428 cusecs,	**N=900 rpm**	

Problem 9.3 A model turbine is provided with a runner 2 ft (or 0·61 *m*) in diameter. It is found to develop 64 HP (or 65 Metric HP) under a head of 100 ft (30·48 *m*) when running at a speed of 4,000 RPM. Calculate the specific speed and unit speed for this model. It is required to build a similar turbine to develop 200 HP (or 203 Metric HP) under a head of 120 ft (or 36·6 *m*). Calculate the required diameter. Prove any formula used. *(Punjab University—1953A)*

Solution

$D_m = 2$ ft (or 0·61 *m*) $\qquad H_m = 100$ ft (or 30·48 *m*)

$P_{t_m} = 64$ HP (or 65 Metric HP) $\qquad N_m = 4{,}000$ RPM

$P_{t_a} = 200$ HP (or 203 Metric HP) $\qquad H_a = 120$ ft (or 36·6 *m*)

$$\text{Specific speed of model } N_{s_m} = \frac{N_m \cdot \sqrt{P_{t_m}}}{H_m^{\frac{5}{4}}}$$

$$= \frac{4{,}000 \times \sqrt{64}}{100^{\frac{5}{4}}} = \mathbf{101{\cdot}3\ (FPS\ Units)} \quad \textit{Answer}$$

$$\left[\text{or } = \frac{4{,}000 \times \sqrt{65}}{30{\cdot}48^{\frac{5}{4}}} = \mathbf{450\ (Metric\ Units)} \quad \textit{Answer}\right]$$

$$\text{Unit speed of model } N_{1_m} = \frac{N_m}{\sqrt{H_m}} = \frac{4{,}000}{\sqrt{100}}$$

$= \mathbf{400\ (FPS\ Units)}$ *Answer*

$$\left[\text{or} \quad N_{1_m} = \frac{4{,}000}{\sqrt{30{\cdot}48}} = \mathbf{726\ (Metric\ Units)} \quad \textit{Answer}\right]$$

Now, the specific powers of the similar turbines are equal,

$$\therefore \quad \frac{P_{t_a}}{D_a^2 \cdot H_a^{\frac{3}{2}}} = \frac{P_{t_m}}{D_m^2 \cdot H_m^{\frac{3}{2}}}$$

or
$$D_a = \sqrt{D_m^2 \cdot \left(\frac{H_m}{H_a}\right)^{\frac{3}{2}} \cdot \left(\frac{P_{t_a}}{P_{t_m}}\right)}$$

$$= \sqrt{2^2 \times \left(\frac{100}{120}\right)^{\frac{3}{2}} \times \frac{200}{64}}$$

$$= \mathbf{3{\cdot}09\ ft \simeq 3\ ft - 1\ in.} \quad \textit{Answer}$$

$$\left[\text{or } D_a = \sqrt{0{\cdot}61^2 \times \left(\frac{30{\cdot}48}{36{\cdot}6}\right)^{\frac{3}{2}} \times \frac{203}{65}} = \mathbf{0{\cdot}94\ m} \text{ or } \mathbf{940\ mm} \quad \textit{Answer}\right]$$

See Art 3.13 and 3.14 for the proofs of the formulae used above.

Problem 9.4 A quarter scale turbine model is tested under a head of 36 ft. The full scale turbine is required to work under a head of 100 ft and to run at 428 RPM. At what speed must the model be run and if it develops 135 HP and uses 38 cu ft/sec of water at this speed, what power will be obtained from the full scale turbine, assuming that its efficiency is 3 per cent better than that of the model ?

State the type of runner used in this turbine and explain briefly why the efficiency of the full scale turbine should be better than the efficiency of the model.

(*London University—July 1949*)

Solution

Scale ratio $n = 4 = \frac{D_a}{D_m}$

$H_m = 36$ ft	$H_a = 100$ ft
$P_{t_m} = 135$ HP	$N_a = 428$ RPM
$Q_m = 38$ cfs	$\eta_a = \eta_m + 3\%$

a) Speed ratios of the model and the prototype are equal.

$$\therefore \quad (K_{u_1})_m = (K_{u_1})_a \qquad \text{...(See Eqn 9.2}a\text{)}$$

or
$$N_m = N_a \cdot \frac{D_a}{D_m} \cdot \sqrt{\frac{H_m}{H_a}} = 428 \times 4 \sqrt{\frac{36}{100}}$$

$$= \mathbf{1{,}027\ RPM} \quad \textit{Answer}$$

b) Efficiency of model :

$$P_{t_m} = \frac{w \cdot Q_m \cdot H_m}{550} \cdot \eta_{t_m}$$

or
$$\eta_{t_m} = \frac{550 \cdot P_{t_m}}{w \cdot Q_m \cdot H_m} = \frac{550 \times 135}{62{\cdot}4 \times 38 \times 36} = 0{\cdot}87$$

$$\therefore \quad \eta_{t_a} = 0{\cdot}87 + 0{\cdot}03 = 0{\cdot}9 \quad \text{or } 90\%$$

Now, specific speeds of the model and the prototype are equal,

$\therefore \quad N_{s_m} = N_{s_a}$ ...(See Eqn 9.1)

$$\text{and} \quad N_{s_m} = \frac{N_m \cdot \sqrt{P_{t_m}}}{H_m^{\frac{5}{4}}} = \frac{1{,}027 \times \sqrt{135}}{36^{\frac{5}{4}}} = \mathbf{136} = N_{s_a}$$

$$\therefore \quad P_t = \left(\frac{N_{s_a} \cdot H_a^{\frac{5}{4}}}{N_a}\right)^2 = \left(\frac{136 \times 100^{\frac{5}{4}}}{428}\right)^2 = 10{,}080 \text{ HP}$$

But $\eta_{t_a} > \eta_{t_m}$ ...(See Eqn 9.9)

$\therefore$ Required Power obtained from the full scale turbine

$$= 10{,}080 \times \frac{90}{87} = \mathbf{10{,}400\ HP} \qquad \textit{Answer}$$

c) As $N_s = 136$, type of runner used is **Kaplan.** *Answer*

d) For explanation to $\eta_{t_a} > \eta_{t_m}$, refer to Art 9.2 (*v*).

B. Selection of Hydraulic Turbines

9.4 Preliminary Data—Three important types of hydraulic turbines *viz*, Pelton, Francis and Kaplan, are in vogue today. Propeller turbines are also employed at some places. A hydraulic turbine is always selected and designed to match the specific conditions under which it has to operate in order to attain a high order of efficiency which is expected in present-day installations. The following preliminary data are required while selecting the different types of water power plant which will ultimately effect the right type of selection of hydraulic turbine.

a) *Selection of site* depends upon—

i) availability of head and discharge,

ii) places of power shortage,

iii) power-deficient periods, such as peak loads.

b) *Field Information* depends upon—

i) Estimation of power available from rainfall and stream flow records,

ii) Ultimate development of power—The power plant should not be constructed just to meet the immediate needs, and ultimate development of the place must be kept in mind,

iii) Proportion of available potential that can be economically developed,

iv) Supply of accurate field data.

Improper and unsuitable selection means—

i) High initial costs,

ii) Low efficiency which will reduce the revenue thus impairing the earning repayment-ability of plant,

iii) Unit may be such as has excessive operating cost and is difficult to control.

9.5 Selection of Type of Hydraulic Turbine—The following points are important for the selection of right type of hydraulic turbine which will be discussed separately.

a) Specific Speed
b) Rotational Speed
c) Efficiency
d) Part Load Operation
e) Overall Cost
f) Cavitation
g) Disposition of turbine shaft
h) Number of units
i) Head

9.6 Specific Speed—From the field the main data available are—

a) Head and b) Discharge.

It has been found from experience that there is a range of head and specific speed at which each type of turbine is most suitable. Certain curves shown in Fig 9.1, representing N_s vs H have been drawn from experimental data for all types of water turbines. The head H being known from the field data, an appropriate value of N_s can be determined from the above curves. However for a particular head different values of specific speed may be available. The selection of right type of specific speed is a matter of experience. It is a common practice to select a high specific speed runner which is always economical because the size of the turbo-generator as well as that of the power house will become smaller.

High specific speed is essential where head is low and output (BHP) is large, because otherwise the rotational speed (N) will be very low which means cost of turbo-generator and power house will be high. On the other hand there is practically no need of choosing a high value of specific speed for high head installations, because even with low specific speed, high rotational speed can be attained with medium-capacity plants.

High specific speed means greater value of cavitation factor, explained later.

9.7 Rotational Speed depends on specific speed. From the equation of specific speed (See Eqn 3.39) it is evident that the specific speed is directly proportional to the rotational speed of the turbine. Also the rotational speed of an electrical generator with which the turbine is to be directly coupled, depends on the frequency and number of pair of poles (See Eqn 6.9). The value of specific speed adopted should be such that it will give the synchronous speed of the generator.

If the rotational speed is high, then for the same power the following considerations are important—

a) *Civil Engineering Considerations*—The hydraulic turbine will have a smaller size, which is less costly. Also the size of the generator is small, because the number of pair of poles is less. Smaller units will occupy less space and the installation and excavation costs are less.

b) *Hydraulic Engineering Considerations*—Higher specific speed turbine is generally more liable to cavitation, thus involving deeper excavation and consequently more costly foundations.

c) *Mechanical Engineering Considerations*—Mechanical design is influenced by centrifugal forces which are set up at higher speed in the revolving parts, which will have to stand more severe stresses.

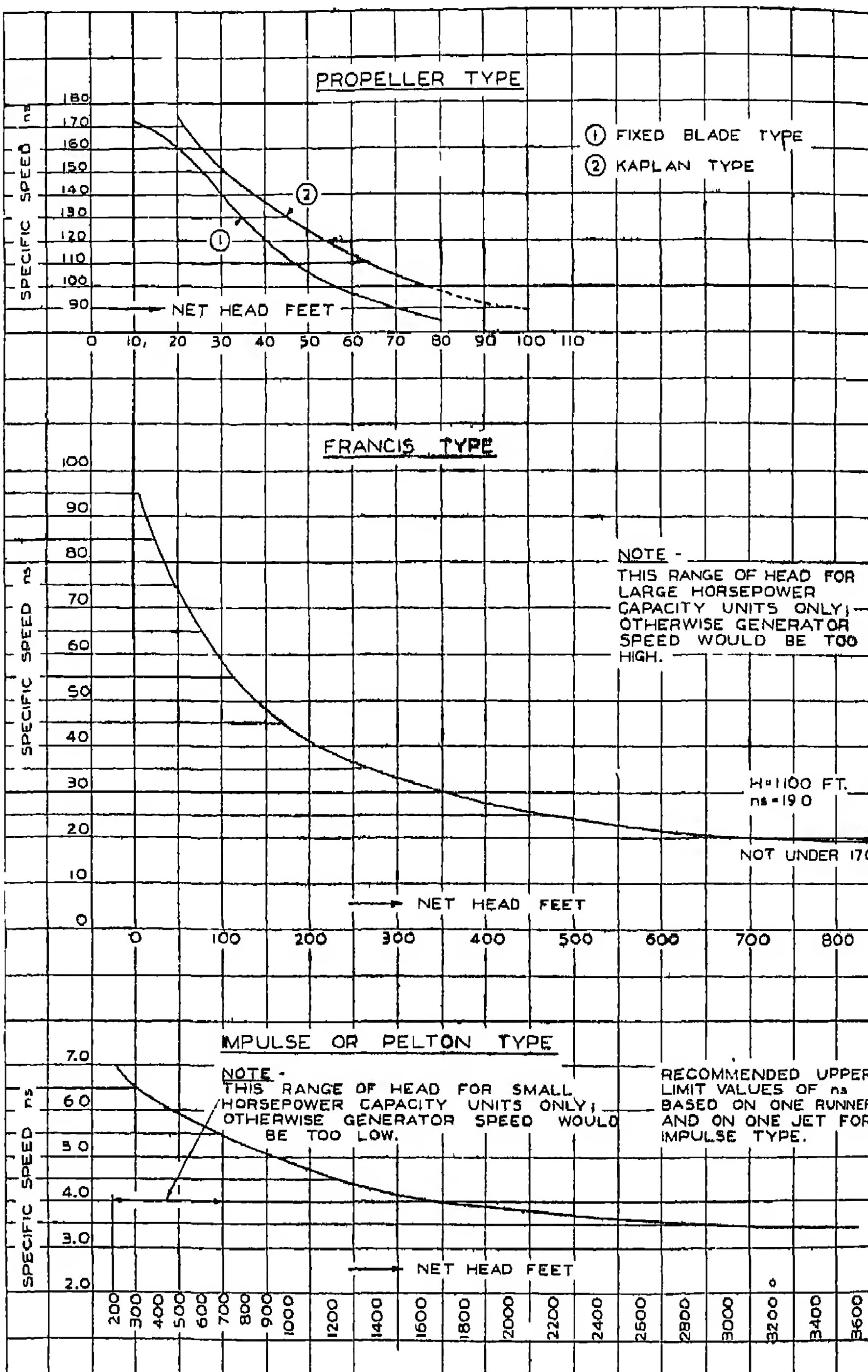

Fig 9.1 Specific Speed vs Head

d) *Metallurgical Considerations*—Suitable material may have to be manufactured by special metallurgical processes.

9.8 **Efficiency**—The turbine selected should be such that it gives the highest overall efficiency for various operating conditions. The turbine is normally designed to give maximum efficiency at the rated head, which is the estimated annual mean net head available.

9.9 **Part-Load Operation**—The turbine may have to cope with considerable load variations. As the load deviates from normal working load, the efficiency would also vary. In general the efficiency at part-loads and over-loads, is less than normal. For the sake of economy the turbine should always run with maximum possible efficiency to get more revenue.

With part-load operations, the maintenance cost is also increased.

9.10 **Overall Cost**—Cost is a major item. Cost may be subdivided broadly into initial cost and running cost. In the estimation of initial cost, both manufacture and installation should be kept in mind. The running cost includes the maintenance and overhead charges, the cost of operation, repair and replacement. The influence of speed, efficiency and part-load operations, described above, is a guiding factor in the selection of water turbines.

9.11 **Cavitation**—Cavitation effects the installation of water turbine of reaction type, over the tail race level. The critical value of cavitation factor must be obtained to see that the turbine works in safe zone. Such a value of cavitation factor also effects the design of turbine, especially of Kaplan type.

High specific speed increases the value of cavitation factor. Therefore the specific speed and the suction head must have proper relation to the net working head, if satisfactory performance and life of vital parts are to be assured. The cavitation factor determines whether the turbine is to be placed above or below tail race water level. It is always advisable to avoid the undue depth of excavation for the power house foundations, so that the cost may not be very high. If the turbine is installed below tail race level, the water has to be pumped out every time the inspection or repair is made. However for Kaplan or propeller turbine, in order to bring the working within safe zone, it becomes essential to install the runner below tail race level. It has been explained above that high specific speed is necessary to reduce the size of turbine, generator and power house, but on the other hand such a turbine has to be installed below tail race level. The experience has shown that the economy secured by increase in specific speed is so high, especially in case of low head plants, where the size of machinery is relatively large, that it becomes necessary and advantageous to adopt high specific speed turbines and to sacrifice the easy access to runner.

9.12 **Disposition of Turbine Shaft**—A vertical shaft turbine will require deeper foundations and a high building. On the other hand horizontal shaft turbine will need a greater floor area. Experience has shown that the vertical-shaft arrangement is better for large sized reaction turbines, therefore it is almost universally adopted. In case of large size impulse turbines, horizontal-shaft arrangement is mostly employed.

9.13 **Number of Units**—The following points must be borne in mind to determine whether a multi-unit plant is required—

a) **Cost**—If the units are large, the cost per kilowatt for hydro-electric power plant of a given capacity does not vary much with the number of units. Generally speaking the cost of power house and its foundations increases with the number of machines installed, but not in direct proportion. When more than one machine is installed they should be of the same capacity so that their parallel running is more stable and also some economy may be achieved by standardization of spare parts.

The running costs will be higher for a multi-unit plant.

b) **Load Supply**—Due to the variation of load, the turbine may be required to operate at low efficiency. Multi-unit plants can efficiently meet large variations of load by varying the number of units in service.

The multi-unit plant is useful for peak load conditions.

For a plant belonging to inter-connected system, the multi-unit plant is more economical.

9.14 Head—*a*) **Very High Head (about 1,100 ft or 350 m and above)** : For heads greater than 1,100 ft (or 350 m), Pelton turbine is generally employed and there is practically no choice except in very special cases.

b) **High Head (450 ft to 1,100 ft or 150 m to 350 m)** : In this range either Pelton or Francis turbine may be employed Fig 9.1 shows approximate values of specific speed (N_s) against head (H). The curve has been drawn for existing installations. For higher specific speeds Francis turbine is more compact and economical than the Pelton which for the same working conditions would have to be much bigger and rather cumbersome.

For heads ranging from 800 to 1,100 ft (or 250 to 350 m), discharge velocity of water is rather high and to restrict the loss of head, necessary draft tube installations become complicated. Pelton turbine is therefore preferable, as it is also stronger mechanically. Further the efficiency of Pelton turbine is less sensitive than that of Francis towards variation of load and head. Fig 9.2 illustrates the variation of turbine efficiency with load. Whenever the operating conditions vary over a wide range, Pelton turbine is preferable.

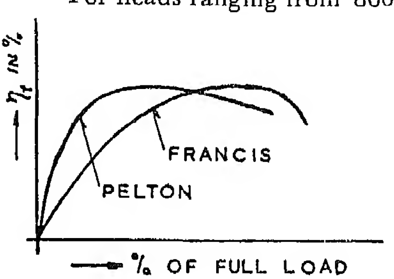

Fig 9 2 Comparison of Performance of Pelton and Francis Turbines

c) **Medium Heads (180 ft to 450 ft or 60 m to 150 m)** : In this range a Francis turbine is usually employed. Whether a high or low specific speed unit would be used, depends on the selection of speed which has been previously discussed.

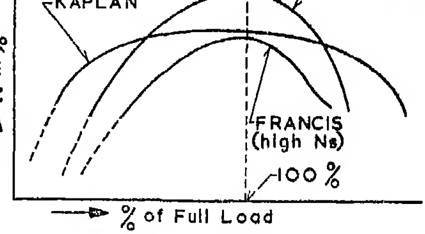

Fig 9 3 Comparison of Performance of Francis and Kaplan Turbines

d) **Low Heads (under 180 ft or 60 m)** : Both Francis and Kaplan turbines may be used for heads from 90 ft to 180 ft (or 30 to 60 m). The latter is more expensive but yields a higher efficiency at part-loads and overloads. It is therefore preferable for variable load. (See Fig 9.3).

Kaplan turbine is generally employed for heads under 90 ft (or 30 m). In this range Francis turbine would be an *exception*. Propeller turbines are, however, commonly used for heads up to 50 ft. They are adopted only when there is practically no load variation (See Fig 9.4). Propeller turbines are definitely cheaper than Kaplan turbines because the former dispense with the runner blade adjusting mechanism.

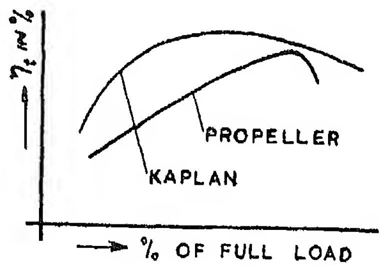

Fig 9.4 Comparison of Performance of Kaplan and Propeller Turbines

Problem 9.5 The quantity of water available for hydro-electric station is 9,200 cusecs (or 260 m^3/sec) under a head of 5.6 ft (or 1.71 m). Assuming the speed of the turbines to be 50 rpm and their efficiency 82%, determine the least number of turbines of Kaplan type, all of the same size, that will be needed if turbine specific speed does not exceed 180 FPS Units (or 800 Metric Units). What would be the output of each turbine ? (*Punjab University—September 1954*)

Solution

Q=9,200 cfs (or 260 m^3/sec) H=5.6 ft (or 1.71 m)

N=50 rpm η_t=82%

N_s=180 FPS Units (or 800 Metric Units)

Specific speed $$N_s=\frac{N\,.\,\sqrt{P_t}}{H^{\frac{5}{4}}}$$

or $$P_t=\left(\frac{N_s\,.\,H^{\frac{5}{4}}}{N}\right)^2=\left(\frac{180\times 5{\cdot}6^{\frac{5}{4}}}{50}\right)^2$$

$$=\textbf{960 HP per turbine} \quad \textit{Answer}$$

$$\left[\text{or}\quad P_t=\left(\frac{800\times 1.71^{\frac{5}{4}}}{50}\right)^2=\textbf{975 Metric HP per turbine}\quad \textit{Answer}\right]$$

Turbine output $$P_t=\frac{w\,.\,Q\,.\,H}{550}\,.\,\eta_t \quad \text{BHP}$$

$$\left[\text{or}\quad P_t=\frac{w\,.\,Q\,.\,H}{75}\,\eta_t \quad \text{Metric HP}\right]$$

or $$Q=\frac{550\,.\,P_t}{w\,.\,H\,.\,\eta_t}=\frac{550\times 960}{62{\cdot}4\times 5{\cdot}6\times 0.82}=1{,}840 \text{ cfs}$$

$$\left[\text{or}\quad Q=\frac{75\,.\,P_t}{w\,.\,H\,.\,\eta_t}=\frac{75\times 975}{1{,}000\times 1{\cdot}71\times 0{\cdot}82}=52\ m^3/\text{sec}\right]$$

But Q=9,200 cfs (or 260 m^3/sec)

$\therefore$ Number of Kaplan turbines required$=\dfrac{9{,}200}{1{,}840}=$**5** *Answer*

$$\left[\text{or}\quad \text{No. of turbines}=\frac{260}{52}=\textbf{5}\quad \textit{Answer}\right]$$

Problem 9.6 A power plant is to be built on a river having rate of flow of 2,370 cusecs and utilising 42 ft of head. The speed fixed by the electrical company is 166·67 rpm. The efficiency of the turbine is 90%. It is further seen that the specific speed of 76·7 rpm would be best for the hydro-plant. Selection of water turbine is to be made from the following data meant for the required specific speed and calculated for the unit head.

$N_s=76{\cdot}7$, $H=1$ ft

Dia of runner in in.	54	57	60	63	66
HP	7·8	8·7	9·6	10·6	11·6
rpm	27·4	26 0	24·7	23·5	22·5

Find out—

a) the type of the turbine runner,

b) the number of turbines required,

and c) the diameter of the runner selected.

Solution

$Q=2{,}370$ cusecs $\qquad H=42$ ft

$N=166{\cdot}67$ rpm $\qquad \eta=0{\cdot}9$

$N_s=76{\cdot}7$

Now specific speed $N_s=\dfrac{N\,.\,\sqrt{P_t}}{H^{\frac{5}{4}}}$

or $$76{\cdot}7=\frac{166\ 67\times\sqrt{P_t}}{24^{\frac{5}{4}}}$$

or $P_t=2{,}420$ HP per turbine

Unit power $$P_1=\frac{P_t}{H^{\frac{3}{2}}}=\frac{2{,}420}{42^{\frac{3}{2}}}=8{\cdot}9 \quad \text{HP ft}^{-\frac{3}{2}}$$

Unit speed $$N_1=\frac{N}{H^{\frac{1}{2}}}=\frac{166{\cdot}67}{42^{\frac{1}{2}}}=25{\cdot}7 \quad \text{rpm ft}^{-\frac{1}{2}}$$

a) Type of runner is **Kaplan,** because specific speed is 76·7 *Answer*

b) Turbine output $P_t=\dfrac{w\,.\,Q\,.\,H}{550}\,.\,\eta_t$

or $$Q=\frac{550\,.\,P_t}{w\,.\,H\,.\,\eta_t}=\frac{550\times2{,}420}{62{\cdot}4\times42\times0{\cdot}9}=565 \text{ cusecs}$$

∴ Number of units$=\dfrac{2{,}370}{565}=$ **4·2≈4 units** *Answer*

c) For a unit power of 8·9 HP, select the diameter of runner as **60 in** from the above table. If an intermediate runner is available, then calculate the diameter by interpolation.

Let x be the required diameter of runner in inches.

Then $$\frac{26\cdot0-24\cdot7}{25\cdot7-24\cdot7}=\frac{60-57}{60-x}$$

or $$\frac{1\cdot3}{1}=\frac{3}{60-x}$$

or $$60-x=\frac{3}{1\cdot3}=2\cdot3$$

$\therefore$ $x=60-2\cdot3=57\cdot7$ in. $\approx$ **57$\frac{3}{4}$ in.** *Answer*

UNSOLVED PROBLEMS

(A) Turbine Models

9.1 Why is the turbine model manufactured ?

9.2 What are the conditions of making the turbine models ?

9.3 Show that the speed of a turbine model is $\sqrt{n}$ times the speed of its prototype. The prototype turbine is n times bigger than that of its model.

9.4 Show which one is correct and give reasons in favour of your answer: "The efficiency of a turbine model is less, equal or more than the prototype."

9.5 A model of a water turbine develops 25 HP when working under a head of 15 ft and running at 480 rpm.

Determine the HP of the actual turbine if the working head is 120 ft and model scale is $\frac{1}{10}$ the actual turbine. What is the rpm of the actual turbine? Assume the efficiency of the model as well as that of actual turbine to be the same.

(56,500 HP ; 136 rpm) (*Jadavpur University—1958*)

9.6 A model Francis turbine made to $\frac{1}{5}$th scale, when tested gave 4·1 BHP at a speed of 360 rpm under a head of 6 ft. Calculate the equivalent speed and power of full size turbine when working under a head of 20 ft. Find also the ratio of quantities of water flowing through the turbine per second.

(131 rpm ; 627 BHP ; 45·7) (*Madras University—1952*)

9.7 A model having a scale ratio of $\frac{1}{10}$ is constructed to determine the best design for a Kaplan turbine to develop 10,000 BHP under a net head of 30 ft when running at 100 rpm. If the head available at laboratory is 20 ft and the model efficiency is 88%, find—

a) the correct running speed of the model,

b) the flow required in laboratory in cusecs,

c) the BHP of the model,

d) the specific speed in each case.

Prove any formula, you use.

(816 rpm ; 27·2 cfs ; 54·2 BHP ; 142·2) (*Delhi University—1957*)

9.8 Laboratory tests are to be carried out on a model to determine the best design speed for a Francis turbine to develop 40,000 HP under a net head of 800 ft when running at 500 rpm. If the available head at the laboratory was 100 ft and discharge 5 cusecs, assuming an overall efficiency of 88%, find—

a) suitable scale ratio for the model,

b) running speed of model,

c) the HP of the model.

Prove any formula you may use.

(6 ; 1,060 rpm ; 49 HP) (*AMI Mech E—1956*)

9.9 A model test of a 16 inches water turbine runner was carried out under a head of 25 ft and gave the following best results :

Discharge = 17 5 cusecs ;
Speed = 400 rpm ;
Output = 39 8 HP.

a) Find the specific speed, unit speed, unit power, unit discharge and efficiency of the runner,

b) State the type of runner,

c) If a 40 inches runner of the same design is used under a head of 150 ft, determine the speed, discharge and horsepower for the same.

[(*a*) 45·3 ; 80 rpm ; 0·3182 HP ; 3·5 cfs ; 80·4% (*b*) Fast Francis (*c*) 392 rpm ; 268 cfs ; 3,650 HP] (*Jadavpur University—1956*)

(B) Selection of Turbines

9.10 What kind of preliminary data is required for the selection of water turbines ?

9.11 On what factors selection of site for hydro-power plant depends ?

9.12 What kind of field information is needed to make the right type of selection of a hydro-power plant ?

9.13 What difficulties will arise, if the selection of turbine is unsuitable ?

9.14 On what factors the selection of different types of water turbine depend ?

9.15 How do the rotational speed and cavitation effect the selection of a water turbine ?

9.16 What type of turbine will you select for head of (*i*) 600 ft (*ii*) 200 ft (*iii*) 60 ft. (*Pilani—1953*)

9.17 Discuss the considerations which govern the type and size of turbines to be installed in a hydro-electric power station. What considerations are favourable for the use of

a) Propeller turbines and

b) Other type of reaction turbines ?

(*Punjab University—1954A*)

9.18 Sketch a typical layout of a low head hydro-electric power station fitted with Kaplan turbines *i.e.*, having variable pitch propeller blades. Compare this type of turbine with a fixed blade propeller turbine and Francis turbine for this application.

(*AMI Mech E—Oct 1954*)

9.19 It is required to develop 1,00,000 BHP by a number of water turbines. Each of the turbine runs at 166·7 rpm when working under a head of 98 ft. Find the number of turbines to be employed if the specific speed of each turbine is 100.

(3) (*Roorkee University—1958*)

9.20 The quantity of water available for a hydro-electric station is 9,200 cusecs under a head of 5·6 ft. Assuming the speed of the turbines to be 50 rpm and their efficiency 82%, determine the number of turbines required. Assume a specific speed of 200.

(*Madras University—1957*)

9.21 A hydro-electric station is installed to utilise 1,000 cfs of water of a river at a head of 25 ft. If 85% turbine efficiency can be counted on,

i) is it feasible to develop this power by two turbines with rpm not less than 50, the turbine having specific speed not greater than 100 rpm ?

ii) what type of runner will be used ?

iii) what is the diameter of runner if speed ratio is 0·85 ?

(*Punjab University—1956A*)

9.22 In a water power site the available discharge is 12,000 cusecs under a net head of 90 ft. Assuming a turbine efficiency of 88% and rotational speed of 166·7 rpm, determine the least number of machines, all of the same size, that may be installed if the selection rests with—

a) Francis turbine with N_s not greater than 60

b) Kaplan turbine with N_s not greater than 180.

What will be the output of each unit ? Which of the two installations will be more economical ?

(11, 2 ; 9,818 HP, 54,000 HP) (*Punjab University—1959A*)

9.23 *a*) Following is the data for a hydro-electric development :

Net Head $=304{\cdot}8\ m$

Turbine Output $=$ 10,000 HP

i) What type of turbine would you suggest ?

ii) Assuming the following specific speeds as the suitable ones, what would be the nearest synchronous speeds ?

$N_s=23{\cdot}8$ per jet for a Pelton wheel with two nozzles,

$N_s=78{\cdot}4$ for a Francis type runner.

b) Which of the two possibilities *viz* ; a twin jet Pelton or a single Francis type turbine as described above under (*ii*) above would you favour ? Support your arguments in the light of the relative advantages and disadvantages.

(*Punjab University—1960A* ; Converted to metric units)

CHAPTER 10

RECENT TRENDS IN THE DEVELOPMENTS OF WATER TURBINES

10.1 Introduction—In the foregoing chapters, fundamentals of water turbines were dealt with. In this chapter a review of recent developments touching upon the design and performance of hydraulic machines will be made. It is mainly concerned with different kinds of hydro-electric schemes which are being carried out in the field.

A Research Committee was appointed by the Institution of Civil Engineers, London to prepare a review outlining the recent developments in the sphere of Hydraulics. The recent developments in different type of hydraulic turbines, described in Art 10.2 to 10.7 have been prepared by the above committee and published in the Proceedings of the Institution of Civil Engineers, Part III Vol 4, December 1955, Number 3.

Pumped storage plants have become quite familiar for taking peak loads. It is necessary, therefore, to describe them in detail. A few articles have been devoted for the underground and underwater power stations. The former is mostly used to save the plant from air raids during war time. The latter is employed for low head installations. Recently P.E. Deriaz, Chief Designer, English Electric Co has developed a new type of runner by modifying the Kaplan runner to suit heads upto 600 ft (or 200 m).

Aerofoil theory is now being used for the design and testing of axial flow machines such as Propeller and Kaplan turbines and pumps. The blade sections are tested in the wind tunnel to arrive at the results yielding the best efficiency. Art 10.21 to 23 deal with the theory in general and the calculations for the determination of power and efficiency of Kaplan turbine.

10.2 Recent Developments in Water Turbines in General—For modern reaction turbines a specific weight of 11 lb/HP (or 5 kg/HP) is obtainable. Runner efficiencies of 96 to 97% with 90% for draft tubes have been achieved giving maximum overall efficiencies of 92 to 93%. For impulse turbines this figure falls to about 90% owing mainly to jet impact bucket losses. Improvement in the zone of partial load and lower efficiency should now attract more attention, particularly for Francis turbines which are at a slight disadvantage compared with the flat efficiency characteristics of impulse and Kaplan turbines.

The present trend of research in water turbines is towards higher speeds, reduced dimensions and simplified designs, investigations into flow conditions, properties of materials, hydraulic forces acting on regulating gear and limitation of runaway speed.

The present limit of 160,000 HP for turbines is fixed by physical dimensions. The optimum output is based on overall economy including conduits, valves and special low-loss high-head manifolds.

10.3 Recent Developments in Pelton Turbines—The present trend of research on Pelton turbines is towards the improvement of efficiency by studying the bucket surface resistance, friction and windage losses, water rebound in turbine casing and residual energy in the water leaving the wheel. Attempts are made to recover the unused head at the runner exit, by hydro-pneumatization, which have not proved successful. The needle and nozzle profiles forming the jet have resulted from experimental work based on minimum surface resistance and the long stroke required for precision discharge adjustment and governing where balancing of needle forces has become important. Short jet lengths have been suggested to avoid energy loss by jet spreading and disintegration.

The water containing large quantities of sand damaging needle and nozzle reduces the turbine efficiency. Now stainless steel designs permit rapid replacement. Welding and annealing allow worn bucket surfaces to be restored to the normal profile. Pressed steel bowls with forged-steel splitters and lugs welded on are a new departure in bucket construction.

10.4 Recent Developments in Francis Turbines—Francis turbines are being used for high heads, upto 1,000 ft (or 300 m), which were formerly confined to Pelton turbines. High rotational speed has reduced the turbine overall dimensions as well as that of the power house. Research on cavitation and availability of material to withstand the stresses has led to increase the turbine rotational speeds.

In order to obtain high efficiency, research has been directed towards the smooth welded construction for spiral casings, stay rings, guide vanes, runner and other components.

Modern measuring instrument technique has been developed to check vibrations causing cracked blades and reduced outputs. Such equipment includes surface recessed pressure cells, strain and acceleration gauges, strain amplifiers and oscillographs.

10.5 Recent Developments in Kaplan Turbines—Kaplan turbines have taken the place of Francis turbines for certain medium head installations. Kaplan turbines with sloping guide vanes to reduce the cover dimensions have been used recently.

Research is being carried out on draft tubes to recover the residual velocity head at the runner exit. The revolving parts of the turbo-set are designed to withstand runaway speeds of 2·5 to 3·5 times the working speeds. Emergency oil-pressure systems and runner braking vanes have been devised to prevent the maximum runaway speed being attained, particularly where intake gates are omitted.

Aerofoil theory has provided the aerodynamic tests to be carried out in the wind tunnels for obtaining the best profile of the Kaplan blade. Research has been carried to find the flow phenomena in the space between guide vanes and the runner, by the aid of conformal transformation. Such investigations have aided the development of efficient cavitation-free high head Kaplan turbine blading for a wide range of specific tip speeds.

10.6 Recent Research on Cavitation—The cavitation has been recognised as one of the greatest dangers in fluid flow. In Pelton turbines, the cavitation is initiated by nozzle induced vortex motion in jet. In reaction turbines, liability to cavitation increases with specific speed. As described earlier (See Art 7.16), the setting of reaction runner mainly depends on cavitation factor.

In reaction turbine runners the cavitation at the discharge side is dependent on vane shape. The draft tube has also received close examination, since it can influence efficiency and runner exit pressures.

Research has also been carried out on materials resistance to cavitation. Pressed plates appear to behave better than castings and whilst aluminium, bronze, and zinc alloys are promising, there is a preference for chrome-nickel stainless steels which can be constituted for cavitation resistance, use in sandy water and welded construction. For relatively small thin-bladed Kaplan turbines, welding distortion can be avoided with a thin cast stainless-steel sheath over a carbon-steel core which requires only light grinding and polishing.

The cavitation problems due to the presence of solid bodies entering the water, are being studied. One modern hypothesis envisages "Cavitation nuclei" consisting of minute dust-sheltering bubbles in crevices of metallic surfaces.

Stroboscope is now extensively used to predict cavitation points in flow, with model tests.

10.7 Recent Development on Model Tests—Model tests predict the performance of actual sized turbine concerning design such as efficiency, flow at runaway speed, influence of draft tube on thrust and runaway speed etc.

Model tests using air as medium instead of water have become very common. Water tests and aerodynamic counterparts have shown satisfactory agreement.

Ackeret formula for efficiency conversion from model tests, is still accepted. It can be utilised for model experiments using either air or water. Reaction turbine prototype efficiencies may be 2 to 5% greater than those predicted from model experiments.

In view of the cost, interrupted plant operation, and difficulty of site tests, model results have been accepted occasionally but before this practice can be universally adopted, assurances on geometrical similarity and scale-effect relations between model and prototype will be demanded.

In pumping installation, model studies have been very useful for the development of intake, as it is necessary to prevent vortex formation in the sump because head falls seriously. It has been seen that as little as 2% of air will reduce the head obtainable by 50% and 10% of air will de-prime the pump. Model tests are also very useful for centrifugal and propeller pumps of large size which are not manufactured on mass production.

10.8 Pumped Storage Plants*—The load on a power station fluctuates within large limits during the twenty-four hours of the day as well as during the twelve months of the year. To supply peak loads the installed capacity of the station needs to be high, most of which would remain idle during off-peak hours. It means that machines which work only during peak hours, do not bring good revenue. It is necessary, therefore, to devise some way to achieve the economical loading of the plant by levelling-up the load curve. The following methods may be employed—

a) *Commercial Method*—By selling electric current at a higher rate during peak hours than during off-peak hours. In Zurich (Switzerland) the current rate from 6-30 to 9-30 PM is more than during the rest of 24 hours of the day. With this system most of the people refrain themselves from using extra electric energy during these three peak hours.

b) *Technical Method*—The following two means may be used—

i) By *installing special peak load power plant* in which the fixed charges are low. Such a station may be either thermal or hydraulic. In thermal stations a relatively long period is needed before they can take up load because both the boiler and the turbine must be made ready for operation as soon as considerable increase in load has to be expected. This fact makes the efficiency of current production very poor. Therefore hydraulic power station is preferred for the purpose.

ii) By *storing energy* produced during off-peak hours. Such a system is known as **Pumped Storage Plants**. A pumped storage hydro-electric plant pumps water at off-peak hours of electric demand by means of surplus power (which is not being supplied), into a high level natural or artificial reservoir, in order to utilise the stored energy at periods when it is most needed. Hence pumped storage does not supply the electric energy to consumers, but it merely transforms the surplus electrical current, which is not utilised during off-peak periods (night during 24 hours and winter during the year) into hydraulic energy, stored to be used during peak periods. Thus pumped storage plants act like large energy flywheels.

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*Pumped Storage Plants by Prof M. L. Mathur, "Beacon," Jodhpur, March 1959, Vol VI. No. 1, Page 26.

10.9 Purposes of Pumped Storage Plants—The following purposes may be served by the pumped storage plants—

a) Combined with steam power stations, they represent the best compensation for it, in that they reduce the load fluctuations to within the narrow limits necessary for their economy and act as economic peak load stations throughout the year.

b) Combined with hydro-electric installations in addition to daily peak load, it can also cater to seasonal variations in water. The firm power of the installation, which is the power available at all times, is increased as during low water periods, water can be drawn from storage. Increase in firm power increases the earning capacity of the installations.

c) In some cases storage plants may not have current generating unit. In may consist of pump and motor and no turbine. The utility of such a plant is that the pump increases the head in the feeder reservoir of a separate hydro-electric plant while the motor acts as phase advancer, improving the power factor in electrical supply net work.

10.10 Classifications of Pumped Storage Plants—The following are the ways by which the pumped storage plants may be classified—

a) *Daily, weekly or seasonal storage plants.*

b) *High, medium or low head plants.* Low head plants are not common.

c) *According to the type of turbine used* in the plant. Francis turbine is very common because of its high efficiency and high speed. Pelton and Kaplan turbines have also been used.

d) *Pure or mixed storage plants*—If the total quantity of water passing through the turbines is equal to the total quantity of pumped water, both the hydraulic machines working under the same gross head, the plant is called a pure storage plant. On the other hand, if the water passing through the turbine is more than the pumped water and/or where the turbine head is more than pumps gross head, the plant is known as mixed storage plant.

e) *Horizontal or vertical storage plants*, according to the disposition of shafts. Generally the horizontal plants (See Fig 10.1) are preferred over the vertical due to good visibility, favourable conditions of erections and dismantling for repairs and check-up.

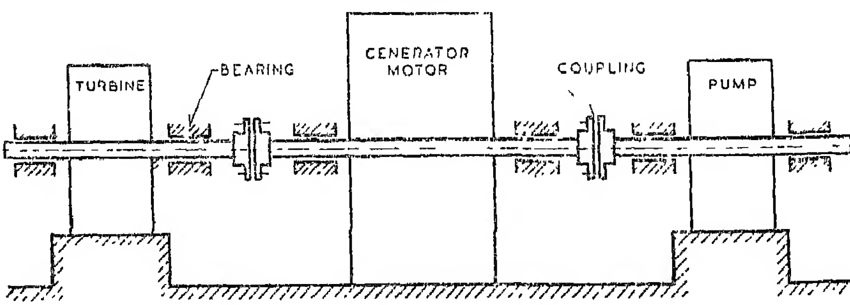
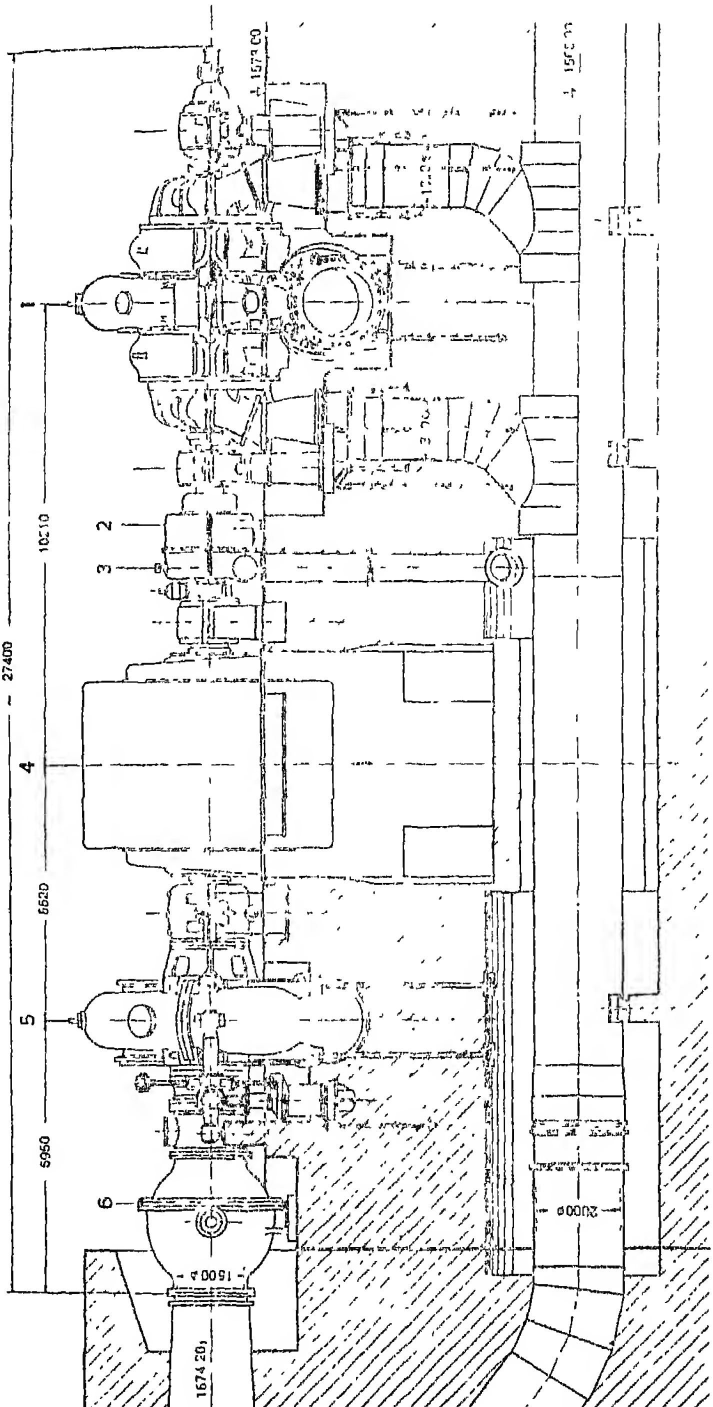


Fig 10.1 Horizontal Arrangement of Pumped Storage Plant



1. Storage Pump 3. Braking Turbine 5. Francis Turbine
2. Toothed Coupling 4. Motor-Generator 6. Rotary Valve

Fig 10.2 Limberg (Austria) Pumped Storage Plant (Escher Wyss)

10.11 Starting of Pumped Storage Plants—The layout of a pumped storage plant is shown in Fig 10.1. The set consists of a turbine, generator/motor and a pump. There are several methods employed for starting the plant. The most common method is given below :

The pump is primed by means of an ejector, and started by means of the turbine, the generator being disconnected from the system and pump delivery valve being closed. When the synchronous speed is reached the generator which acts as a motor now, is closed to the system, the turbine inlet valve shuts off and pump delivery valve is gradually opened. Turbine shaft with runner will be revolving idle while pumping takes place.

In some cases there is a small Pelton turbine coupled with the same shaft to which the main turbine, generator/motor and pump are connected. The Pelton turbine rotates the shaft in the opposite direction. As soon as the generating is to be shut off, the Pelton turbine nozzle is opened which first stops the main turbine and then by revolving the shaft in opposite direction, brings the pump and motor to a synchronous speed. The motor is, then connected to the system and the Pelton turbine inlet is closed. In high head installations, the Pelton turbine brake nozzle serves the above purpose. The research* conducted by the author on this subject showed that the process of "Generating to Pumping" can be carried out within 60 to 80 seconds.

10.12 Notable Installations of Pumped Storage Plants—One of the biggest installation of pumped storage plant is at Limberg Power Station (Austria), which is equipped with the brake turbine of Pelton type. This plant has pumps, toothed coupling, braking turbine, motor-generator, Francis turbine and rotary valve all six connected, horizontally with a total length of 27,400 *mm* (about 90 ft) as shown in Fig 10.2. The hydraulic machines were manufactured by Escher Wyss & Co Ltd. The pump rotor, comprising two single impellers and one impeller with double admission weighs 41 tons. The length of the shaft is 9 *m* (about 30 ft) with a maximum diameter of 820 *mm* (2·7 ft). The input of the pump is 83,000 HP. The main turbine is of Francis type developing 79,000 HP under 364 *m* (1,195 ft) head at 500 rpm, using 18·75 m^3/sec (661·5 cusecs).

Fig 10.3 and 10 4 show one of four two-stage pumps, manufactured by Escher Wyss, of Witznau (Switzerland) Power Plant, which are arranged vertically. They lift 8 to 10·5 m^3/sec (282 to 370 cusecs) of water against a mano-metric delivery head of 248 to 272·5 *m* (814 to 894 ft), the corresponding input being 34,600 to 39,100 HP and the speed 333·3 rpm. This plant is also equipped with Pelton turbine and a clutch arrangement to start pumping as described above. The operation of changeover from generating to pumping takes 60 seconds. The windage loss of Pelton runner, when it is revolving idle is about 37 kw.

Table 10.1 shows particulars of some important pumped storage plants operating at present.

10.13 Reversible Turbine-Pump—In a pumped storage plant, there is only one eletrical machine which acts as a generator as well as a motor. However the plant, as described in the above articles

* See "Characteristics of Free Jet Water Turbine in Working and Brake Regions" (in German), by Dr Jagdish Lal, Published by Springer-Verlag, Vienna, 1952.

TABLE 10.1

Some More Pumped Storage Plants

(Arranged according to Pump Delivery Head)

Sl No.	Pumped Storage Station	Country	TURBINE: No. of turbines	TURBINE: Type of turbine	TURBINE: Turbine HP each	STORAGE PUMP: No. of Pumps	STORAGE PUMP: No. of pump stages	STORAGE PUMP: Pump HP each	STORAGE PUMP: Max. Delivery Head in ft	STORAGE PUMP: Pump speed in rpm	Ratio turbine HP / pump HP	Type of unit	Type of Coupling	Year of Operation
1	Risseck	Austria	—	—	—	3	—	7,550	3,510	—	—	—	—	Under construction
2	Oberems	Switzerland	2	—	5,300	1	—	7,200	3,304	—	—	—	—	1942
3	Luenersee	Austria	6	—	48,500	6	—	57,500	3,200	—	1·185	—	—	Under construction
4	Tremorgio	Switzerland	1	Pelton	15,000	2	9	6,400	2,960	1,000	1·17	Horizontal	Gear	1926
5	Belleville	France	2	Pelton	4,500	2	6	3,800	1,730	1,000	1·19	Horizontal	Hydraulic	—
6	Etzel	Switzerland	6	Pelton	20,000	5	2	21,500	1,611	500	0·94	Vertical	Hydraulic	1947
7	Pfestiniog	UK	4	—	100,000	4	—	83,000	1,070	—	1·2	—	—	Under construction
8	Providenza	Italy	3	—	67,000	2	—	61,600	940	—	1·09	—	—	1950
9	Sallre	Sweeden	1	Francis	10,000	1	2	8,500	740	600	1·175	Vertical	Rigid	—
10	Haeusern	Germany	4	Francis	47,000	4	2	28,000	720	333	1·8	Vertical	Hydro-Mechanical	—
11	Our	Luxembourg	4	—	107,000	4	—	53,750	120	—	1·99	—	—	Under construction
12	Flatiron	USA	1	Francis	10,720	1	—	11,793	300	300	0·91	Reversible turbine/pump	—	1954
13	Sron Mor (Glenshire)	UK	1	—	5,000	1	—	7,000	160	—	0·715	—	—	Under construction
14	Niagara	Canada	6	—	46,300	6	—	46,000	90	—	1·005	—	—	Under construction

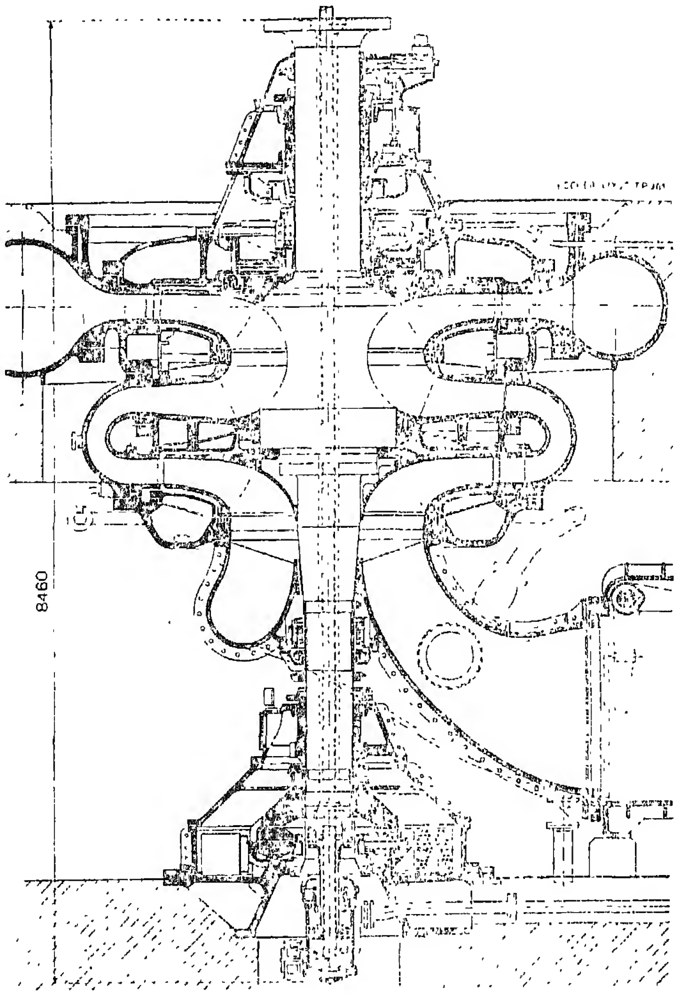


Fig 10.3 Section Through Witznau Pumped Storage Plant Vertical Type (Escher Wyss). Pump Input 40,000 HP per unit, From top are the motor-generator (not shown), Francis Turbine, Coupling with Starting Turbine, Two Storage Pumps and Thrust Bearing.

consists of two hydraulic machines *viz*, a turbine and a pump. If these two machines are combined in one pump/turbine unit, it will not only save the cost of one full machine, but also eliminates elaborate hydraulic connections and couplings etc. Such a machine is possible because Francis turbine is just a reverse of a centrifugal pump. It functions in one direction as a motor-driven pump and in the reverse direction as a turbine-driven alternator. According to model tests, the efficiencies of pumping and generating have exceeded 85%. As the specific speed of a pump is greater than that of geometrically similar turbine two different rotational speeds are necessary for the two machines if the best efficiency is to be obtained in each direction of rotation. The same speed is possible only with some sacrifice of efficiency.

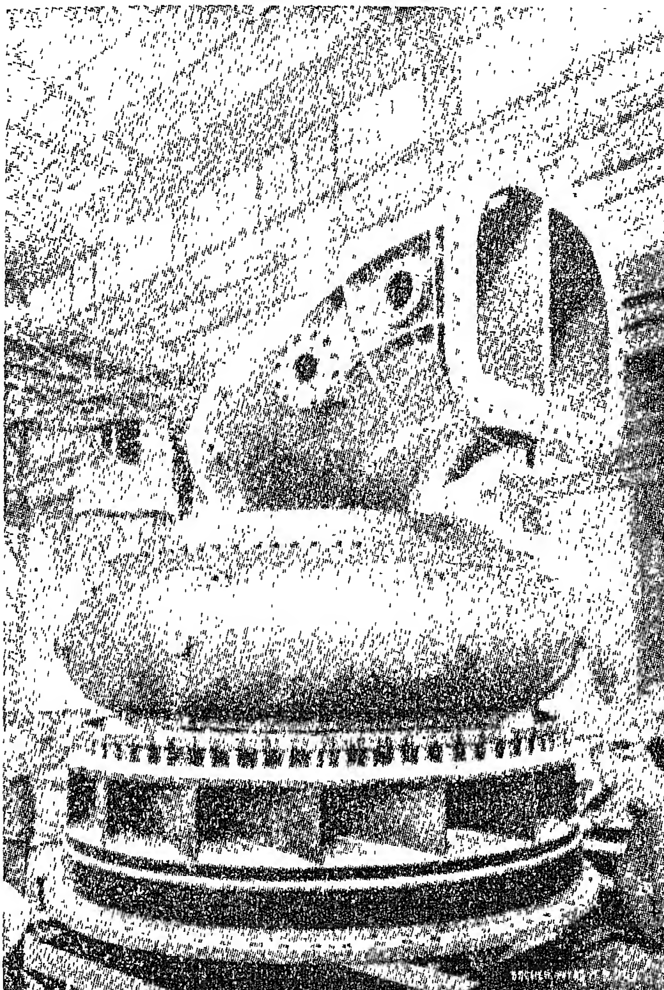


Fig 10.4 Pump of Witznau Pumped Storage Plant, without Spiral Casing, during Shop Erection of Escher Wyss

Comparing the Francis turbine and the pump for the same head and horsepower, the turbine runner size is larger than the pump impeller and the turbine casing is smaller than the pump casing. The resulting overall dimensions of reversible turbine/pump are taken as greater of the two. For pump/turbine the impeller is submerged to maintain proper flow while pumping. If the centre line is set above the tail race, priming is necessary, therefore impeller-runner is submerged and depth of submergence depends upon the critical value of cavitation factor.

10.14 Reversible Turbine-Pump Installations—

a) Flatiron Station (USA)—The first large-capacity reversible turbine-pump was installed in USA at the Flatiron Power Station of Colorado Big Thomson Project, about 50 miles (or 80 *km*) north of Denver. In

addition to one reversible turbine-pump there are two high-head conventional Francis turbines. The water discharged from these turbines is used for irrigation all the year round. To meet the seasonal irrigation requirements the discharged water is stored by the reversible turbine/pump in Caster Lake about one mile south of the plant.

The Flatiron station is a peak load plant. It is designed for two speed operations and generator/motor of special design, to realise high efficiency. The specifications of the unit are given below :

	PUMP		TURBINE	
	FPS	Metric	FPS	Metric
Rotation	Counter-clockwise	Counter-clockwise	Clockwise	Clockwise
Speed	300 rpm	300 rpm	257 rpm	257 rpm
Head	240 ft (170 to 300 ft)	73·3 m (51·8 to 91·5 m)	250 ft (290 to 140 ft)	76 3 m (88·5 to 42·7 m)
Discharge	394·5 cfs	11·17 m^3/sec	431 cfs	122 m^3/sec
Power	11,793 (input) HP	12,000 Metric HP	10,720 (output) HP	10,900 Metric HP
Efficiency	91%	91%	88%	88%

Runner impeller diameter = 101·25 in. (or 2,570 *mm*), 6 blades.

b) *Hiwassee Dam TVA* in South Western North Carolina (USA) is the world's largest pump-turbine unit till today. It is built for the following specifications—

	PUMP		TURBINE	
Rotation	Counter-clockwise		Clockwise	
Speed	105·9 rpm		105 9 rpm	
Head	250 ft	(or 62·5 m)	190 ft	(or 58 m)
Discharge	3,900 cfs	(or 110 m^3/sec)	3,900 cfs	(or 110 m^3/sec)
Power	102,000 HP		70,000 KVA	
Efficiency	75%		75%	

Runner impeller diameter = 266 in. (or 6,760 *mm*).

10.15 Economical Aspects of Pumped Storage Plants—More energy is required to pump the water upto the lake. Also some energy

is lost when hydraulic energy is converted into electrical energy. The overall efficiency of dual conversion of pumped storage energy is about 67% excluding transmission losses, which means the load can be supplied at about two-third the efficiency of direct generation. Now one-third energy losses must be compensated to make the plant economical. This can be done by charging more for the on-peak energy than for the off-peak energy as explained in Art 10.8, or otherwise the sum of the generating costs, interest on capital and sinking fund, cost of power used for pumping, must be less than the cost of producing peak load.

10.16 Underground Power Stations—Underground hydro-electric power plants have been extensively used in Europe. More interest has recently been developed in this direction due to the following advantages*—

a) They are safe against bomb attacks, rock and earth slips, and snow avalanches,

b) Air-conditioning to maintain comfortable temperature all the year round is possible,

c) They allow greater operating heads,

d) Since excavation is in the hard rock, draft tubes, penstocks and surge tanks may be cut in it, thus effecting a considerable savings in the cost of power houses and the fabrication and erection of accessories,

e) They do not disfigure the scenic beauty of the site.

Some of the important underground power stations are—

a) *Kemano* (*Canada*)—This power station has a record size underground plant, constructed for the Aluminium Co of Canada Ltd, located in Central British Columbia. The pressure tunnel and the penstock system about 11 miles (or 17·7 *km*) long, laid underground throughout, conducts water from Nechako River to Kemano power house which discharges to the Pacific Ocean. The underground power station chamber is 982 ft (or 300 *m*) long, 81·5 ft (or 24·85 *m*) wide and 139 ft (or 42·4 *m*) high from the bottom of turbine pits to the crown of roof arch. This station is equipped with 16 sets of Pelton turbines each developing 140,000 BHP (See also Art 6.8).

b) *Innert Kirchen and Handeck II* (*Switzerland*)—These two stations belong to the system of four power houses of Oberhasli Power Station Co, Innert Kirchen. Fig 10.5 shows the layout of Innert Kirchen Underground Power House (built in 1940–42) together with Handeck I which is not underground. The pressure tunnel has a length of 2 *km* (or 1·24 miles) and diameter of 2·4 to 2·6 metres (or 7·87 to 8·52 ft). It is reinforced along its whole length with steel plates. The power house is equipped with five Pelton turbines supplied by Escher Wyss, each developing 65,000 HP under a gross head of 664 metres (or 2,160 ft) and running at 428·6 rpm.

The Handeck II underground power station, built in 1947 to 1950, has four vertical Pelton turbines, each developing an average output of 40,000 HP under a net head of 450 *m* (or 1,480 ft) and at 300 rpm.

* Journal of the Institute of Engineers (India), December 1956 Vol XXXVII No. 4 Part II Page 396, "Modern Hydraulic Turbine Practice" by Prof N. S. Govinda Rao and K. Seetharamiah.

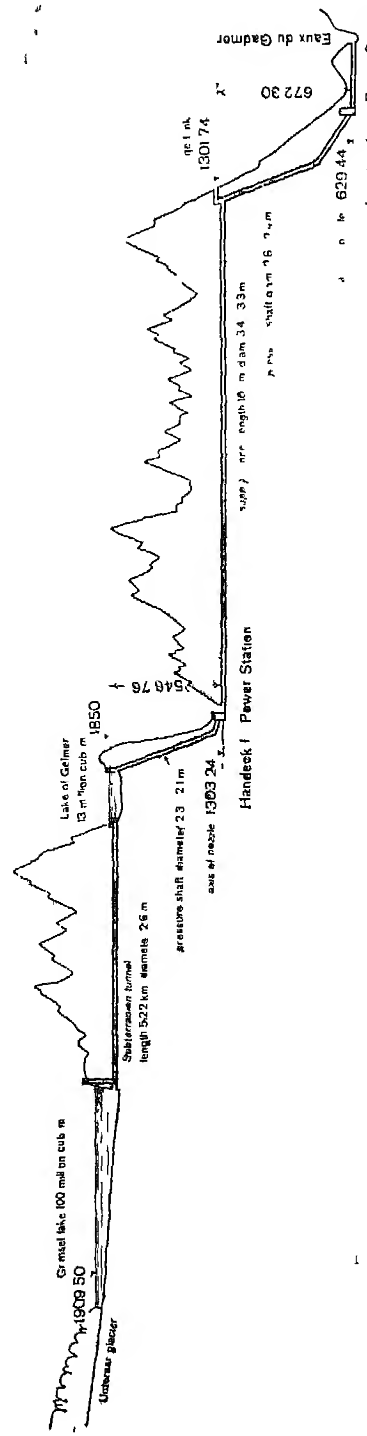


Fig 10.5 Layout of Innert-Kirschen (Switzerland) Underground Power Station together with Handeck I Power House

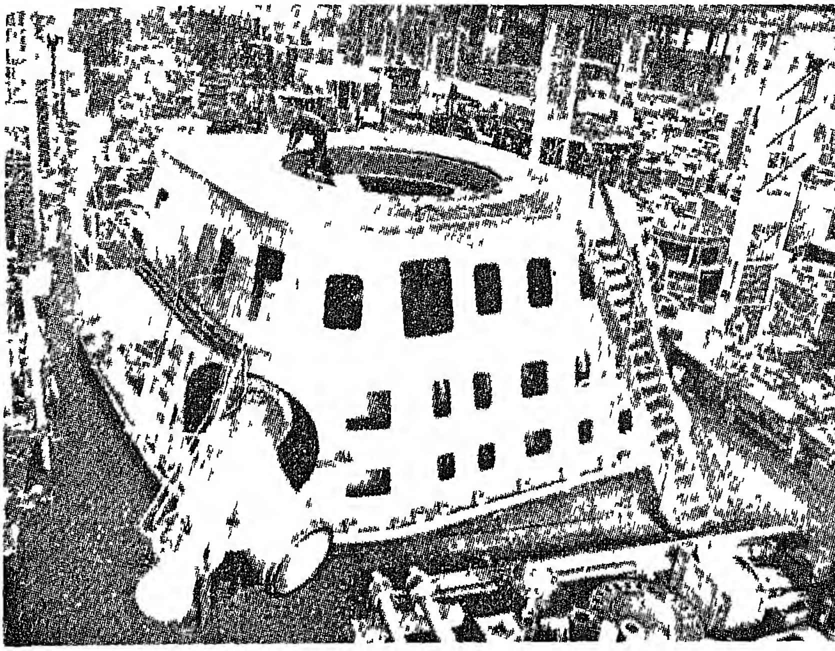


Fig 10 6 Casing of a 42,000 HP Vertical Pelton Turbine for Handeck II Power Station, lying in Escher Wyss Workshops

Fig 10 6 shows a casing of 42,000 HP vertical Pelton turbine lying in the Workshops of Escher Wyss. This turbine works under a net head of 460 *m* (or 1,510 ft), having two jets (also seen in the casing), deflectors, one piece cast steel runner with overall diameter of 3·56 *m* (or 11 ft 4 in.) and weighing 16 tons.

10.17 Underwater Power Stations or Tubular Turbines Power Stations—The underwater power stations are those which are constructed below the upper level of water. It has been discussed that water turbine (see Art 7.17*a*), especially Kaplan turbine is sometimes installed below the tail race level to create negative suction head H_s in order to avoid cavitation. In a conventional low head Kaplan installation there are a number of bends at inlet, casing, draft tube of elbow type, through which the water has to flow describing a 'Z'—form path, leading to head losses. In addition to this disadvantage such a power plant is of vertical type to reduce the dimensions of buildings. Whenever the turbine has to be dismantled, the generator has to be removed first. To overcome the above two disadvantages, Arno Fischer in 1936 developed in Germany a modified turbine which was known as *tubular turbine* and power stations employing such a turbine were known as *under water power stations* Such an installation consists of a weir across the river to be harnessed and housing the power station within the weir. The head is connected to the tail race by a straight passage in which the Kaplan runner works The turbine is connected to the generator by a horizontal or inclined shaft, depending on convenience. Fig 10.7a shows a Kaplan turbine coupled to a generator by a horizontal shaft. The water flows axially without having deflection losses between inlet

a)

b)

c)

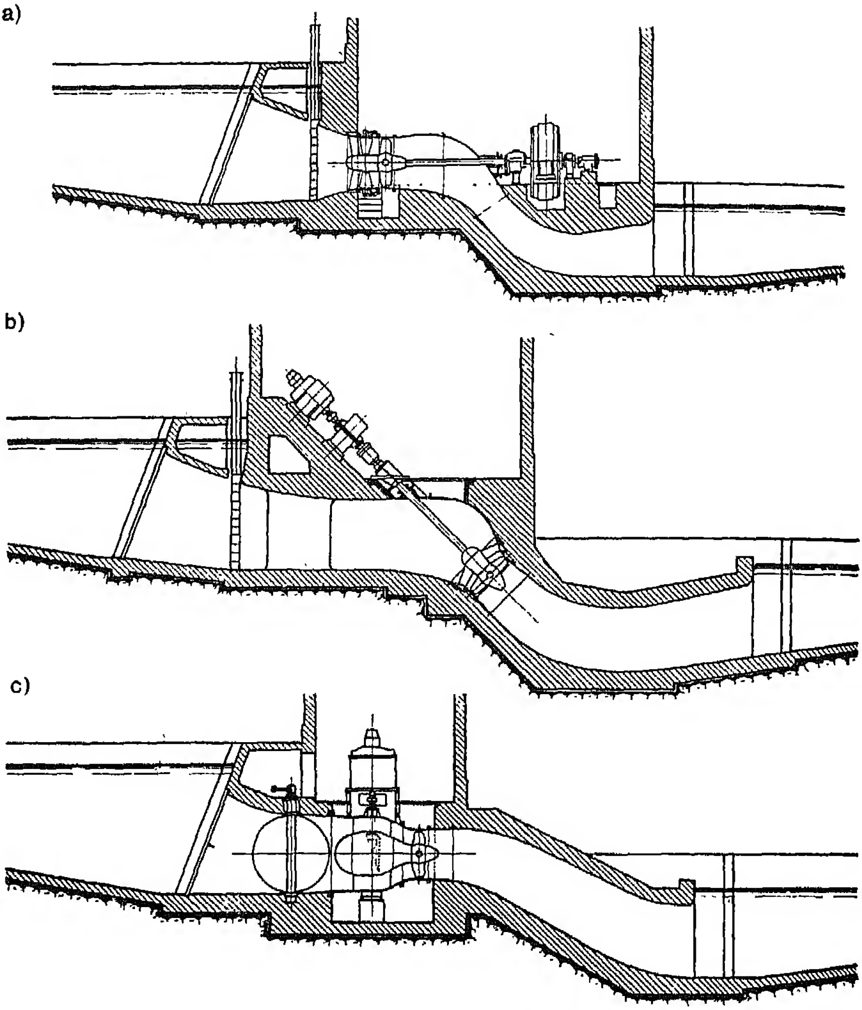


Fig 10.7 Different Layouts of Tubular Turbine Power Stations (*a*) Horizontal (*b*) Inclined (*c*) Horizontal Turbine and Vertical Generator.

and outlet. The dismantling of turbine can be carried out by opening the coupling, without touching the generator. Fig 10.7*b* shows the coupling of the turbine and generator by an inclined shaft. In Fig 10.7*c* horizontal turbine drives a vertical generator through bevel gears. In such a case the generator being vertical, the dimensions of the power house building will be less, however the gearing is to be properly sealed as shown in the figure. Fig 10.8 shows a Kaplan turbine of Escher Wyss make installed at Rostin Power Station, Pomerania coupled to generator which is arranged in the guide wheel boss. The generator in such types of plant has to be sealed against water. This turbine works under a head of 3·75 *m* (or 12·3 ft) utilising 6·3 m^3/sec (or 222 cusecs), developing 265 HP. Fig 10.9 shows underwater power station at Sealach using tubular turbines.

The advantages of tubular turbines can be summarised as follows—

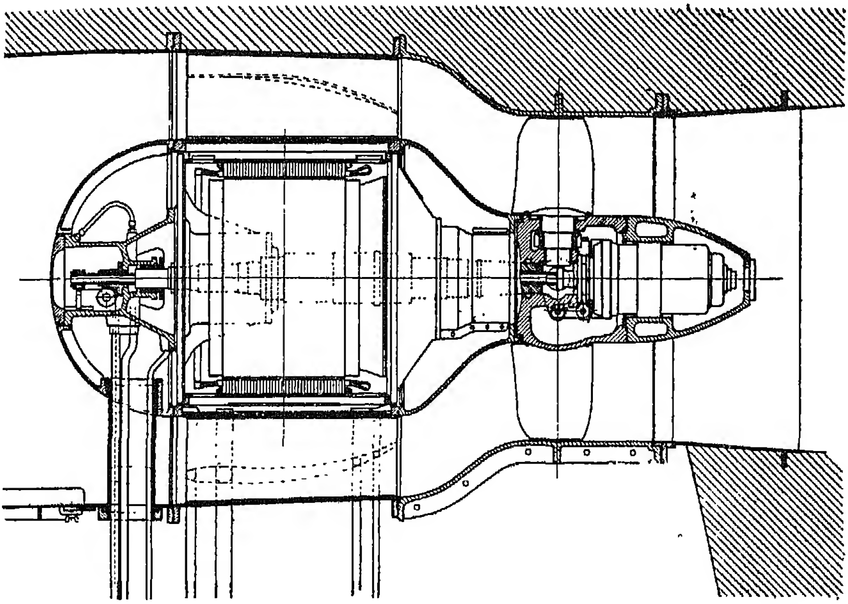


Fig 10.8 Tubular Turbine Coupled with Generator arranged in Guide Wheel Boss, Supplied by Escher Wyss for Rostin Power Station, Pomerania.

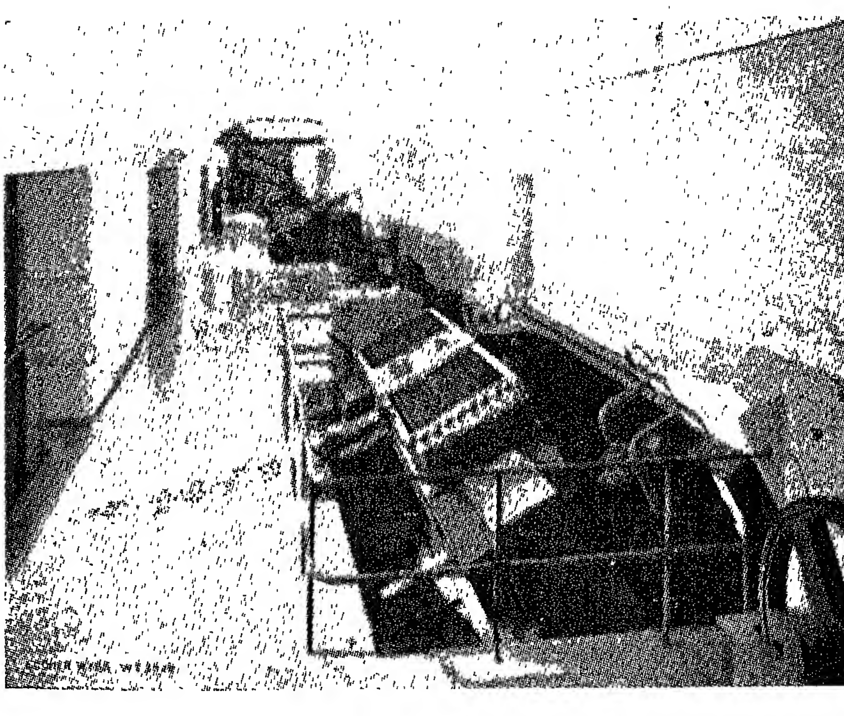


Fig 10.9 Inside View of Underwater Power Station at Sealach.

1) Large saving of overall building cost compared to conventional Kaplan turbine installation, since space needed for a unit is less and the excavation is reduced.
2) No head loss due to turning of flow between entrance and exit.
3) Turbine accessible without removing the generators.
4) Best suited to surrounding scenery.

The tubular turbines are becoming very popular in low head installations. From 1936 to 1951 only Escher Wyss built 75 tubular turbines for various power stations having a total output of 145,000 HP. Tauren Power Station has alone installed tubular turbines of a capacity of 163,000 HP.

Proposals are made to use tubular turbines for tidal power plants.

10 18 Deriaz Runner* **(Mixed-Flow Variable Pitch Pump-Turbine Runner)**—The difference between fixed blade axial flow turbine (*i.e.* propeller turbine) and the Kaplan turbine is that the former is used where there is no load variation (See Art 7.20 and 7 24). On the other hand the Kaplan turbine is useful where the load varies because it operates over a range from 40% load to 120% at practically maximum efficiency (See Fig 10.10). However, Kaplan turbines can be used only upto about 200 ft head till today. For higher heads Francis turbine has to be employed. This is a fixed runner-blade turbine, which again will not be much useful where there is load variation ; because like Propeller turbine it gives its maximum efficiency at a particular load (See Fig 10.11). Further experience has shown that fixed-runner-blade turbines of Francis type generally suffer from an unstable hydraulic condition in the draft tube at gate openings between approximately 35 to 60% (See Fig 10.11). In some critical cases operation at certain fractional gate openings is impossible, because of the severity of the pulsing output or the consequent periodic pressure surges in the penstock or both together. Paul Deriaz Dipl. Mech. Engg. (ETH-Zurich), Chief Designer, Water Turbine Development, The English

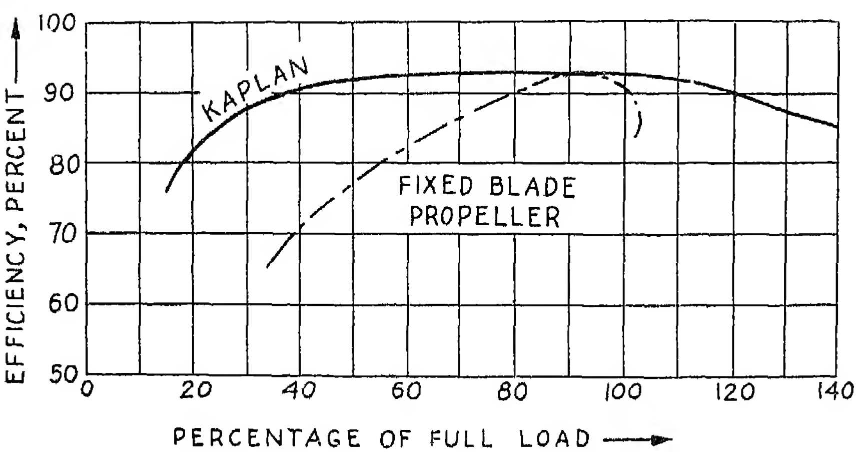


Fig 10.10 Comparison of Efficiencies of Propeller (Fixed-Blade) and Kaplan Turbines.

*See "Mixed-flow Variable-Pitch Pump-Turbine" by P. Deriaz ; Journal of "Water Power", Volume 12, No. 2, February, 1960, Page 49.

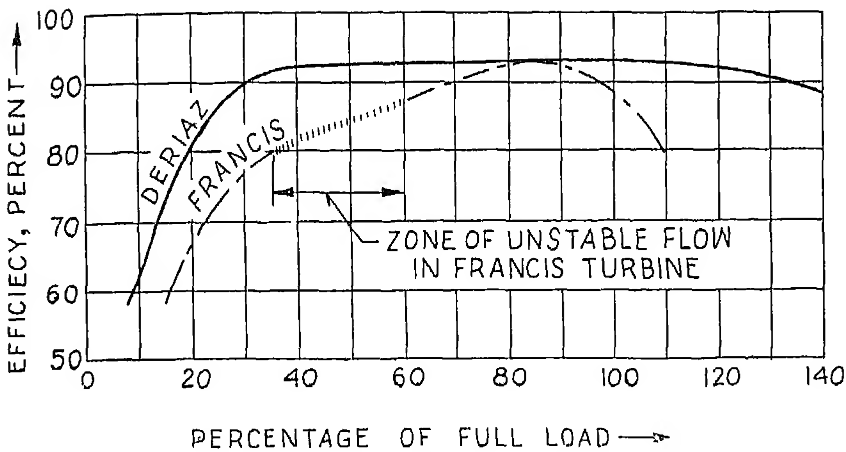


Fig 10.11 Comparison of Efficiencies of Francis (Fixed Blade) and Deriaz Turbines.

Electric Co, Ltd, gave the complete solution of the above instability to make the runner blades adjustable, so as to vary the pitch according to the turbine output. The concept of Kaplan runner has been applied to Francis runner, thus making the Deriaz runner useful for the part load and overload also. (See Fig 10 11).

In case of Kaplan turbines, with the increase of head, the hub diameter (d) increases and the blade width decreases. It is seen that with a head of 120 ft (or 40 metres) the ratio of hub diameter to runner diameter $\frac{d}{D_1}$ will be 0·52 and with 240 ft (or 80 metres) head it increases to about 0·65. The runner vanes or $\triangle R_1$ (See Fig 10.12) become narrow which decreases the efficiency. Deriaz suggested to revert to mixed flow arrangement of the Francis turbine in such cases. In a Deriaz runner the blades instead of being at right angles to the hub will be inclined at an angle of 45°. It is stated that a runner of this type can be used for a head upto 600 ft (or 200 m).

The runner is surrounded by a conventional gate mechanism, which in conjunction with the rotatable blades, gives a flat efficiency curve and overload capacity better than Kaplan turbine, designed for the same specifications. The principal features of Deriaz Runner (See Fig 10.12) are—

1. Oblique position of the runner-blade trunnions. Then the large hub-runner ratio can be accommodated or the radial extension $\triangle R_1$ of runner blade is small.

2. The oblique position of the blade trunnions Y—Y gives a significant reduction in the loading of the outer trunnion journal, because of the centrifugal force on the blade, acting partly against hydraulic loading.

3. The spherical surfaces B-B of the hub and of the skirt permit rotation of the vanes while maintaining close clearances.

4. Securing advantage of longer levers L_2 and less crowded construction in the boss.

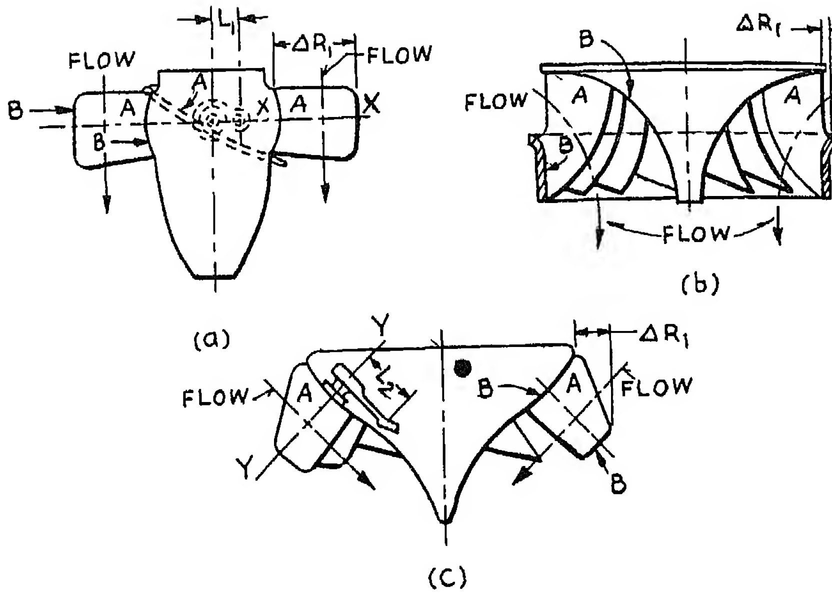


Fig 10.12 Comparison of the Construction of (a) Kaplan, (b) Francis and (c) Deriaz Runners.

5. Simpler runner blades, than those of Kaplan. If desirable the blades can be made to cover the whole cross-sectional area, in the closed position. This is the essential difference between Deriaz and Kaplan designs. In the closed position of Kaplan runner the vanes come in contact with each other at the periphery, but a large gap remains between the vanes near the hub.

10 19 Use of Deriaz Runner in Pumped Storage Plants—

1. Deriaz runner can be used as one machine turbine-pump (See Art 10.13) without lowering the efficiency of either turbine or pump.

2. It can be employed for peak load conditions where its efficiency as part load even at 30% (See Fig 10.11) is nearly the maximum.

3. Deriaz-runner turbine can be started at a very short notice. This feature forms the basis of storage schemes. The advantage of short-notice availability can be extended to an instantaneous availability of keeping the water turbine running at part load in readiness for a sudden increase in demand. This requires good part-load efficiency which Deriaz-runner has achieved.

4. For operation as a turbine, the machine rotates in the opposite direction to that for operation as a pump. The change to pump operation necessitates starting in reverse from rest. In view of the very large size of the motor-generator, synchronous machines as a rule have a relatively low torque near synchronism, resulting in difficulty in running up to speed because of the heavy torque required by the fixed-blade pump when submerged, even with shut gate. This difficulty can be overcome by pneumatisation, which consists in using compressed air to lower the water level in the pump suction pipe so as to permit

starting the pump with the runner clear of the water. With the variable-Deriaz runner this complication disappears because the torque required to drive the turbine runner with the blades in the shut position is a very small fraction of the normal load torque. Provision for compressed air, with its bulky tanks and compressors, becomes then unnecessary, and starting can be accomplished in a shorter time.

Deriaz-Runner Turbine Installations—The first Deriaz turbines were installed at the Sir Adam Beck-Niagra Pumping-Generating Station in Canada. Each is rated at 45,500 HP as a turbine under 83 ft head and discharges as a pump 4,000 to 5,000 cu ft/sec under heads from 59 to 90 ft. Six such pump-turbines for the above Power Station of Hydro-Electric Power Commission of Ontario have been supplied by English Electric Canada, a division of John Inglis Co, Ltd and manufactured at Toronto. Fig 10.13 and 10.14 show the assembly of the Deriaz reversible runner hubs with the main shaft assembled and the blades in position.

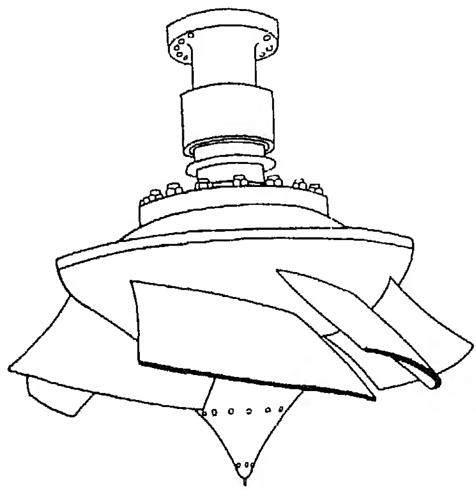


Fig 10.13 Assembly of the Deriaz Reversible Runner (Pump-Turbine Runner) for Sir Adam Beck Pumping-Generating Station in Canada.

The reversible version of the Deriaz machine with its oblique spiral casing shown in Fig 10.14 is based on mixed flow pump practice. This figure is a sectional elevation of the Niagara reversible pump-turbine. The oblique runner passages are continued straight through the pump diffuser into the spiral casing, which has a conical speed ring. The conical spiral casing reduces the overall diameter of the set and permits a reduction in the distance between sets. It also improves the streamlining of the water passages. As the runner vanes can be made to close completely, the guide wheel mechanism is not essential and has been omitted.

10.20 Tidal Power Projects—On the sea coasts, the rise and fall of tidal water can be utilised for generating power. A barrage consisting of a number of gates is constructed along the long coast. During the flood-tide, the gates of barrage are kept open and the water enters, filling the basin. The gates are closed when the flood is at its top. As the water-level in the sea falls, the basin being full of water acts as a reservoir. The low head turbines of Kaplan type or may be of tubular type, can be employed to utlise the head available. The power station constructed may be of under-water type.

Passamaquoddy and Cobscook Bay Project* close to the frontier between the USA and Canada is proposed to capture energy from the

* World Power Conference held in June 1960 in Madrid, Spain.

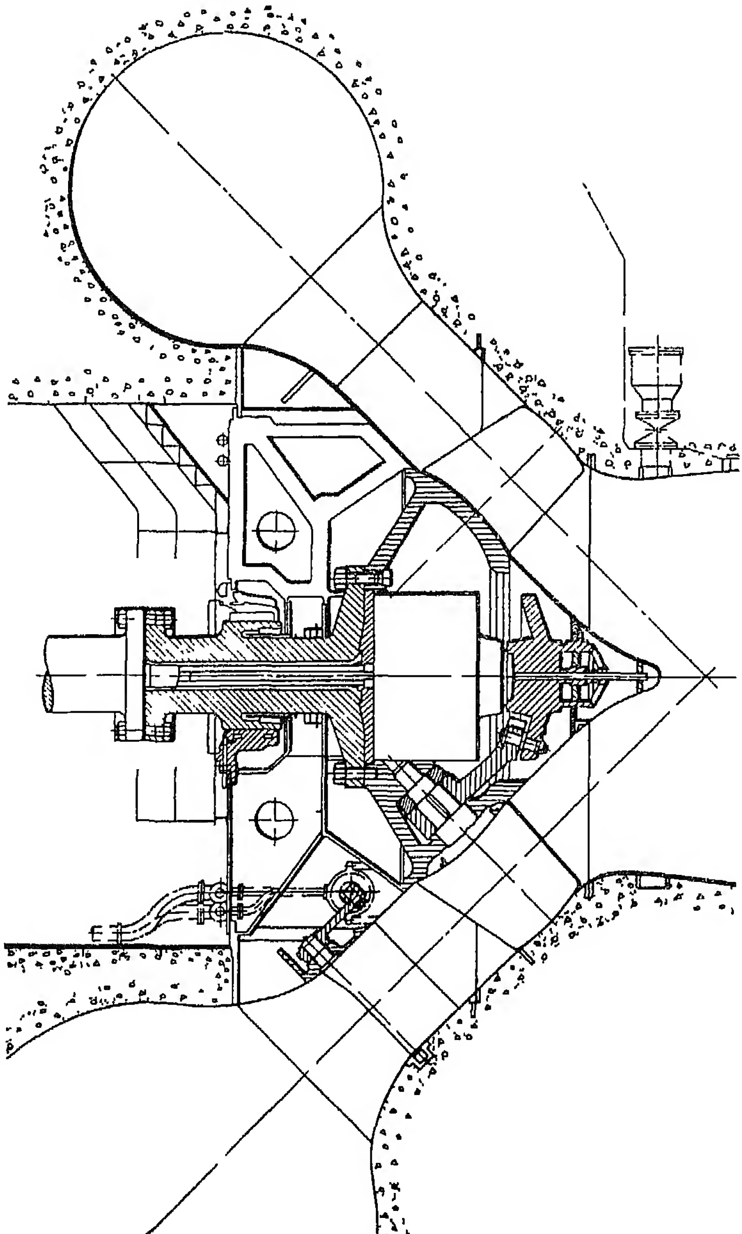


Fig 10 14 Sectional Elevation through Reversible Pump-Turbine at Sir Adam Beck-Niagara Pumping-Generating Station (Ontario)

sea. Near the mouth of the Bay of Fundy, the tides ranged from 11 3 ft (or 3·45m) at minimum neap tide, to 25 7 ft (or 7 84m) at maximum spring tide and averaged 18 1 ft (or 5 52m) The 100 sq miles (or 259km^2) of Passamaquoddy Bay will be controlled by 90 filling gates and will be used as the high pool. The 40 sq miles (or 103 5km^2) of Cobscook Bay will be controlled by 70 emptying gates which will act as low pool. The upper pool will be filled during the high tide and the lower pool emptied during low tide. The power will be generated by flow from the high pool to the low pool, through a power house containing 30 propeller turbines, each rated at 10,000 KW.

The Rance Tidel Project in France will have six sluices of 15$m\times$10m (or 49 2 ft $\times$32·8 ft) each. The installed power capacity will be 2,40,000 KW. The construction on this project is going to start soon

Due to the following disadvantages, no tidel power station has come up so far—

1) Initial capital cost is very high ;
2) Power output fluctuates during the day. During the tide which takes place twice a day, the turbines have to be shut down completely for two to three hours.

10.21 Aerofoil Theory—Let a solid body, in the form of a flat plate, be totally immersed in a fluid stream having a velocity w relative to plate motion (See Fig 10.15). Let the plate be inclined at angle α to the direction of flow. The plate then will be subjected to dynamic or kinetic pressure resulting from momentum changes induced in the liquid. Let the pressure be represented by P acting below the plate and normal to its surface as shown in Fig 10 15. Resolve P into two components vertical and horizontal. The vertical component $P\cos\alpha$ is called *Lift* and is denoted by L. The horizontal component $P\sin\alpha$ is called *Drag* and is denoted by D.

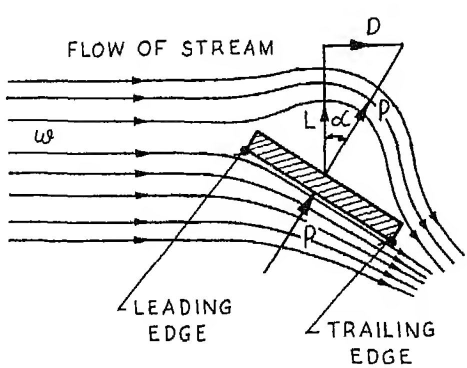


Fig 10 15 Aerofoil Theory

Lift is induced due to difference of pressure on the two sides of the plate. The stream lines strike the leading edge of the plate. Some are diverted towards the bottom of the plate and the rest are raised above the upper surface and then drop, joining again at the trailing end of the plate. Vortices are produced on the upper surface of the plate which make the pressure negative at that place. Then the resultant pressure P is the sum of the positive pressure acting below the plate and the negative pressure acting on the upper surface of the plate.

The resultant pressure P is proportional to the projected area A of the body normal to the flow and the square of the kinetic energy of the stream, thus

$$P \propto \frac{w_a \cdot w^2}{2g} \cdot A$$

or $$P = C \cdot \frac{w_d \cdot w^2}{2g} \cdot A$$

$$= C \cdot \rho \cdot \frac{w^2}{2} \cdot A \qquad \ldots(10.1)$$

where

w_d = specific weight of fluid,
w = relative velocity of stream,
A = area of plate,
C = co-efficient of pressure,
ρ = density of fluid,
$= \frac{w_d}{g}$

The lift will thus be equal to :

$$L = C^L \cdot \rho \cdot \frac{w^2}{2} \cdot A \qquad \ldots(10.2)$$

where C_L = lift co-efficient

and Drag will be written as

$$D = C_D \cdot \rho \cdot \frac{w^2}{2} \cdot A \qquad \ldots(10.3)$$

where C_D = drag co-efficient

The co-efficients C_L and C_D depend upon the shape of body (or blade), roughness of surface, Reynolds' number, aspect ratio and angle of attack (See Art 10.22).

Lift is responsible for the motion of the body, while drag is the resistance and should be the least.

10.22 Aerofoil Section—The plate immersed in fluid, shown in Fig 10.15 has too much drag. A typical section known as aerofoil section has been used in practice which has the least drag. Fig 10.16 shows such a section. The front nose of the section is rounded off smoothly and the tail is made a sharp edge The top surface is curved smoothly. The bottom surface is not very important.

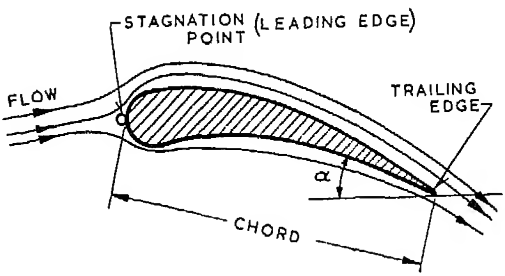


Fig 10.16 Aerofoil Section

Some definitions concerning aerofoil section are given below.

Stagnation Point is a leading edge where the stream strikes first and is bifurcated smoothly. The two parts of stream meet again at the trailing edge without formation of eddies.

Angle of Attack—If a horizontal line is drawn from the trailing edge, the angle α (See Fig 10.16) it makes with the bottom surface of section, is known as angle of attack.

Aspect Ratio is the ratio of the lengths of the two sides of plate and is equal to $\frac{b}{l}$ (See Fig 10.17).

Problem 10 1 An aeroplane is flying at a speed of 350 mph. The chord area of each of the two wings is 80 sq ft. The air resistance of all the parts other than the two wings is 30% of the total wing resistance. The propulsion efficiency of propeller is 80%. The lift and drag co-efficients C_L and C_D at the angle of incidence of the flights are 0·36 and 0 02 respectively. The atmospheric pressure and temperature at the altitude of the flight are 10 lb/sq in. and 240° K respectively. Determine the lift and the power required to drive the aeroplane. Assume $PV=96\ w_a T$.

Solution

$$w=350 \text{ mph}=350\times\frac{88}{60}=513 \text{ ft/sec}$$

Total area of wing $A=80\times2=160$ sq ft

$$C_L=0{\cdot}36 \text{ and } C_D=0{\cdot}02$$

$$PV=96\ w_a T$$

$$=96\ \rho\ g\ T$$

$$\therefore \qquad \rho=\frac{PV}{96\ gT}=\frac{10\times144\times1}{96\times32\ 2\times240}=0{\cdot}00194 \text{ slug}$$

Lift $\qquad L=C_L\cdot\rho\cdot\frac{w^2}{2}\cdot A$

$$=0{\cdot}36\times0{\cdot}00194\times\frac{513^2}{2}\times160\times\frac{1}{2{,}240} \text{ tons}$$

$=$**6·55 tons** *Answer*

Drag $\qquad D=C_D\cdot\rho\cdot\frac{w^2}{2}\cdot A$

$$=0\ 02\times0\ 00194\times\frac{513^2}{2}\times160 \text{ lb}$$

$$=816 \text{ lb}$$

$\therefore$ Total drag $=1{\cdot}3\times816=1{,}060$ lb

Total power absorbed $=\frac{D\cdot w}{550}$

$$=\frac{1{,}060\times513}{550} \text{ HP}$$

Power required to drive the plane $=\frac{1{,}060\times513}{550\times0{\cdot}8}$

$=$**1,240 HP** *Answer*

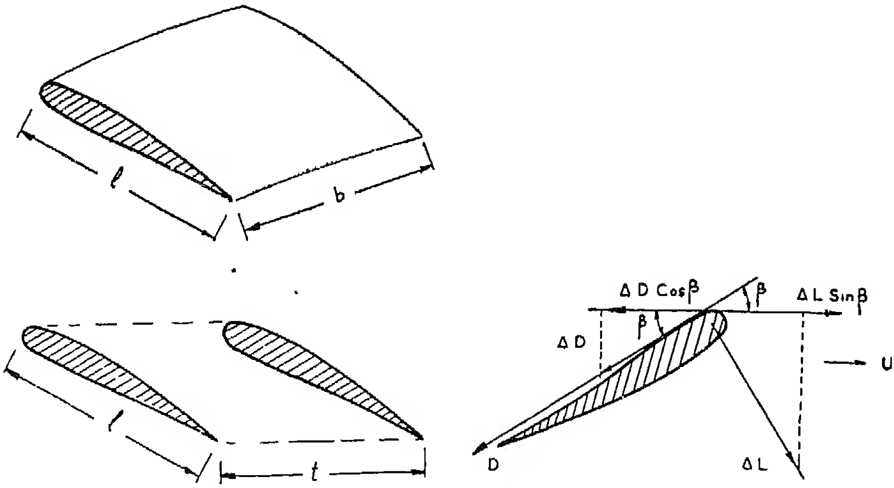


Fig 10.17 Kaplan and Propeller Turbine Blade Sections

10.23 Application of Aerofoil Theory to Axial Flow Turbo-machinery—Aerofoil section is stream lined and causes the least resistance to flow. Hence the guide blades of all reaction water turbines and runner blades of propeller and Kaplan turbines have an aerofoil section. Fluid mechanics draws no distinction between two cases giving rise to the relative motion whether the body (or blade) moves through the stationary fluid or fluid passes by the stationary body. Therefore it is possible to test the still aeroplane models in the wind tunnel and the towing torpedo model in the still water tank, for the determination of the characteristics of prototype machines. Model experiments of Kaplan turbine bladings are also made in the wind tunnel to find the best and efficient blade section. Spacing between the two blades or the blade pitch should be such that the fluid may pass without interference and that it does the maximum amount of work.

Fig 10.17 shows a section of Kaplan or propeller turbine blade.

Let l=length of blade chord,

t=circumferential pitch of blade.

The blade length b is divided into a number of blade elements. The lift and drag of each blade element are given by—

Lift $\quad \triangle L = C_L \cdot \triangle b \cdot l \cdot \rho \cdot \frac{w^2}{2}$

Drag $\quad \triangle D = C_D \cdot \triangle b \cdot l \cdot \rho \cdot \frac{w^2}{2}$

Force on blade element (See Fig 10.17)—

$$\triangle F_u = \triangle L \cdot \sin\beta - \triangle D \cdot \cos\beta$$

or $\quad \triangle F_u = l \cdot \triangle b \cdot \rho \cdot \frac{w^2}{2}\,(C_L \cdot \sin\beta - C_D \cdot \cos\beta)$

$$= l \cdot \triangle b \cdot \rho \cdot \frac{w \cdot w \cdot \sin\beta}{2}\,(C_L - C_D \cdot \cot\beta)$$

but $w \sin \beta = v_m$ = radial velocity or velocity of flow

$$\therefore \quad \Delta F_u = l \cdot \Delta b \cdot \rho \cdot \frac{w \cdot v_m}{2} (C_L - C_D \cot \beta) \qquad \ldots(1)$$

On the other hand—

$$\Delta F_u = \frac{w_d \cdot \Delta q}{g} \cdot \Delta v_u$$

where w_d = specific weight of fluid

Δv_u = peripheral velocity of blade element

and $\Delta q = \Delta b \cdot t \cdot v_m$

$$\therefore \quad \Delta F_u = \frac{w_d \cdot \Delta b \cdot t \cdot v_m}{g} \cdot \Delta v_u \qquad \ldots(2)$$

Equalising (1) and (2)

$$\frac{w_d \cdot \Delta b \cdot t \cdot v_m \cdot \Delta v_u}{g} = l \cdot \Delta b \cdot \rho \cdot \frac{w \cdot v_m}{2} (C_L - C_D \cdot \cot \beta)$$

$$= \frac{l \cdot \Delta b \cdot w_d \cdot w \cdot v_m}{2g} (C_L - C_D \cdot \cot \beta)$$

$$\therefore \quad \Delta v_u = \frac{l}{t} \cdot \frac{w}{2} (C_L - C_D \cdot \cot \beta)$$

Denoting $\frac{l}{t} = \lambda$,

$$\Delta v_u = \lambda \cdot \frac{w}{2} (C_L - C_D \cot \beta) \qquad \ldots(3)$$

Practical Data—

$$\lambda_a = \frac{l_a}{t_a}$$

and $\lambda_a = N_s^{\frac{2}{3}} = 80$

where N_s = specific speed in metric units

$R_a = 0{\cdot}95\, R_1$ (Fig 10.18)

$$\lambda = \lambda_a + \nu \left(1 - \frac{R}{R_1}\right)$$

where $\nu = \frac{r_1}{R_1}$

$$t = \frac{\pi \cdot D}{z_2}$$

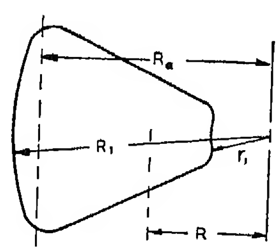


Fig 10.18 Outline of Propeller Blade

Power developed ΔP_H by Blade Element—

$$\Delta P_H = \Delta F_u \cdot u$$

$$=l \cdot \Delta b \cdot \rho \cdot \frac{w \cdot v_m}{2} (C_L - C_D \cdot \cot\beta) \cdot u$$

$$=l \cdot \Delta b \cdot \rho \cdot w_a \cdot \frac{w \cdot v_m}{2g} (C_L - C_D \cdot \cot\beta) \cdot u \qquad \ldots(10.4)$$

Head Efficiency η_H of Blade Element—

$$\eta_H = \frac{\Delta P_H}{\Delta P_a} \text{ where } \Delta P_a = w_a \cdot \Delta q \cdot H$$

$$= w_a \cdot \Delta b \cdot t \cdot v_m \cdot H$$

$$\therefore \quad \eta_H = \frac{l \cdot \Delta b \cdot \frac{w_a \cdot w \cdot v_m}{2g} (C_L - C_D \cdot \cot\beta) \cdot u}{w_a \cdot \Delta b \cdot t \cdot v_m \cdot H} \qquad \ldots(10.5)$$

$$= \frac{l}{t} \cdot \frac{w}{\sqrt{2g\,H}} \cdot \frac{u}{\sqrt{2g\,H}} \cdot (C_L - C_D \cdot \cot\beta)$$

$$= \lambda \cdot K_w \cdot K_u \cdot (C_L - C_D \cdot \cot\beta) \qquad \ldots(10.6)$$

Problem 10.2 A Kaplan turbine is fitted with four aerofoil blades and has a speed of 120 rpm. The mean radius of blade circle is 5 ft and the blade length in a radial direction is 2 ft. The chord of the aerofoil blade is inclined at 25° to the direction of the motion. The chord length is 8·2 ft. The values of C_L and C_D for the angle of incidence, used are 0·7 and 0·04 respectively. The turbine is supplied with water under a head of 25 ft. Neglecting the area occupied by the blade thickness and assuming a velocity of flow as 15 ft/sec, calculate the horsepower developed and the theoretical efficiency of the turbine.

Solution

$Z_2 = 4$	$N = 120$ rpm
$R_1 = 5$ ft	$b = 2$ ft
$\beta_1 = 25°$	$l = 8{\cdot}2$ ft
$H = 25$ ft	$v_{m_1} = 15$ ft/sec

From velocity triangle (See Fig 1.17*a*)

$$v_{m_1} = w_1 \sin\beta_1 = 15 \text{ ft/sec}$$

or $$w_1 = \frac{15}{\sin 25°} = \frac{15}{0{\cdot}4226} = 36 \text{ ft/sec}$$

Force $$F = \rho \cdot A\, \frac{w_1^2}{2} (C_L \cdot \sin\beta_1 - C_D \cos\beta_1)$$

$$= \frac{62{\cdot}4}{32{\cdot}2} \times (2 \times 8{\cdot}2) \times \frac{36^2}{2} (0{\cdot}7 \times 0{\cdot}4226 - 0{\cdot}04 \times 0{\cdot}9063)$$

$$= 5{,}340 \text{ lb}$$

Power developed by the turbine

$$P_H = \frac{F \cdot u \cdot Z_2}{550} = \frac{5,340}{550} \times \left(2\pi \times \frac{120}{60} \times 5\right) \times 4$$

$= \mathbf{2,446\ HP}$ *Answer*

Efficiency of the turbine

$$\eta_H = \frac{F \cdot u \cdot Z_2}{w \cdot Q \cdot H}$$

where $Q = (k \cdot 2\pi \cdot R_1 \cdot b)\ v_{m_1}$ and $k = 1$

$= (1 \times 2\pi \times 5 \times 2) \times 15$ cfs

$$\therefore \quad \eta_H = \frac{2,446}{62\ 4 \times (2\pi \times 5 \times 2 \times 15) \times 25} \times 100$$

$= \mathbf{91\ 6\%}$ *Answer*

10.24 Boundary Layer Theory—The flow of fluid over a surface of a body is considered to be made up of a number of layers. The velocity of flow of the layer of fluid, which is adjacent to the surface, varies from zero to maximum, zero being on the side of the surface. This layer is called *boundary layer*. It was first known by Hele-Shaw.

Prandtl Theory—Prandtl was the first to publish his theory of boundary layer in 1904. According to this theory the fluid flow may be divided into two portions—

a) a thin layer close to the surface of the body in which whole of the resistance due to skin friction occurs. The variation of velocity between the surface of the body and the fluid is transmitted through this layer,

b) the remaining portion of the fluid, outside the layer described under (*a*), has a large value of Reynolds' number because of the high velocity of flow. Therefore viscous forces may be neglected in this region as the influence of viscosity is very little.

The *thickness* of the boundary layer is defined arbitrarily as the distance from the surface of the body to a place where the velocity of flow differs by 1% from its maximum value, as shown in Fig 10.19.

The *variation of velocity* in a boundary layer with the distance normal to the surface is given by (See Fig 10 20)—

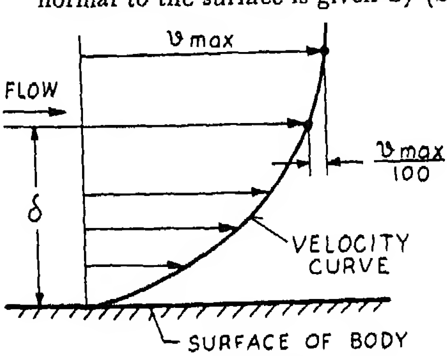


Fig 10 19 Thickness of Boundary Layer = δ

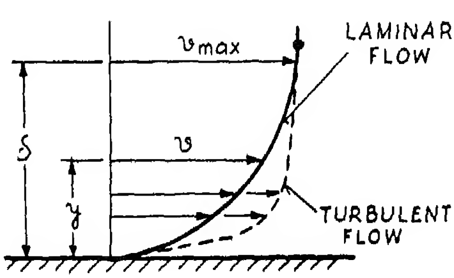


Fig 10.20 Velocity Distribution in Boundary Layer in Laminar and Turbulent Flow

$$v = v_{max} \frac{y}{\delta}\left(2-\frac{y}{\delta}\right) \text{...for laminar flow} \qquad ...(10.7)$$

and $$v = v_{max}\left(\frac{y}{\delta}\right)^n \text{...for turbulent flow} \qquad ...(10.8)$$

where $n=\frac{1}{5}$ to $\frac{1}{7}$ depending upon the Reynolds' number.

10.25 Separation of Boundary Layer—The boundary layer remains in contact with the surface of the body if

a) the inclination of the body to the direction of motion does not change,

b) the ratio of the boundary layer thickness to the length is not large.

In case the inclination or the ratio described under (*a*) and (*b*), exceeds certain limits, the boundary layer does no longer remain in contact with the surface and it detaches itself from the body at its rear portion. The detached layer has the tendency to roll itself up into vortices which are left behind the body and form what is called a **wake.** This phenomenon is known as **separation**, which increases the resistance to the flow. Fig 10.21 shows the separation of the boundary layer,

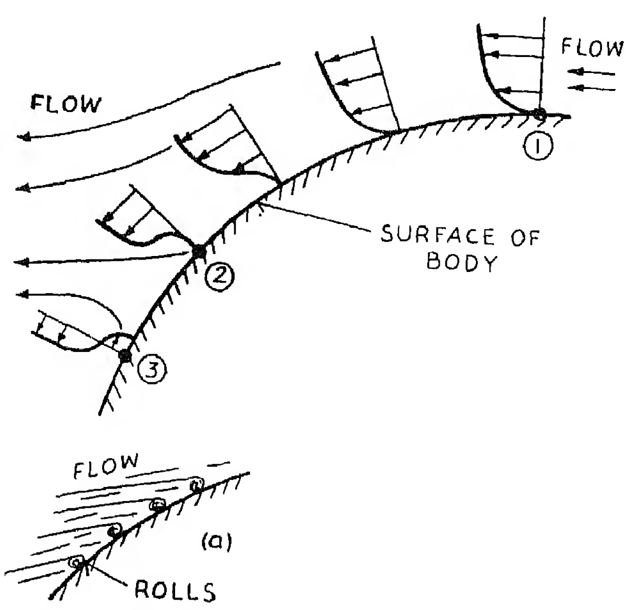


Fig 10.21 Separation Phenomenon of Boundary Layer. Rolling Up of Layer is shown in (*a*)

which is self-explanatory. The separation starts at point (2) which is known as *separation point.* It is defined as the point where the tangent to the velocity distribution curve at the surface becomes normal to the surface of the body. It has been seen that the separation occurs sooner in laminar boundary layer than in case of turbulent one.

10.26 Prevention of Separation—The separation of boundary layer, which causes friction, can be prevented by one of the following methods—

a) **By Suction of Air.**

i) *Turbine Draft Tube*—For a draft tube the maximum diverging angle is 4° as explained in Art 7.7, Fig 7.6 (*a*). If the value of this angle is increased the water layer is detached from the inside surface of the tube. Sometimes in order to decrease the length of the draft tube the diverging angle has to be made more than 4°, in which case the air is sucked from the inside surface of the draft tube as shown in Fig 10.22 so that the water may not detach itself from the tube. Experiments conducted by Prof Ackeret showed that the efficiency of draft tube was raised from 50 to 81% in certain cases by this process. However the water equal to 5% of the total quantity was also withdrawn from the draft tube by this method.

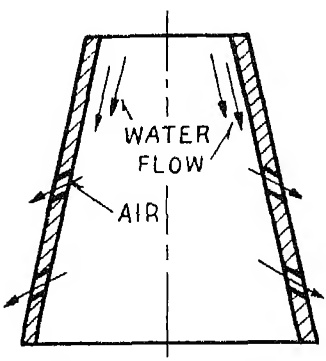


Fig 10.22 Suction of Air from Draft Tube.

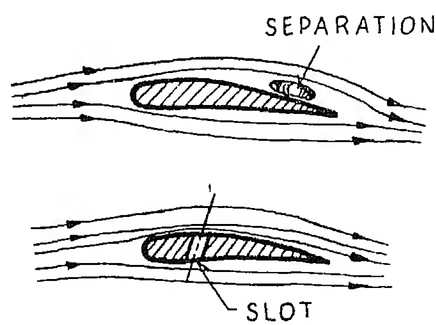


Fig. 10.23 Provision of Slot in an Aerofoil Section.

ii) *Venturimeter*—The length of diverging cone is made larger than the converging cone, taking in view the maximum angle of divergence of 4°. In case this angle has to be increased, the above method for sucking the air from the inside walls has to be applied.

iii) *Spiral Casing of Centrifugal Pump* has to be designed keeping in view the separation of boundary layer.

b) **By Providing Slot** (See Fig 10.23)—If a slot is provided in the aerofoil section, it causes suction by which the separation is controlled.

UNSOLVED PROBLEMS

10.1 Write brief notes on recent developments in

a) Pelton turbines

b) Francis turbines

c) Kaplan turbines

d) Cavitation

e) Model tests on turbo-machinery.

10.2 Describe different methods to supply peak load of the power stations. What are the different methods for achieving economical loading of the plant ?

10.3 What are pumped-storage plants? Do they actually supply electric energy to the consumers?

10.4 Classify pumped storage plants.

10.5 What is the purpose of a pumped-storage installation used in conjunction with an extensive electricity distribution system? Explain the working of a typical system, showing how the pumps, the turbines, and storage reservoirs are related.

Why is it customary to instal separate pump and turbine units, although a single rotating electrical machine will serve either as motor or as generator? Describe any system you know in which this difficulty is surmounted, and in which, therefore, the rotating hydraulic element will operate either as a pump or as a turbine.

[*AMI Mech E (Lond)—Oct 1958*]

10.6 What is a reversible turbine-pump? Where do you use such a machine?

10.7 Is it economical to instal pumped storage plants?

10.8 Where is it necessary to have underground power stations? Name some such stations.

10.9 What are underwater power stations?

10.10 What are tubular turbines? What are their advantages over the ordinary Kaplan turbines?

10.11 What led to the evolution of Deriaz runner? Where will such a runner be useful? How will Deriaz runner be helpful in increasing the efficiency of pumped storage plants?

10.12 Write a note on the design features of Deriaz runner.

10.13 Write a note on Tidal Power projects. Why is tidal power not utilised so far?

10.14 What is aerofoil theory? How is the lift of an aeroplane induced?

10.15 Define the terms,

a) lift co-efficient, K_L and

b) drag co-efficient, K_D used in connection with the aerofoil theory for determination of forces on curved surfaces.

10.16 What is meant by boundary layer? Show the velocity distribution in laminar and turbulent boundary layers.

10.17 What is a boundary layer? Do you expect separation to take place in a decelerating or an accelerating flow? Give reasons.

(*Punjab University—1957A*)

10.18 An aeroplane is travelling at 180 mph and its rudder consists of a flat surface of area 2·5 sq ft. Find the force on the rudder normal to the direction of flight when turned through an angle of 30°. Assume K_L for a flat surface of an angle of incidence of 36° to be 0 35. (142·2 lb)

10.19 Show that the horsepower required to maintain uniform motion of an aerofoil in a horizontal direction depends upon the value of $K_D \, (K_L)^{-1.5}$

An aerofoil having an area of 100 ft^2 carries a vertical load 2,000 lb. Values of K_L and K_D are as follows—

Angle of incidence	0°	2°	4°	6°	8°	10°
K_L	0·17	0·24	0·31	0·38	0·445	0·50
K_D	0·0080	0·0112	0·0155	0 0212	0·028	0 035

Find the minimum horsepower required to maintain horizontal motion through air having density 0·075 lb/ft^3, together with the corresponding velocity and angle of incidence.
[*AMI Mech E (Lond)—Oct 1958*)

10.20 Sketch a graph showing how the lift and drag coefficients of typical aerofoil section vary with the angle of incidance.

Over the incidence range $-4° < \alpha < +4°$ the lift and drag coefficients of an aerofoil are given by

$$C_L = 0{\cdot}1\ \alpha + 0{\cdot}45$$

$$C_D = 0{\cdot}01\ \alpha + 0{\cdot}1$$

If the aerofoil has to lift 5 lb per sq ft of surface at an incidence of $-1°$, determine the minimum velocity of flight and the horsepower required for a total lift of 1,000 lb in air with a specific weight of 0·07 lb/ft^3. [*AMI Mech E (Lond)—Oct 1959*]

10.21 A large airship is travelling at a speed of 54 miles per hour. For a length of 800 ft and diameter of 130 ft, area given on the chord is 90,000 sq ft, find the Reynolds' number and the drag of the hull and also the horsepower required to overcome the hull-drag at the above speed. For air $\rho = \dfrac{0{\cdot}075}{g}$ ft lb units and kinematic viscosity=0·000167 sq ft per second. Take $K_D = 0{\cdot}0375\left(\dfrac{1}{R_e}\right)^{\frac{1}{7}}$ where R_e is Reynolds' number. (*Punjab University—1954A*)

10.22 The aerofoil section wings of a monoplane are 36 ft long, chord length 6 ft and angle of incidence 3°. The lift co-efficient at this angle of attack is 0·35 and the drag co-efficient 0·02. Assuming the propeller efficiency to be 75%, and the air resistance of all parts other than the wings to be 40% of the total wing resistance, calculate the horsepower required by the monoplane when it is travelling at 300 miles per hour. What is the lift of the plane at this speed ? (*Punjab University—1955 Sept*)

10.23 A water turbine of Kaplan type is supplied with water under a head of 30 ft. The runner of the turbine has four blades of aerofoil section, and moves at 75 rpm. The blade length in radial

direction is 2 ft, and the mean radius of blade circle is 5 ft. The chord of the blade is inclined at 30° to the direction of motion, and the chord length is 8 5 ft. The velocity of flow is 16 ft/sec. Calculate the horsepower developed by the turbine, and the theoretical efficiency of the turbine. Neglect the area occupied by the blade thickness. C_L and C_D for the angle of incidence used are 0·75 and 0 05 respectively. (3,480 HP ; 46·7%)

10.24 The thickness of a boundary layer is 2 in. at a certain point on a body when the velocity at the outer edge of the layer is 10 ft/sec. Draw the curves showing the velocity distribution for laminar and turbulent flow by plotting the values of velocity at intervals of 0·5 inch. (*I.I.T. Kharagpur—1955*)

SECTION III

Pumps

CHAPTER 11

RECIPROCATING PUMPS

11.1 Pumps – Pump is a mechanical device to increase the pressure energy of a fluid. In most of the cases pump is used for raising fluids from a lower to a higher level. This is achieved by creating a low pressure at the inlet or suction end and high pressure at the outlet or delivery end of the pump. Due to the low inlet pressure the fluid rises from a depth where it is available and the high outlet pressure forces it up to a height where it is required. Of course, work has to be done by a prime-mover on the pump to enable it to impart energy to the fluid.

A liquid pump is generally placed at a certain height above the liquid surface in the reservoir. The depth from which liquid has to be sucked by a pump is equivalent to its *suction head* H_s (See Fig 11.1). The surface from where the water is drawn is usually exposed to the atmosphere and theoretical suction head is then equal to atmospheric pressure (14·7 lb per sq in.) *i.e.*, 34 ft of water. In practice, however, it is never more than 25 ft partly because of frictional losses and partly because of

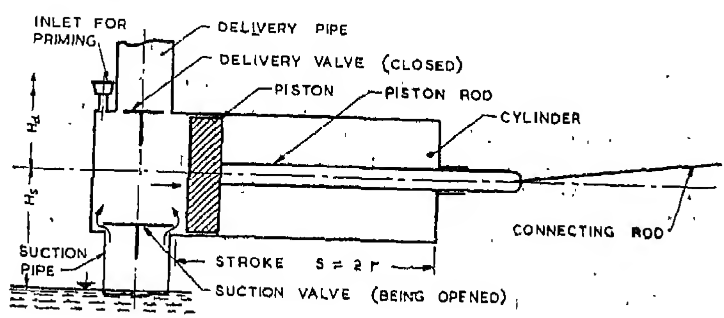


Fig 11.1 Reciprocating Single Acting Piston Pump. (Piston may be directly connected to the connecting rod.)

the fact that under a definite minimum absolute pressure, water would evaporate. The *delivery head* H_d is equivalent to the vertical height to which the liquid can be raised above the centre line of the pump.

11.2 Classification of Pumps—

- Pump
  - Reciprocating or positive displacement pump
    - Piston pump
      - Single acting
      - Double acting etc
    - Plunger pump
      - Single acting
      - Double acting etc
    - Bucket pump
    - Rotary pump (positive displacement with circular motion)
  - Rotodynamic or dynamic pressure pump
    - Centrifugal pump (Radial type)
      - Volute type pump
        - Single stage pump
        - Multi-stage pump
      - Turbine or diffusion type pump
        - Single stage pump
        - Multistage pump or Deepwell pump (deepwell pump is always vertical)
    - Axial or propeller pump
    - Semi-axial or mixed flow pump

Reciprocating pumps are now-a-days out of date and rotodynamic pumps especially of centrifugal type are replacing them. Reasons for this are given in Art 12.1. The chapter on reciprocating pumps is, therefore, altogether omitted in many modern text books. Here the author has, however, decided in favour of including it.

11.3 Reciprocating Pumps—A reciprocating pump consists primarily of a piston or a plunger reciprocating inside a close fitting cylinder, thus performing the suction and delivery strokes. The reciprocating pump is a positive acting type which means it is a displacement pump which creates lift and pressure by displacing liquid with a moving member or piston. The chamber or cylinder is alternately filled and emptied by forcing and drawing the liquid by mechanical motion. This type is called "positive" inasmuch as the only limitation on pressure which may be developed is the strength of the structural parts. Suction and

delivery pipes are connected to the cylinder as shown in Fig 11.1. Each of the two pipes is provided with a non-return valve. The function of the non-return or one way valve is to ensure a unidirectional flow of liquid. Thus the suction pipe valve allows the water only to enter the cylinder while the delivery pipe valve permits only its discharge from the cylinder.

Volume or capacity delivered is constant regardless of pressure, and is varied only by speed changes.

Reciprocating pump generally operates at low speeds and is therefore to be coupled to an electric motor with V-belts.

Applications—The reciprocating pump is best suited for relatively small capacities and high heads. In oil drilling operations this type of pump is very common.

The reciprocating pump is used generally for—

Pneumatic pressure systems, feeding small boilers, condensate return and light oil pumping.

11.4 Working Principle—Movement of the piston or plunger creates a vacuum and atomspheric pressure forces the water up through the suction pipe into the cylinder. Suction pipe and clearance volume of the cylinder are first filled with water to replace the air. This is known as *priming* of the pump. Once the pump has been primed, water follows closely the piston or the plunger on its forward stroke. In the return or backward stroke water is pushed upwards into the delivery pipe. Delivery pipe valve must be opened before starting the pump.

If H_s and H_d be the suction and delivery heads respectively of the pump, then (H_s+H_d) is known as its "*static head*".

11.5 Piston Pumps—

a) **Single Acting**—It consists of one suction and one delivery pipe simply connected to the cylinder (Fig 11.1).

Let A=cross-sectional area of the piston in sq ft (or m^2)

a=cross-sectional area of the piston rod in sq ft (or m^2)

S=stroke of the piston in ft (or m),

N=speed of crank in RPM.

Then average rate of flow $Q=\frac{A \cdot S \cdot N}{60}$ cusecs (or m^3/sec) ...(11.1)

Force on piston in forward stroke

$$=w \cdot H_s \cdot A \quad \text{lb (or Kg)} \qquad ...(11.2)$$

Force on piston in backward stroke

$$=w \cdot H_d \cdot A \quad \text{lb (or Kg)} \qquad ...(11.3)$$

Neglecting head losses in transmission and at valves, horse power of the pump

$$=\frac{w \cdot Q \cdot H}{550}=\frac{w \cdot (A \cdot S \cdot N)(H_s+H_d)}{33{,}000} \qquad ...(11.4)$$

$$\left[\text{or} = \frac{w \,.\, Q \,.\, H}{75} = \frac{w \,.\, (A \,.\, S\, N)(H_s + H_d)}{4{,}500} \text{ metric HP} \quad .. (11\ 4a)\right]$$

b) **Double Acting**—It has two suction and two delivery pipes. Fig 11.2 and 11 3 show a slow speed double acting piston pump.

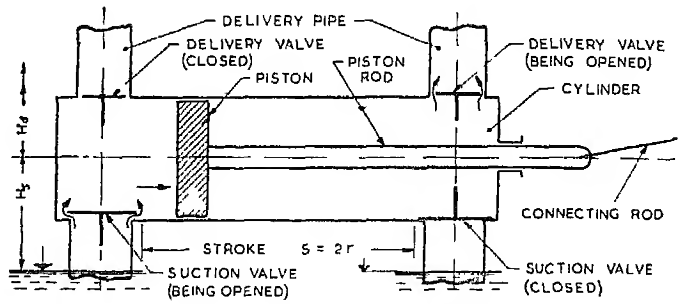


Fig 11 2 Double Acting Piston Pump

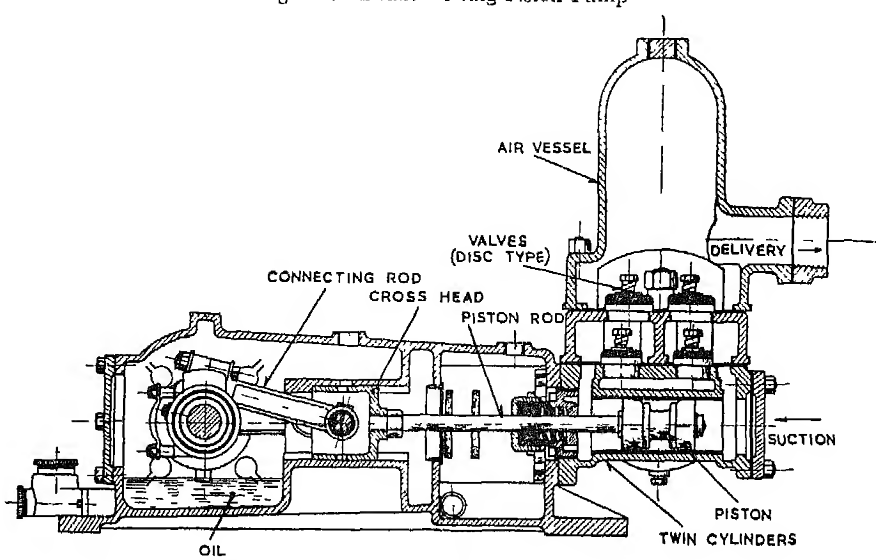


Fig 11 3 (*a*) Section Through a Low Speed, Heavy Duty Double Acting Piston Pump

$$Q = A \,.\, S \,.\, \frac{N}{60} + (A-a) \,.\, S \,.\, \frac{N}{60}$$

$$= \frac{S \,.\, N\,(2A-a)}{60} \text{ cusecs (or } m^3/\text{sec)} \qquad ...(11.5)$$

$$\approx \frac{2\,A \,.\, S \,.\, N}{60} \text{ cusecs (or } m^3/\text{sec)}$$

Force acting on piston in forward stroke

$$= w \,.\, H_s \,.\, A + w \,.\, H_d\,(A-a) \quad \text{lb (or Kg)} \qquad .. (11.6)$$

Force acting on piston during backward stroke

$$= w \,.\, H_s\,(A-a) + w \,.\, H_d \,.\, A \quad \text{lb (or Kg)} \qquad .. (11.7)$$

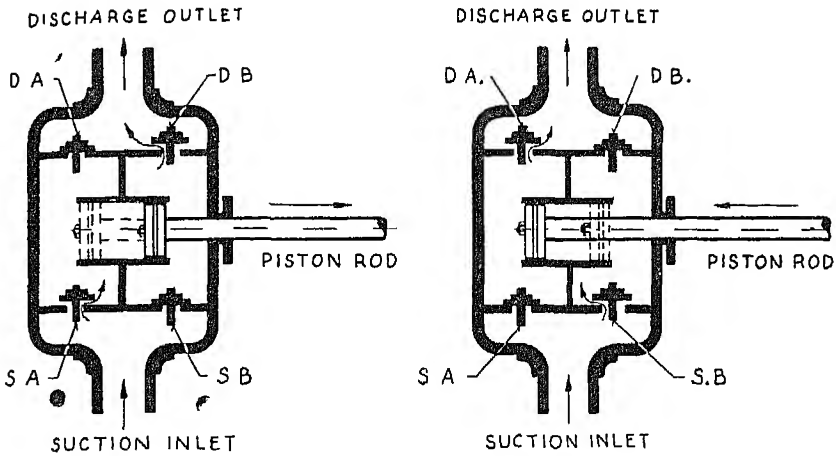


Fig 11.3 (b) Operation of Valves for Double Acting Reciprocating Pump

Average rate of flow for a triple acting piston pump is similarly

$$Q=3 \cdot A \cdot S \cdot \frac{N}{60} \text{ cusecs (or } m^3/\text{sec)}$$

(neglecting cross-sectional area of piston rod).

Problem 11.1 A single acting reciprocating pump has its piston diameter as 6 in. and stroke 10 in. The piston moves with simple harmonic motion and makes 50 double strokes per minute. The suction and delivery heads are 15 ft and 45 ft respectively. Find the force required to work the piston during the suction as well as delivery stroke Assume the efficiency of the suction and delivery strokes as 60% and 75% respectively. Determine the HP required by the pump.

Solution

$D=6$ in. $\quad S=10$ in.

$H_s=15$ ft $\quad H_d=45$ ft

$\eta_s=0{\cdot}6 \quad \eta_d=0{\cdot}75$

$N=50$ double strokes per min. $=50$ rpm

a) Average force for suction $= \dfrac{w \cdot H_s \cdot A}{\eta_s} = \dfrac{62{\cdot}4 \times 15 \times \frac{\pi}{4} \times 0{\cdot}5^2}{0{\cdot}6}$

$=$ **307 lb** *Answer*

b) Average force for delivery $= \dfrac{w \cdot H_d \cdot A}{0{\cdot}75} = \dfrac{62{\cdot}4 \times 45 \times \frac{\pi}{4} \times 0{\cdot}5^2}{0{\cdot}75}$

$=$ **735 lb** *Answer*

HP required by the pump $= \dfrac{\text{total force} \times \text{distance moved per sec}}{550}$

$$= \frac{\text{Force (suction+delivery)}}{550} \times \frac{S \cdot N}{60}$$

$$=\frac{(307+735)}{550}\times\frac{10}{12}\times\frac{50}{60}$$

$=$**1·315 HP** *Answer*

Problem 11.2 A double acting reciprocating pump has a piston of 10 in. (or 254 *mm*) diameter and piston rod of 2 in. (or 50·8 *mm*) diameter which is on one side of the piston only. Length of the piston stroke is 15 in. (or 381 *mm*) and the speed of the crank moving the piston is 60 rpm. The suction and discharge heads are 15 ft (or 4·57 *m*) and 60 ft (or 18·3 *m*) respectively. Find the force required to work the piston during the 'in' and 'out' strokes. Neglect friction. Determine the quantity of water in gpm (or litres per min) raised by the pump and the HP (or metric HP) required.

Solution

$D=10$ in. (or 254 *mm*) $d=2$ in. (or 50·8 *mm*)

$S=15$ in. (or 381 *mm*) $N=60$ rpm

$H_s=15$ ft (or 4·57 *m*) $H_d=60$ ft (or 18·3 *m*)

Cross-sectional area of piston $A=\frac{\pi}{4}\times D^2=\frac{\pi}{4}\times\left(\frac{10}{12}\right)^2=0{\cdot}545$ sq ft

$$\left[\text{or}\quad A=\frac{\pi}{4}\times(0{\cdot}254)^2=0{\cdot}0506\ m^2\right]$$

Cross-sectional area of piston rod $a=\frac{\pi}{4}\times d^2=\frac{\pi}{4}\times\left(\frac{2}{12}\right)^2$

$=0{\cdot}0218$ sq ft

$$\left[a=\frac{\pi}{4}\times(0{\cdot}0508)^2=0{\cdot}00202\ m^2\right]$$

Force required to work the piston :

a) during 'in' stroke—

i) for suction$=w\,.\,A\,.\,H_s=62{\cdot}4\times0{\cdot}545\times15=510$ lb

[or$=1{,}000\times0\ 0506\times4{\cdot}57=231$ Kg]

ii) for delivery$=w\,.\,(A-a)\,.\,H_d=62{\cdot}4\times(0{\cdot}545-0{\cdot}0218)\times60$

$=1{,}960$ lb

[or$=1{,}000\times(0{\cdot}0506-0{\cdot}00202)\times18{\cdot}3=890$ Kg]

∴ Total force during 'in' stroke$=510+1{,}960=$**2,470 lb** *Answer*

[or $231+890=$**1,121 Kg** *Answer*]

b) Force during 'in' stroke—

i) for suction $=w\,.\,(A-a)\,.\,H_s=62{\cdot}4\times(0{\cdot}545-0{\cdot}0218)\times15$

$=490$ lb

[or$=1{,}000\times(0{\cdot}0506-0{\cdot}00202)\times4{\cdot}57=222$ Kg]

ii) for delivery$=w\,.\,A\,.\,H_d=62{\cdot}4\times0{\cdot}545\times60=2{,}040$ lb

[or$=1{,}000\times0{\cdot}0506\times18{\cdot}3=926$ Kg]

∴ Total force during 'in' stroke$=490+2{,}040=$**2,530 lb** *Answer*

[or $222+926=$**1,148 Kg** *Answer*]

Discharge during 'in' stroke $= A \cdot S \cdot N = 0{\cdot}545 \times \frac{15}{12} \times 60 \times \frac{62{\cdot}4}{10}$

$= 255$ gpm

[or $= 0{\cdot}0506 \times 0{\cdot}381 \times 60 \times 1{,}000 = 1{,}156$ lit/min]

Discharge during 'out' stroke $= (A-a) \cdot S \cdot N$

$= 0{\cdot}5232 \times \frac{15}{12} \times 60 \times \frac{62{\cdot}4}{10} = 245$ gpm

[or $= 0{\cdot}04858 \times 0{\cdot}381 \times 60 \times 1{,}000 = 1{,}110$ lit/min]

$\therefore$ Total quantity of water raised by the pump $= 245 + 255 =$ **500 gpm**

[or $= 1{,}156 + 1{,}110 =$ **2,266 lit/min**]

$$\text{HP required by the pump} = \frac{(w \cdot Q) \cdot H}{550} = \frac{\left(\frac{500 \times 10}{60}\right) \times (15+60)}{550}$$

$=$ **11·35 HP** *Answer*

$$\left[\text{or} = \frac{\frac{2{,}266}{60} \times (4{\cdot}57 + 18{\cdot}3)}{75} = \textbf{11·5 metric HP} \;\; \textit{Answer} \right]$$

11.6 Plunger Pump—It is just a piston pump with a plunger replacing the piston and its rod. Generally used for rough work, it can build up a very high pressure. It is generally preferred if the water contains sand. Single acting and double acting plunger pumps are illustrated in Fig 11.4 and 11.5 respectively. Rate of flow, forces acting on plunger and the horsepower of the pump can be easily determined as in the previous cases.

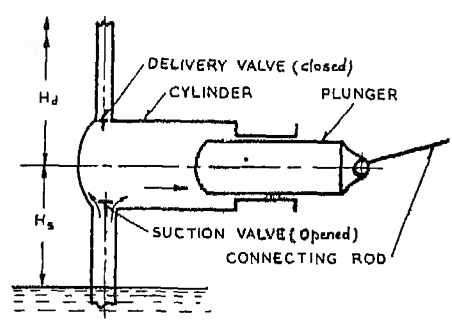


Fig 11.4 Single Acting Plunger Pump

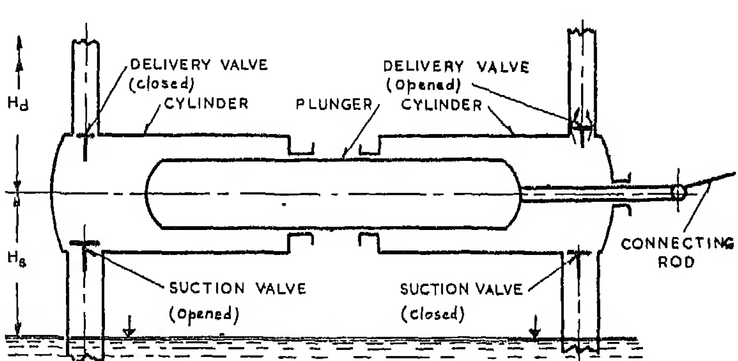


Fig 11.5 Double Acting Plunger Pump

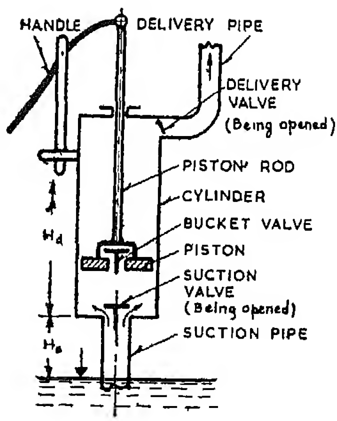


Fig 11.6 Bucket Pump or Hand Pump

11.7 Bucket Pump—This is a vertical piston pump (Fig 11.6). The piston which is provided with a valve is called a bucket. It is essentially a low speed pump. When driven by hand it is popularly known as a *hand pump*.

When the bucket rises, piston valve remains closed. This is the suction cum delivery stroke and during this stroke water is drawn into the space under the bucket and simultaneously the water above the bucket is forced into the delivery pipe. When the bucket falls neither delivery nor suction takes place. As the piston valve is open, water flows across the piston from the suction to the delivery side of the cylinder.

Problem 11 3 A bucket pump lifts water from a depth of 15 ft and forces up to a height of 75 ft. The diameters of piston and piston rod are 10 inches and 3 inches respectively It has a stroke of 2 ft. Determine the force required to raise and to lower the bucket. Find the volume of water delivered during up and down strokes. Neglect friction.

Solution

$H_s = 15$ ft $\quad\quad H_d = 75$ ft

$D = 10$ in. $\quad\quad d = 3$ in.

$S = 2$ ft

Cross-sectional area of piston $\quad A = \frac{\pi}{4} \times \left(\frac{10}{12}\right)^2 = 0{\cdot}545$ sq ft

Cross-sectional area of piston rod $a = \frac{\pi}{4} \times \left(\frac{3}{12}\right)^2 = 0{\cdot}049$ sq ft

Force required to raise the bucket=Force required to produce suction+Force required to force up the water (because the suction and the delivery take place at the same time).

Force required to produce suction$= w \,.\, H_s \,.\, A = 62{\cdot}4 \times 15 \times 0{\cdot}545$
$= 510$ lb

Force required to deliver the water$= w \,.\, H_d \,.\, (A - a)$
$= 62{\cdot}4 \times 75 \times (0{\cdot}545 - 0{\cdot}049) = 62{\cdot}4 \times 75 \times 0{\cdot}496$
$= 2{,}320$ lb

$\therefore$ Force required to raise the bucket$= 510 + 2{,}320 =$ **2,830 lb** *Answer*

During down stroke the water from the lower side of the piston goes to the top of the piston, but as on the top of piston some volume is occupied by the piston rod, therefore, some force is required to raise this excess water against the delivery head.

$\therefore$ Force required to lower the bucket$= w \,.\, H_d \,.\, a = 62{\cdot}4 \times 75 \times 0{\cdot}049$
$=$ **229 lb** *Answer*

Volume discharged during up stroke $=(A-a) \cdot S=0\ 496 \times 2$
$=0\ 992$ **cu ft** *Answer*

Volume discharged during down stroke $=a \cdot S=0{\cdot}049 \times 2$
$=0{\cdot}098$ **cu ft** *Answer*

11.8 Slip and Co-efficient of Discharge—*Slip* is the difference of volume swept through the piston and the actual discharge per stroke.

or Slip=Volume swept/stroke *minus* actual discharge/stroke ...(11.8)

The value of the slip is generally positive. However in practice sometimes the delivery valve opens before the suction stroke is completed, thus delivering a greater volume than actually swept by the piston. Hence the slip will be *negative* in such a case.

$$\text{Co-efficient of Discharge } C_a = \frac{\text{actual discharge/stroke}}{\text{volume swept/stroke}}.$$

The value of C_a is generally less than unity, but in case the slip is negative, C_a will be more than one.

11.9 Rate of Delivery—

a) **Single-Acting Piston or Plunger Pump**—The rate of delivery into or out of this type of pump is not uniform. Rate of delivery can be plotted against crank angle. During the first half revolution of the crank there is only suction and during the second half only delivery. Afterwards the same cycle is repeated. If rates of flow into and out of the pump are regarded as having positive and negative signs respectively, then since the motion of the pisoton is approximately simple harmonic, rate of delivery vs crank angle curve will be a sine curve (See Fig 11.7). The part of the curve on one side of the axis represents suction and that on the other delivery.

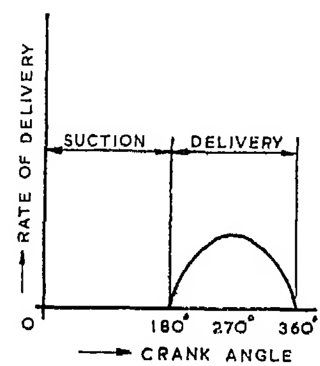


Fig 11.7 Rate of Delivery vs Crank Angle for Single Acting Piston or Plunger Pump

Velocity of discharge of water at any instant is proportional to the velocity of the piston or plunger at that instant. Therefore the sine-curve obtained above also represents the velocities of discharge to some scale.

b) **Double Acting Piston or Plunger Pump**—Each stroke is a suction cum delivery stroke. Curve of rate of delivery against angle of rotation of crank is therefore the resultant of two sine-curves drawn at a phase difference of 180°, only the delivery being considered (See Fig 11.8).

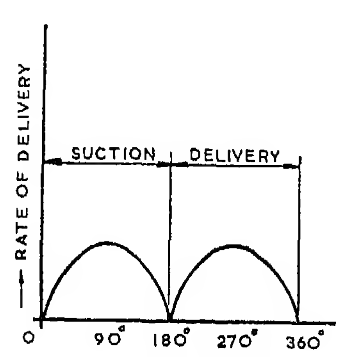


Fig 11.8 Rate of Delivery vs Crank Angle for Double Acting Piston or Plunger Pump

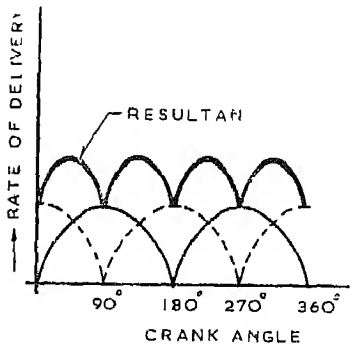


Fig 11.9 Rate of Delivery vs Crank Angle for Double Acting Driven by Two Cranks at Right Angles Piston or Plunger Pump

Rate of delivery is still variable. To make it somewhat uniform two equal cylinders with pistons connected to the perpendicular cranks of a common crank-shaft are employed. The rate of delivery curve is then the resultant of two similar curves drawn at a phase difference of 90° (See Fig 11.9).

c) **Triple Acting Piston Pump** (Fig 11.10)—This gives an even more uniform rate of delivery by using three equal cylinders with

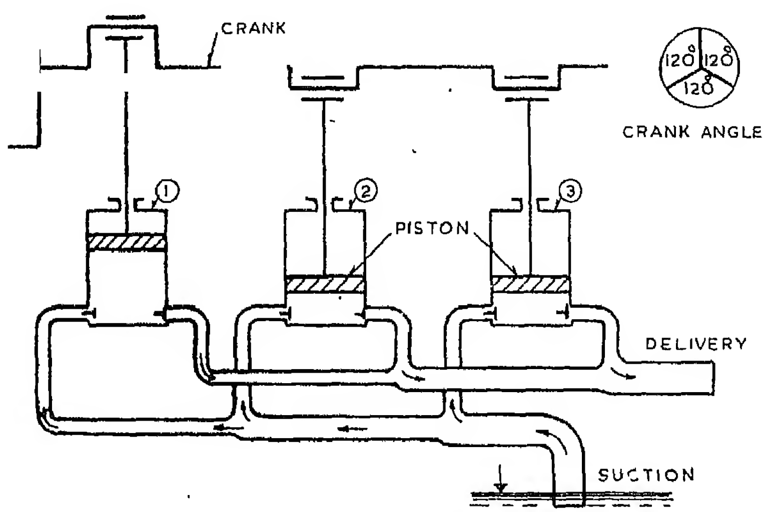


Fig 11.10 Three-Throw (Triple Acting) Piston Pump

pistons connected to cranks 120° apart from one another but driven by a common shaft. Rate of delivery curve is the resultant of three similar curves at intervals of 120° (See Fig 11.11). Pump with this arrangement is known as "*Three-Throw*" pump.

11.10 Velocity and Acceleration of Water in Reciprocating ·ps—It has been stated earlier that after the pump has been ·l, water follows the piston closely. It is of the utmost importance ·re should be no discontinuity of flow *i.e.*, there should be no

separation of the flow of water in suction pipe, cylinder or delivery pipe. If at any instant separation takes place, it will result in a sudden change of momentum of the moving water which has been separated from the rest. This causes an impulsive force which is responsible for the phenomenon of "*water hammer*" in reciprocating pumps. Pump is liable to fracture under the heavy shocks sustained as a result of this.

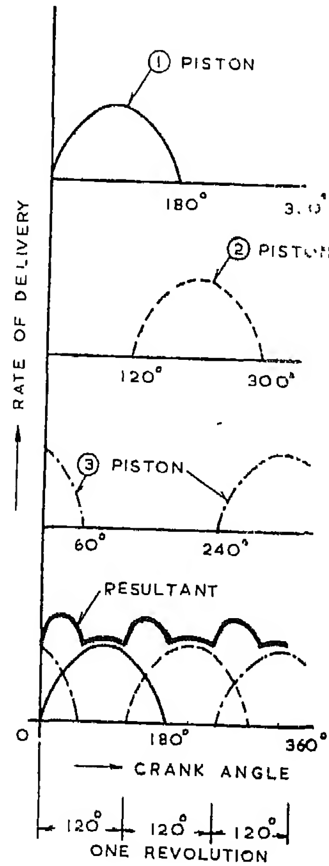


Fig 11.11 Rate of Delivery vs Crank Angle for Three Throw Pump

To eliminate the cause of water hammer *viz*, separation on the suction side of pump, driving force must be sufficient to accelerate the mass of water following the piston at the same rate as the piston itself. Assuming that the pressure inside the cylinder is zero when the piston moves forward, total suction pressure is equal to *atmospheric pressure* and it has to work against the following forces :

i) Work against gravity equivalent to suction height H_s

ii) Work against inertial forces equivalent to head H_{a_s}

iii) Work against frictional forces equivalent to head H_{f_s}

iv) Work against force required to open the non-return valve H_{v_s}

v) Work against friction in the valve $H_{v_{f_s}}$

vi) Work against vapour pressure equivalent to head H_{vap}

$$\therefore \quad H_{atm}=H_s+H_{a_s}+H_{f_s}+H_{v_s}+H_{v_{f_s}}+H_{vap} \qquad \ldots(11.9)$$

Let f_p=acceleration of piston,

A=cross-sectional area of piston,

a_s=cross-sectional area of suction pipe.

Then acceleration of water in suction pipe

$$f_s=f_p \cdot \frac{A}{a_s}$$

Accelerating force=mass×acceleration

$$F_{a_s}=\frac{w \cdot a_s \cdot L_s}{g} \cdot f_s \quad \text{lb (or kg)}$$

where L_s=length of suction pipe

Force per unit cross-sectional area

$$p_{a_s} = \frac{w \cdot L_s}{g} \cdot f_s \quad \text{lb/sq ft} \quad (\text{or Kg}/cm^2)$$

Head due to this force

$$H_{a_s} = \frac{L_s}{g} \cdot f_s \quad \text{ft} \quad (\text{or } m) \text{ of water}$$

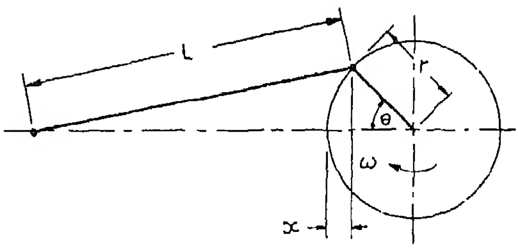


Fig 11.12 Determination of Velocity and Acceleration of Piston

Acceleration of Piston and Water—Let the crank in Fig 11.12 be at an angular distance θ from its zero position at a time t.

Then the displacement of the piston from its zero position, if the connecting rod is very long,

$$x = r - r\cos\theta, \quad \text{where } r \text{ is the radius of the crank.}$$

or $\quad x = r - r\cos\omega t \qquad \ldots(\text{as } \theta = \omega t)$

$\therefore$ velocity of piston $\quad \dfrac{dx}{dt} = \omega \cdot r \cdot \sin\theta$

and acceleration of piston $\quad \dfrac{d^2x}{dt^2} = \omega^2 \cdot r \cdot \cos\theta$

Now, $\quad f_s = f_p \cdot \dfrac{A}{a_s} = \omega^2 \cdot r \cdot \cos\theta \;\dfrac{A}{a_s}$

and $\quad H_{a_s} = \dfrac{L_s}{g} \cdot f_s = \dfrac{L_s}{g} \cdot \omega^2 \cdot r\cos\theta \cdot \dfrac{A}{a_s} \qquad \ldots(11.11)$

This is maximum when $\cos\theta = 1$ or $\theta = 0°$, *i.e.*, when piston is at its dead centre.

$$\therefore \left(H_{a_s}\right)_{max} = \frac{L_s}{g} \cdot \frac{A}{a_s}\,\omega^2 \cdot r \qquad \ldots(11.11a)$$

To be more accurate, if connecting rod is n times as long as the crank,

$$H_{a_s} = \frac{L_s}{g} \cdot \omega^2 \cdot r \cdot \cos\theta \cdot \frac{A}{a_s} \cdot \left(1 \pm \frac{1}{n}\right)$$

and $\quad \left(H_{a_s}\right)_{max} = \dfrac{L_s}{g} \cdot \dfrac{A}{a_s} \cdot \omega^2 \cdot r \cdot \left(1 \pm \dfrac{1}{n}\right) \qquad \ldots(11.11b)$

(where $+ve$ and $-ve$ signs indicate beginning and end of strokes respectively).

11.11 Speed—The resultant of the six forces enumerated above against which atmospheric pressure has to drive the water is maximum when the piston is at its dead centre, even though the velocity and consequently the frictional loss is zero at this point. If this maximum

value is higher than the atmospheric pressure, separation will occur. The pump should, therefore, be so designed that separation does not occur when the piston is at its dead centre. Of the six heads listed above, H_{vap} depends only on temperature and altitude, and H_s is automatically determined when the site of installation of pump is chosen. But the rest depend upon the length (L_s) of the suction tube and speed (N) of the crank. In general, the pump should have L_s a minimum possible under the circumstances, and the speed N should then be selected so as to avoid separation.

Hence the reciprocating pumps are designed for low speed and cannot be coupled directly to modern prime movers or electric motors which operate at high speed.

Problem 11.4 The plunger in reciprocating pump moves with simple harmonic motion. The diameter of plunger is 10 in. and stroke 18 in. The suction pipe is 5 in. in diameter and 40 ft long. The suction lift is 10 ft. Calculate the speed at which the pump can operate without separation occurring at the beginning of the stroke. There is no air vessel on the suction side. The barometer reads 30 ft of water.

(*AMIE—April, 1950*)

Solution

$D=10 \text{ in.}=\frac{10}{12} \text{ ft}$ $\qquad S=18 \text{ in.}=1{\cdot}5 \text{ ft}$

$d_s=5 \text{ in.}=\frac{5}{12} \text{ ft}$ $\qquad L_s=40 \text{ ft}$

$H_s=10 \text{ ft}$

Assume that the separation occurs at absolute pressure of 8 ft of water. Now, at beginning of the suction stroke the velocity is zero, therefore, the friction losses on the suction side, H_{f_s} are equal to zero. The accelerating head, however, would then be maximum.

$$\therefore \quad H_{atm}=H_s+H_{a_{s\,max}}+H_{sep}$$

$$\text{or} \quad H_{atm}=H_s+\frac{L_s}{g}\cdot\frac{A}{a_s}\cdot\omega^2\cdot r+H_{sep}$$

$$\text{or} \quad 30=10+\frac{40}{32{\cdot}2}\times\frac{10^2}{5^2}\times\left(\frac{2\pi\times N}{60}\right)^2\times\frac{1{\cdot}5}{2}+8$$

$$\ldots\left(\because \; r=\frac{S}{2}=\frac{1{\cdot}5}{2}\right)$$

$$\text{or} \quad 12=\frac{40\times100\times4\pi^2\times N^2\times0{\cdot}75}{32{\cdot}2\times25\times3{,}600}$$

$$\therefore \quad N=\sqrt{\frac{12\times32{\cdot}2\times25\times3{,}600}{40\times100\times4\times\pi^2\times0{\cdot}75}}$$

or $\quad N=$ **17·14 rpm** *Answer*

This speed is very low. Therefore it is difficult to couple the pump directly with the modern electric motors which generally run at 1,450 rpm (assuming 50 cycles frequency), hence it necessitates an increase in speed.

Problem 11.5 A single acting reciprocating pump has a plunger diameter of 8 inches (or 203 *mm*) and stroke of 1 ft (or 304·8 *mm*). The suction pipe is 4 inches (or 101·5 *mm*) in diameter and 24 ft (or 7·32 *m*) long. The water surface in the sump from which the pump draws

water is 12 ft (or 3·66 *m*) below the pump cylinder axis. If the pump is working at 30 rpm, find the pressure head on the piston at the beginning, middle and end of the suction stroke. Take $f=0{\cdot}01$.

(*AMIE—May 1953*)

Solution

$D=8$ in. (or 203 *mm*) $\quad d_s=4$ in. (or 101·5 *mm*)

$S=1$ ft (or 304·8 *mm*) $\quad \therefore\ r=\frac{1}{2}$ ft$=0{\cdot}5$ ft (or 152·4 *mm*)

$L_s=24$ ft (or 7·32 *m*) $\quad H_s=12$ ft (or 3·66 *m*)

$N=30$ rpm $\quad f=0{\cdot}01$

The atmospheric pressure has to supply :

$$H_{atm}=H_s+H_{a_s}+H_{f_s}+H_{piston}$$

a) Pressure head on the piston at the beginning of suction stroke—

$$H_{f_s}=0 \quad (\because \text{ Velocity is zero})$$

$$\therefore \quad H_{atm}=H_s+\frac{L_s}{g}\cdot\frac{A}{a_s}\cdot\omega^2\cdot r+H_{piston}$$

(assuming connecting rod length to be very long in comparison to crank radius)

$$34=12+\frac{24}{32{\cdot}2}\times\frac{8^2}{4^2}\times\left(\frac{2\pi\times 30}{60}\right)^2\times 0{\cdot}5+H_{piston}$$

$$\left[\text{or}\quad 10{\cdot}36=3{\cdot}66+\frac{7{\cdot}32}{9{\cdot}81}\times\left(\frac{203}{101{\cdot}5}\right)^2\times\left(\frac{2\pi\times 30}{60}\right)^2\times 0{\cdot}1524+H_{piston}\right]$$

$$=12+14{\cdot}7+H_{piston}$$

$$[\text{or}\quad =3{\cdot}66+4{\cdot}48+H_{piston}]$$

$$\therefore\quad H_{piston}=34-(12+14{\cdot}7)=\mathbf{7{\cdot}3\ ft\ of\ water\ absolute}$$

or **26·7 ft of water vacuum**

[or $H_{piston}=10{\cdot}36-(3{\cdot}66+4{\cdot}48)=$ **2·22 m of water absolute**

or **8·14 m of water vacuum**]

or $\quad 7{\cdot}3\times\frac{62{\cdot}4}{144}=$ **3·16 lb/sq in. absolute** *Answer*

$\left[\text{or } 2{\cdot}22\times\frac{1{,}000}{100\times 100}=\mathbf{0{\cdot}222\ Kg/cm^2}\right.$ *Answer*]

b) Pressure head on the piston at the middle of the suction stroke—

$$H_{a_s}=0 \qquad (\because \cos\theta=\cos 90°)$$

$$\therefore\quad H_{atm}=H_s+H_{f_s}+H_{piston}$$

$$=H_s+\frac{4fL_s}{d_s}\cdot\frac{v_s^2}{2g}+H_{piston}$$

$$=H_s+\frac{4fL_s}{d_s}\cdot\frac{\left(\omega\cdot r\cdot\frac{A}{a_s}\right)^2}{2g}+H_{piston}$$

or $$34 = 12 + \frac{4 \times 0{\cdot}01 \times 24}{\frac{1}{12} \times 64{\cdot}4} \times \left[\frac{2\pi \times 30}{60} \times 0{\cdot}5 \times \frac{8^2}{4^2}\right]^2 + H_{piston}$$

$$\left[10{\cdot}36 = 3{\cdot}66 + \frac{4 \times 0{\cdot}01 \times 7{\cdot}32}{0{\cdot}1015 \times 2 \times 9{\cdot}81} \times \left[\frac{2\pi \times 30}{60} \times 0{\cdot}1524 \times \left(\frac{203}{101{\cdot}5}\right)^2\right]^2 + H_{piston}\right]$$

or $34 = 12 + 1{\cdot}76 + H_{piston}$

[or $10{\cdot}36 = 3{\cdot}66 + 0{\cdot}537 + H_{piston}$]

$\therefore$ $H_{piston} = 34 - 13{\cdot}76 =$ **20·24 ft of water absolute**

or **13·76 ft of water vacuum**

[or $H_{piston} = 10{\cdot}36 - 4{\cdot}197 =$ **6·163 m of water absolute**

or **4·197 m of water vacuum**]

or $\frac{20{\cdot}24 \times 62{\cdot}4}{144} =$ **8·78 lb/sq in. absolute** *Answer*

[or $6{\cdot}163 \times \frac{1{,}000}{100 \times 100} =$ **0·6163 Kg/cm² absolute** *Answer*]

c) Pressure head on the piston at the end of suction stroke—

At the instant when suction valve is just going to be closed and delivery valve has not yet opened, suction head H_s is still acting on the piston. Also $\theta = 180°$.

$\therefore$ $\cos 180° = -1$ *i.e.*, H_{a_s} is negative

$\therefore$ $H_{atm} = H_s + (-H_{a_s}) + H_{f_s} + H_{piston}$

Further $H_{f_s} = 0$ ($\because$ velocity is zero)

$\therefore$ $H_{atm} = H_s - H_{a_s} + H_{piston}$

or $34 = 12 - 14{\cdot}7 + H_{piston}$

.. [H_{a_s} is same as in case of (*a*)]

[or $10{\cdot}36 = 3{\cdot}66 - 4{\cdot}48 + H_{piston}$]

or $H_{piston} = 34 - 12 + 14{\cdot}7$

$=$ **36·7 ft of water absolute**

[or $H_{piston} = 10{\cdot}36 - 3{\cdot}66 + 4{\cdot}48$

$=$ **11·18 m of water absolute**]

i.e., more than atmospheric pressure

or **2·7 ft of water gauge**

[or **0·82 m of water gauge**]

or $36{\cdot}7 \times \frac{62{\cdot}4}{144} =$ **15·9 lb/sq in. absolute** *Answer*

[or $11{\cdot}18 \times \frac{1{,}000}{100 \times 100} =$ **1·118 Kg/cm² absolute** *Answer*]

11.12 Indicator Diagrams—

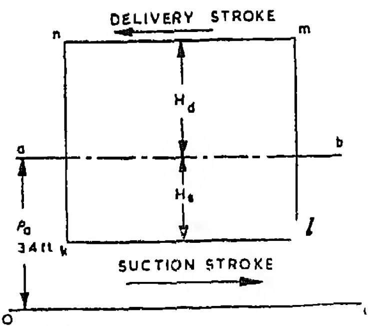


Fig 11.13 Theoretical Indicator Diagram

a) **Theoretical Indicator Diagram**—Fig 11.13 shows the theoretical indicator diagram of a reciprocating pump. The stroke and the pressure of the piston are representated by *x*-and *y*-axes on the diagram respectively. The area *klm n* represents the work done per revolution for a *single* acting pump.

In the above diagram line *ab* represents the atmospheric pressure. The pressure during suction stroke is below atmosphere by an amount equal to the suction head H_s and therefore the line *kl* represents the suction stroke. Similarly the line *mn* represents delivery stroke which is above atmospheric pressure by an amount equal to delivery head H_d.

b) **Effect of Acceleration in Suction Pipe on Indicator Diagram**—The accelerating head H_a depends upon cos θ as proved in Eqn 11.11, therefore accelerating head curve is a cosine curve. Hence the accelerating head is maximum at the beginning, zero at the centre and minimum (negative) at the end of suction stroke of piston. This makes the shape of the indicator diagram as shown in Fig 11.14, that is at the beginning of the suction stroke the negative pressure becomes high, equal to H_s+H_a and at the end of the suction stroke the negative pressure is reduced by H_a.

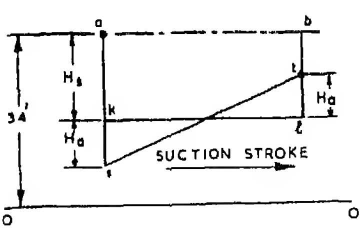


Fig 11·14 Effect of Acceleration in Suction Pipe on Indicator Diagram

The area *stba* of the diagram remains the same, therefore the work done is unaltered.

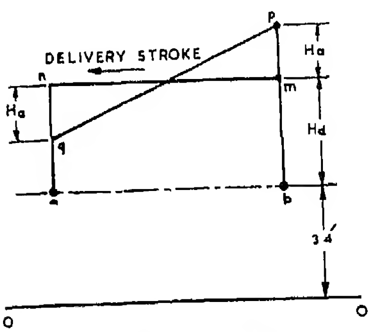


Fig 11.15 Effect of Acceleration in Delivery Pipe on Indicator Diagram

c) **Effect of Acceleration in Delivery Pipe on Indicator Diagram**—As shown in Fig 11.15, the acceleration head is added at the beginning and is subtracted at the end of the delivery stroke. The total area *pqab* of the diagram remains same and hence the work done is also not changed.

The minimum pressure head in the cylinder$=H_d-H_a$ above atmosphere.

(This is represented by point *q* in Fig 11.15).

or Absolute minimum pressure head$=34+(H_d-H_a)$

The separation is likely to occur at the end of delivery stroke if this value is less than 8 ft of water,

or $34+H_d-H_a > 8$

$\therefore \quad H_a < 26+H_d$

The pump may be delivering water with a delivery pipe of shape shown in Fig 11.16*a* or Fig 11.16*b*. In Fig 11.16*a*, the delivery pipe is vertical first and then it is bent to be horizontal. In this case the delivery head will become zero at the bend, after which there is still a long horizontal pipe which will have a considerable value of accelerating head H_a. Therefore according to the above equation the separation will take place in such a case at the pipe bend. In Fig 11.16*b*, the pipe is horizontal first and then it becomes vertical. There is no possibility of separation in this case as at the bend, there is still considerable H_d that is always greater than H_a.

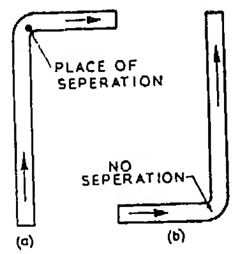


Fig 11.16 Separation in Bent Delivery Pipes (*a*) First Vertical Then Horizontal (*b*) First Horizontal Then Vertical

d) **Effect of Friction in Pipes on Indicator Diagram**—The friction head in suction as well as delivery pipes will be added to the indicator diagram, such that the inclined lines *st* (Fig 11.14) and *pq* (Fig 11.15) will become parabolic curves as shown in Fig 11.17. As the area of the diagram will now increase, the total work done will also increase.

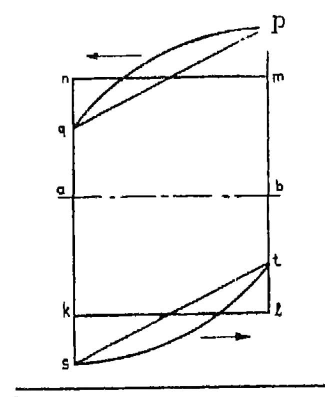


Fig 11.17 Effect of Friction in Pipes on Indicator Diagram.

11.13 Air Vessel—Air vessel is a closed chamber fitted on the suction as well as on the delivery side, near the pump cylinder, to reduce the accelerating head.

Functions :

a) *Suction side*—

1) Reduces the possibility of separation,

2) Pump can be run at a higher speed,

3) Length of suction pipe below the air vessel can be increased.

b) *Delivery side*—

4) A large amount of power consumed in supplying accelerating head can be saved.

5) Constant rate of discharge can be ensured.

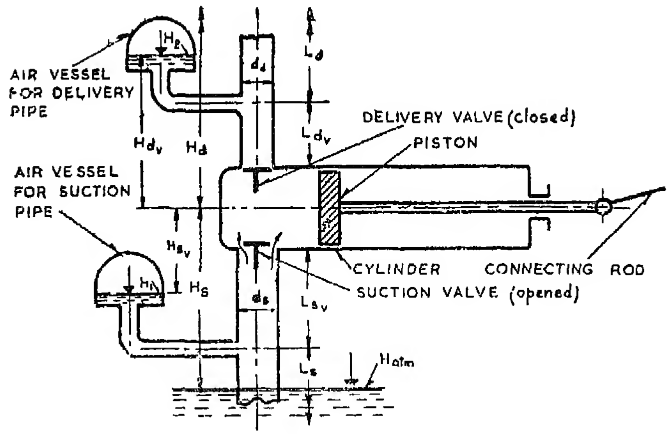


Fig 11.18 Air Vessel on Suction and Delivery Side

In order to eliminate the possibility of separation, the length of the suction pipe where the fluctuation of acceleration takes place can be effectively reduced by inserting the air vessel in the suction pipe. The air vessel is fitted near the pump cylinder, thus the length of the fluctuating column is considerably reduced, allowing the pump to run at a higher speed without the danger of separation.

In some cases where the pump has to be installed away from the sump, a long suction pipe can be safely used by fitting the air vessel near the pump cylinder.

Air vessel is also fitted on the delivery side to minimise the length of the fluctuating water column. Since the length is usually large, accelerating head produced will be considerable in the absence of an air vessel. Therefore the air vessel will save the power consumed in supplying the accelerating head.

Without air vessel, the rate of discharge varies according to the sine curve (See Fig 11.7). By fitting the air vessel constant rate of discharge will be ensured.

Working—An air vessel in a reciprocating pump acts like a fly-wheel of an engine. The top of the vessel contains compressed air which can contract or expand to absorb most of the pressure fluctuations.

Whenever the pressure rises, water in excess of the mean discharge is forced into the air vessel, thereby compressing the air held therein. When the water pressure in pipe falls, the compressed air again ejects the excess water out.

The air vessel acts like an intermediate reservoir. On suction side, the water first accumulates here and is then transferred to the cylinder of the pump. On delivery side, the water first goes to the vessel and then flows with a uniform velocity. The column of water which is now fluctuating, is only between the pump cylinder and the air vessels which

is very small due to the vessels being fitted as near to the pump cylinder as possible.

11.14 Theory of Working of Air Vessel*—Let v_p be the velocity of piston which is moving with S.H.M., assuming a long connecting rod such that the ratio of connecting rod length to crank radius is large. The piston velocity at any instant will be given by

$$v_p = v_{p\ max} \cdot \sin \theta \qquad \text{(See Fig 11.19)}$$

where $v_{p\ max}$ = maximum piston velocity

θ = crank angle

Let $v_{p\ mean}$ = mean velocity of piston

= mean velocity of discharge

For Single-acting pump—

$$\text{Then} \quad v_{p\ mean} = \frac{\int_0^\pi v_p \cdot d\theta}{2\pi} = \frac{v_{p\ max} \int_0^\pi \sin \theta \cdot d\theta}{2\pi}$$

$$\therefore \quad \frac{v_{p\ mean}}{v_{p\ max}} = \frac{\int_0^\pi \sin \theta \cdot d\theta}{2\pi} = \frac{-(\cos \pi - \cos 0°)}{2\pi} = \frac{-(-1-1)}{2\pi}$$

$$= \frac{1}{\pi} \qquad \text{...(11.12)}$$

Similarly for double-acting and triple-acting pump, it can be proved that the value $\frac{v_{p\ mean}}{v_{p\ max}} = \frac{2}{\pi}$ and $\frac{3}{\pi}$ respectively.

Considering the delivery side of the pump, Fig 11.19 shows rate of delivery vs crank angle for a single-acting pump. This is sine-curve. The value of maximum piston velocity $v_{p\ max}$ is shown by a horizontal line which cuts the sine-curve at points A and B, when the crank angles are θ_1 and θ_2 respectively. Starting from crank angle 0 to θ_1, the piston velocity is less than the piston mean velocity, therefore, water is being

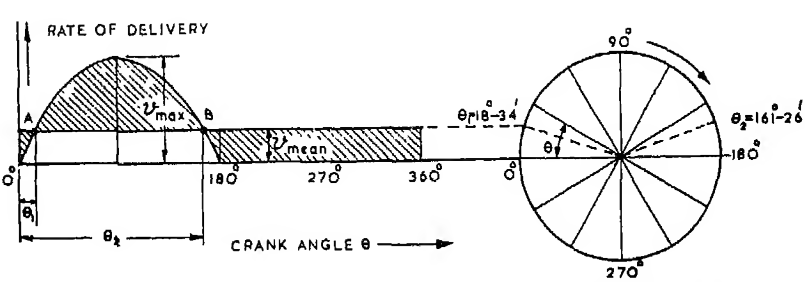


Fig 11.19 Theory of Working of Air Vessel (Rate of Delivery Curve for Single Acting Pump)

discharged from the cylinder to the air vessel placed on the delivery side. This means that at the point A, volume of water in the air vessel is

*Art 11.14 to 11.16 are helpful in designing of reciprocating pumps.

minimum and the volume of air at A is maximum. From A to B, the discharge from the cylinder to the air vessel is more than its mean value, therefore at point B, the volume of air in the air vessel is minimum.

Let V_{max} = maximum volume of air in the air vessel

V_{min} = minimum volume of air in the air vessel

then $V_{max}-V_{min}$ = fluctuating volume of air in the air vessel

if V_{mean} = mean volume of air in the air vessel,

then $\dfrac{V_{max}-V_{min}}{V_{mean}}$ = **co-efficient of fluctuation** $=\delta_f$ ...(11 13)

also $V_{max}-V_{min}$ = maximum volume of water − mean volume of water delivered by the piston

$$=\int_0^T \left[A\left(v_{p\ max}\ .\ \sin\theta\right)dt-A\left(v_{p\ mean}\right)dt\right]$$

where A = cross-sectional area of piston

T = time taken by the piston to move from A to B

$$=\int_0^T \left[A\left(v_{p\ max}\sin\theta\right)dt-A\left(\frac{v_{p\ max}}{\pi}\right)dt\right]$$

(for a single acting pump)

If ω = angular velocity of crank

then $\theta=\omega\ .\ t$

and $d\theta=\omega\ .\ dt$ or $dt=\dfrac{d\theta}{\omega}$

$$\therefore \quad V_{max}-V_{min}=\frac{A\ .\ v_{p\ max}}{\omega}\int_{\theta_1}^{\theta_2}\left(\sin\theta\ .\ d\theta-\frac{1}{\pi}\,d\theta\right)$$

$$=\frac{A\ .\ v_{p\ max}}{\omega}\left(-\cos\theta-\frac{\theta}{\pi}\right)_{\theta_1}^{\theta_2}$$

$$=\frac{A\ .\ v_{p\ max}}{\omega}\left\{\left(-\cos\theta_2-\frac{\theta_2}{\pi}\right)-\left(-\cos\theta_1-\frac{\theta_1}{\pi}\right)\right\}$$

$$=\frac{A\ .\ v_{p\ max}}{\omega}\left(\cos\theta_1-\cos\theta_2-\frac{\theta_2-\theta_1}{\pi}\right)$$

For a single-acting pump,

$v_{p\ max}\ .\ \sin\theta=v_{p\ mean}$ at points A and B

$\therefore \quad v_{p\ max}\ .\ \sin\theta=\dfrac{v_{p\ max}}{\pi}$

or $\sin\theta=\dfrac{1}{\pi}=0{\cdot}3183$

$\therefore \quad \theta_1=18°-34'$ or $0{\cdot}3246$ radians

...(See Fig 11.19)

and $\theta_2 = 180 - (18° - 34')$

$= 161° - 26'$ or $(\pi - 0{\cdot}3246)$ radians

Substituting the values of θ_1 and θ_2,

$$V_{max} - V_{min} = \frac{A \cdot v_{p\ max}}{\omega} \times$$

$$\left[\cos(18° - 34') - \cos(161° - 24') - \frac{\pi - 0{\cdot}3246 - 0\ 3246}{\pi} \right]$$

$$= \frac{A \cdot v_{p\ max}}{\omega} \left(0{\cdot}9478 + 0{\cdot}9478 - \frac{\pi - 0{\cdot}6492}{\pi} \right)$$

$$= \frac{A \cdot v_{p\ max}}{\omega} \times 1{\cdot}1022 \qquad \text{...(11.14)}$$

Now volume of stroke of piston $= A \cdot S$

$= A \cdot v_{p\ mean} \times$ time for one revolution of crank

$= A \cdot \frac{v_{p\ max}}{\pi} \times \frac{2\pi}{\omega}$ (for single acting pump)

$= \frac{2\ A \cdot v_{p\ max}}{\omega}$

$$\therefore \quad \frac{V_{max} - V_{min}}{\text{Piston Stroke Volume}} = \frac{\frac{A}{\omega} \times v_{p\ max} \times 1{\cdot}1022}{2\ \frac{A}{\omega} \cdot v_{p\ max}} = 0{\cdot}55 \qquad \text{.. (11.15)}$$

Similarly it can be shown that for a double-acting pump,

$$\frac{V_{max} - V_{min}}{\text{Piston Stoke Volume}} = 0\ 21 \qquad \text{...(11.15a)}$$

and for a triple acting pump,

$$\frac{V_{max} - V_{min}}{\text{Piston Stroke Volume}} = 0\ 009 \qquad \text{...(11.15b)}$$

This is the proportion of total water discharge per ıevolution which enters and leaves the air vessel per cycle.

11.15 Volume of Air Vessel—The fluctuations of volumes of air and water cause the variations of pressure which should be within the limits. As the air is in contact with water surface, the change of volume of air in the air vessel takes place according to Boyle's Law *i.e* at constant temperature,

or $p \cdot v = \text{constant}$

$\therefore \quad p_{min} \cdot V_{max} = p_{max} \cdot V_{min}$

$= p_{mean} \cdot V_{mean} = \text{constant}$

where $p_{mean} =$ mean air volume in air vessel.

$$\therefore \quad p_{min} = \frac{p_{mean} \cdot V_{mean}}{V_{max}} \quad \text{and} \quad p_{max} = \frac{p_{mean} \cdot V_{mean}}{V_{min}}$$

Now δ_f the co-efficient of fluctuation$=\dfrac{p_{max}-p_{min}}{p_{mean}}$

$$\therefore \qquad \delta_f=\frac{\dfrac{p_{mean}\cdot V_{mean}}{V_{min}}-\dfrac{p_{mean}\cdot V_{mean}}{V_{max}}}{p_{mean}}$$

$$=\frac{V_{mean}\cdot(V_{max}-V_{min})}{V_{max}\cdot V_{min}}$$

Approximately $V_{max}\cdot V_{min}=V^2_{mean}$

$$\therefore \qquad \delta_f\approx\frac{V_{max}-V_{min}}{V_{mean}}$$

This has already been defined under Eqn 11.13

$$\therefore \qquad \delta_f=\frac{p_{max}-p_{min}}{p_{mean}}=\frac{V_{max}-V_{min}}{V_{mean}} \qquad \text{.. (11.16)}$$

$\therefore$ Mean volume of air vessel can be written as

$$V_{mean}=\frac{V_{mean}}{V_{max}-V_{min}}\times\frac{V_{max}-V_{min}}{\text{Piston Stroke Volume}}\times\text{Piston Stroke Volume}$$

$$=\frac{1}{\delta_f}\times\frac{V_{max}-V_{min}}{\text{Piston Stroke Volume}}\times\text{Piston Stroke Volume}$$

(11.17)

The volumes of the delivery and suction air vessels are about 40 to 60 times and 20 to 30 times respectively the volume of water entering it per cycle, these proportions increasing with the rotational speed and the length of the respective pipes.

Effect of Connecting Rod Length—In deriving Eqn 11.17, a large length of connecting rod was assumed, such that the movement of the piston was simple harmonic. With a finite length of connecting rod, the value $\dfrac{V_{max}-V_{min}}{\text{Piston Stroke Volume}}$ increases. Table 11.1 shows such variations.

TABLE 11.1

Effect of Connecting Rod Length on $\dfrac{V_{max}-V_{min}}{\textbf{Piston Stroke Volume}}$

$\dfrac{r}{l}$	0	$\frac{1}{5}$	$\frac{1}{4}$	$\frac{1}{3}$
$\dfrac{V_{max}-V_{min}}{\text{Piston Stroke Volume}}$	0 21	0 25	0 27	0 30

Problem 11 6 Determine the size of air vessel for a differential pump if the permissible fluctuation of pressure is 3%.

Solution

The differential pump acts like a single pump on suction stroke (See Art 11.21, Fig 11.22), therefore the suction air vessel should be designed accordingly.

∴ Suction air vessel mean volume

$$=\frac{1}{\delta_f}\times\frac{V_{max}-V_{min}}{\text{Piston Stroke Volume}}\times\text{Piston Stroke Volume}$$

$$=\frac{100}{3}\times 0.55\times\text{Piston Stroke Volume}$$

$=18.3\times$ **Piston Stroke Volume** *Answer*

On delivery side, the differential pump acts like a double acting pump,

∴ Mean volume of delivery air vessel

$$=\frac{100}{3}\times 0{\cdot}21\times\text{Piston Stroke Volume}$$

$=7\times$ **Piston Stroke Volume** *Answer*

11.16 Resonance in Reciprocating Pumps—It is seen that in delivery pipe, connected to the air vessel and running away from it, the time period of free oscillation of water equals the time period of impulse of the pump. Such a phenomenon is known as *resonance*, which if it occurs, gives rise to a very high pressure. The pipe may not be made to withstand such a high pressure and it may burst.

Time period of impulse of pump—If the speed of a pump is n rpm, there are n impulses in a minute for a single-acting pump. Then the time period of the impulse of the pump is $\frac{1}{n}$ minute or $\frac{60}{n}$ sec for a single-acting pump. For a double-acting pump the time period of one impulse is $\frac{30}{n}$ sec and for a triple-acting pump, the time period of impulse is $\frac{20}{n}$ sec.

Time period of free oscillation of water—In Fig 11.20, water column in the delivery pipe is held in equilibrium by the pressure of air in the air

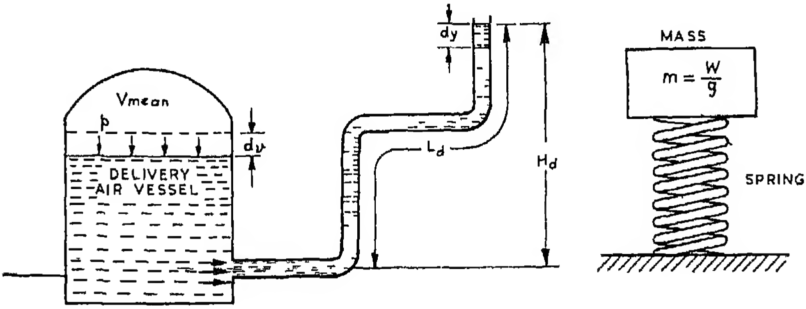


Fig 11.20 Determination of Time-Period of Free Oscillation of Water in Delivery Pipe

vessel, such that it acts like a spring carrying a mass, shown in the Figure. There will be a time period of oscillation of water in the delivery pipe, analogous to that of the mass placed on the spring.

For a mass m supported by a spring of stiffness k lb per ft length, the time period is given by—

$$T=2\pi\sqrt{\frac{m}{k}} \quad \text{seconds} \qquad \ldots(11.18)$$

where $k=\dfrac{\text{Force}}{\text{deflection}}=\dfrac{dP}{dy}$

In an air vessel, the mass moved is the mass of water in the delivery pipe (See Fig 11.20)

$$=\frac{w\,.\,a_d\,.\,L_d}{g}$$

Force acting at the base of this water column $=p\,.\,a_d$

where p=pressure of air at equilibrium position

Let V_{mean}=volume of air in the air vessel at equilibrium position

then $p\,.\,V_{mean}$=constant ...(Boyles' Law)

If water column in the delivery pipe is depressed by dy, the pressure in the air vessel will rise by dp and at the same time the volume is diminished by dV

$\therefore \quad (p+dp)(V_{mean}-dV)=\text{constant}$

or $\quad p\,.\,V_{mean}+dp\,.\,V_{mean}-p\,dV=\text{constant}$

(neglecting $dp\,.\,dV$, being infinitesimal of higher order)

or $\quad dp\,.\,V_{mean}=p\,.\,dV$ ...($p\,.\,V_{mean}$=constant)

or $$\frac{dp}{dV}=\frac{p}{V_{mean}} \qquad \ldots(11.19)$$

Additional force acting on the base of water column due to depression$=a_d\,.\,dp$

$$\therefore \quad \text{Stiffness } k=\frac{a_d\,.\,dp}{dy}=\frac{a_d\,.\,a_d\,.\,dp}{a_d\,.\,dy}=\frac{a_d\,.\,a_d\,.\,dp}{dV}$$

$$=a_d\,.\,\frac{a_d\,.\,p}{V_{mean}} \qquad \ldots(11.20)$$

$\therefore$ time period of free oscillation of water

$$=2\pi\sqrt{\frac{m}{k}}$$

$$=2\pi\sqrt{\frac{\frac{w\,.\,a_d\,.\,L_d}{g}}{}}=2\pi\sqrt{\frac{w\,.\,L_d\,.\,V_{mean}}{}}$$

$\therefore$ To avoid resonance

$$\frac{60}{n} \neq 2\pi \sqrt{\frac{L_d \cdot V_{mean}}{g \cdot a_d \cdot H_d}} \qquad ...(11.22)$$

In case of resonance, the values of either a_d or V_{mean} are altered. From Eqn 11.21, it is seen that time period of free oscillation of water depends on factor $\frac{L_d}{H_d}$ which varies, to a large extent, according to the kind of service, the pump has to perform.

Practical data for $\frac{L_d}{H_d}$

For	Water works pump	$=100$ approx
	Water reservoir pump	$=10$ approx
	Water storage pump	$=1$ approx
	Boiler feed pump	$=\frac{1}{10}$ approx
	Press pump	$=\frac{1}{100}$ approx

If for a pump the value $\frac{L_d}{H_d}$ is large, the time period of free oscillation is also large, which means there is little chance of resonance in water works and reservoir pumps. However there is every likelihood of resonance in the case of the boiler feed and press pumps, therefore, they are not equipped with any air vessel on the delivery side. The air vessels for such pumps are not needed as the length of the delivery pipe will be very small.

That the boiler feed pump as well as the press pump are not provided with air vessels can be explained from the consideration of accelerating head. When no air vessel is fitted, the pressure developed in the pump has also to supply the accelerating head of water arising out of the fluctuation of speed of whole of water column. In case of boiler feed pump or press pump, the accelerating head on the delivery side is *just a fraction of the total delivery head. Hence no air vessel is required* on the delivery side of such pumps.

11.17 Suction in Pump with Air Vessel (Fig 11.18)—When an air vessel is used, suction takes place in two steps. First the water flows through the suction pipe into the air vessel and then it is raised from the air vessel to the cylinder.

Considering the portion between air vessel and cylinder, let H_{s_v} be the static head between the centre line of cylinder and water-level in air vessel, L_{s_v} be the length of the suction pipe between them. Then if H_{v_1} be the pressure head acting on water surface in the air vessel, it must work against the forces resisting opening of the non-return valve, the vapour pressure and frictional resistance to flow of water. Besides,

or $H_{v_1}=H_{s_v}+H_{a_{s_v}}+H_{f_{s_v}}+H_{v_s}+H_{v_{f_s}}+H_{vap}$ ...(11.23)

The acceleration f_s is maximum when piston is at its dead centre *i.e.*, at the beginning of suction stroke when velocity is zero. Then

$$H_{v_1}=H_{s_v}+(H_{a_{s_v}})_{max}+0+H_{v_s}+H_{v_{f_s}}+H_{vap}$$

or $$H_{v_1}=H_{s_v}+\frac{L_{s_v}}{g}\cdot\frac{A}{a_s}\cdot\omega^2\cdot r+H_{v_s}+H_{v_{f_s}}+H_{vap} \quad ...(11.24)$$

Similarly deriving equation for the portion between the sump and the air vessel :

There is no fluctuating head in this portion as the flow is uniform.

$\therefore$ Atmospheric head=static head+head in air vessel+frictional head due to *uniform* flow+kinetic head due to uniform velocity.

or $$H_{atm}=(H_s-H_{s_v})+H_{v_1}+\frac{4fL_s}{d_s}\cdot\frac{v_s^2}{2g}+\frac{v_s^2}{2g} \quad ...(11.25)$$

Therefore adding the two steps (Eqn 11.24 and 11.25), the atmospheric pressure has to work against the following forces :

$$H_{atm}=H_s+\frac{4fL_s}{d_s}\cdot\frac{v_s^2}{2g}+\frac{L_{s_v}}{g}\cdot\frac{A}{a_s}\cdot\omega^2\cdot r+H_{v_s}+H_{v_{f_s}}+H_{vap}+\frac{v_s^2}{2g} \quad ...(11.26)$$

From this equation it is seen that since L_{s_v} has become smaller because of the introduction of an air vessel, H_s can, therefore, be increased. Thus, the pump can be placed at a greater height above the available water surface. This is obviously an advantage in many cases.

$\therefore$ Pressure head on the piston at the beginning of suction stroke

$$=H_s+\frac{4fL_s}{d_s}\cdot\frac{v_s^2}{2g}+\frac{L_{s_v}}{g}\cdot\frac{A}{a_s}\cdot\omega^2\cdot r+H_{v_s}+H_{v_{f_s}}+\frac{v_s^2}{2g} \quad ...(11.27)$$

$$\approx H_s+\frac{4fL_s}{d_s}\cdot\frac{v_s^2}{2g}+\frac{L_{s_v}}{g}\cdot\frac{A}{a_s}\cdot\omega^2\cdot r+\frac{v_s^2}{2g} \text{ (neglecting valve losses)} \quad ...(11.27a)$$

At the middle of suction stroke, $\theta=90°$, which means $\cos\theta=0$ or accelerating head H_{a_a} is also equal to zero. But the velocity of flow at this point is maximum and equal to $\frac{A}{a_s}\cdot\omega\cdot r$, therefore the frictional head loss $H_{f_{s_v}}$ for the length L_{s_v} will have to be considered.

$\therefore$ Pressure head at the middle of suction stroke

$$=H_s+H_{f_s}+H_{f_{s_v}}+H_{K.E}$$

$$=H_s+\frac{4fL_s}{d_s}\cdot\frac{v_s^2}{2g}+\frac{4fL_{s_v}}{d_s}\cdot\left(\frac{A}{a_s}\cdot\omega\cdot r\right)^2+\frac{v_s^2}{2g} \quad ...(11.28)$$

Pressure head at the end of suction stroke

(when $\theta=180°$, $\cos\theta=-1$ *i.e.* H_{a_s} is negative)

$$=H_s+H_{f_s}-(H_{a_s})_{max}+H_K \quad E$$

$$=H_s+\frac{4fL_s}{d_s}\cdot\frac{v_s^2}{2g}-\frac{L_{s_v}}{g}\cdot\frac{A}{a_s}\cdot\omega^2\cdot r+\frac{v_s^2}{2g} \qquad ...(11.29)$$

Problem 11.7 The single acting pump in Problem 11.4 is now equipped with an air vessel on the suction side such that length of the suction pipe from the cylinder to the air vessel is 5 ft and the mean water level in the air vessel is 2 ft below the centre line of the cylinder. Find the speed at which the pump can operate without separation occurring at the beginning of suction stroke. Assume $f=0{\cdot}01$.

Solution

$D=10$ in.	$S=18$ in.	
$d_s=5$ in.	$L_{s_v}=5$ ft	$L_s=35$ ft
$H_s=10$ ft	$H_{s_v}=2$ ft	

Now, the length of water column where the fluctuation of acceleration takes place, has been decreased from 40 ft to 5 ft. In 35 ft length of pipe on suction side the velocity of water is uniform. This velocity is found as follows :

$$v_s=\frac{Q}{a_s}=\frac{A\cdot S\cdot\frac{N}{60}}{\frac{\pi}{4}\times d_s^2}=\frac{\frac{\pi}{4}\times\left(\frac{10}{12}\right)^2\times 1{\cdot}5\times N}{60\times\frac{\pi}{4}\times\left(\frac{5}{12}\right)^2}$$

$$=\frac{100\times 1{\cdot}5\times N}{60\times 25}=0{\cdot}1\ N\ \text{ft/sec}$$

As shown in equation 11.26, the atmospheric pressure has to supply the accelerating head for the length of water column between the cylinder and the air vessel and the pipe frictional losses for the rest of the length below the air vessel, in addition to the static suction head and valve losses etc.

$\therefore$ Equation 11 27 for the flow at the beginning of the suction stroke :

$$H_{atm}=H_s+\left(H_{a_{s_v}}\right)_{max}+H_{sep}+\left[\frac{4fL_s}{d_s}\times\frac{v_s^2}{2g}\right]+\frac{v_s^2}{2g}$$

or
$$34=10+\left[\frac{5}{32{\cdot}2}\times\frac{10^2}{5^2}\times\left(\frac{2\pi N}{60}\right)^2\times\frac{1{\cdot}5}{2}\right]+8$$

$$+\left[\frac{4\times 0{\cdot}01\times 35}{\frac{5}{12}}\times\frac{(0{\cdot}1\ N)^2}{64{\cdot}4}\right]+\frac{(0{\cdot}1\ N)^2}{64\ 4}$$

$$=10+\left[\frac{5\times 4}{32{\cdot}2}\times\frac{4\ \pi^2 N^2}{3{,}600}\times 0{\cdot}75\right]+8$$

$$+\left[\frac{0{\cdot}48\times 35\times 0{\cdot}01\ N^2}{5\times 64{\cdot}4}\right]+\frac{0{\cdot}01\ N^2}{64{\cdot}4}$$

or $34-18=N^2\,(0{\cdot}00514+0{\cdot}000521+0{\cdot}000155)$

or $16=0{\cdot}095816\,N^2$

$\therefore \quad N=\sqrt{\dfrac{16}{0{\cdot}095816}}=\mathbf{52{\cdot}1\ rpm}$ *Answer*

Therefore with the air vessel the speed has been increased from 17·14 to 52·1 rpm.

11.18 Pressures in Cylinder on Delivery Stroke with Air Vessel—Let the length of delivery pipe between cylinder and air vessel be L_{d_v} (See Fig 11.18) and the length of delivery pipe beyond air vessel be L_d. By fitting an air vessel on the delivery side fluctuation of pressure takes place between the cylinder and the air vessel and the flow becomes uniform beyond the air vessel. Therefore the velocity of water will also be uniform in length L_d.

The frictional head due to the uniform flow in delivery pipe of length L_d would be

$$H_{f_d}=\frac{4\,f\,L_d}{d_d}\cdot\frac{v_d^2}{2g}$$

where the uniform velocity v_d in the delivery pipe

$$=\frac{Q}{a_d}=\frac{A\,.\,S\,.\,N}{a_d\,.\,60}\quad\text{(for single acting pump).}$$

Due to the uniform velocity, the above frictional head H_{f_d} would be taken into account when calculating the pressure at the beginning, middle or end of the stroke.

Accelerating head in the delivery pipe between cylinder and air vessel where the fluctuation of pressure takes place:

$$H_{a_d}=\frac{L_{d_v}}{g}\cdot\frac{A}{a_d}\,.\,\omega^2\,.\,r\cos\omega t$$

Pressure head on the piston at the beginning of delivery stroke

$$=H_d+H_{f_d}+H_{a_{d_v}}+H_{K.E.}$$

$$=H_d+\frac{4\,f\,L_d}{d_d}\cdot\frac{v_d^2}{2g}+\frac{L_{d_v}}{g}\cdot\frac{A}{a_d}\,.\,\omega^2\,.\,r+\frac{v_d^2}{2g}\qquad\ldots(11.30)$$

($N.B.$ This equation on delivery side is similar to Eqn 11.27*a* which is derived for suction side.)

This is the pressure when $\theta=0$ or $\cos\theta=1$ *i.e.*, maximum occuring at the beginning of delivery stroke.

At the middle of delivery stroke, $\theta=90°$ which means $\cos\theta=0$ or accelerating head H_{a_d} is also equal to zero. However, the velocity at this point is maximum and is equal to $\dfrac{A}{a_d}\,.\,\omega\,.\,r$, therefore the frictional head loss $H_{f_{d_v}}$ for the length L_{d_v} will have to be considered.

$\therefore$ Pressure head at the middle of stroke

$$=H_d+H_{f_d}+H_{f_{d_v}}+H_{K.E.}$$

$$=H_d+\frac{4fL_d}{d_d}\cdot\frac{v_d^2}{2g}+\frac{4fL_{d_v}}{d_d\,.\,2g}\cdot\left(\frac{A}{a_d}\cdot\omega\cdot r\right)^2+\frac{v_d^2}{2g} \quad ...(11.31)$$

Pressure head at the end of delivery stroke

$$=H_d+H_{f_d}-(H_{a_d})_{max}+H_{K.E.}$$

$$=H_d+\frac{4fL_d}{d_d}\cdot\frac{v_d^2}{2g}-\frac{L_{d_v}}{g}\cdot\frac{A}{a_d}\cdot\omega^2\cdot r+\frac{v_d^2}{2g} \quad ...(11.32)$$

11.19 Theoretical Power Required to Drive the Pump Fitted with Air Vessels on Suction and Delivery Sides—In order to determine the theoretical power required to drive the pump, when it is fitted with air vessels both on suction and delivery sides, the maximum pressure head on the piston must be known first. The maximum pressure occurs at the beginning of suction and delivery strokes, (See Eqn 11.27*a* and 11.30). Adding the heads given by these two equations—

Total head against which the pump has to work

$$=H_s+H_d+\frac{4f\,.\,L_s}{d_s}\cdot\frac{v_s^2}{2g}+\frac{4f\,.\,L_d}{d_d}\cdot\frac{v_d^2}{2g}+\frac{L_{s_v}}{g}\cdot\frac{A}{a_s}\cdot\omega^2\,.\,r$$

$$+\frac{L_{d_v}}{g}\cdot\frac{A}{a_d}\cdot\omega^2\,.\,r+\frac{v_s^2}{2g}+\frac{v_d^2}{2g}$$

If Q cusecs of water are required to be raised, the work done per second by the pump

$$=w\,.\,Q\,.\left(H_s+H_d+\frac{4f\,.\,L_s}{d_s}\cdot\frac{v_s^2}{2g}+\frac{4f\,.\,L_d}{d_d}\cdot\frac{v_d^2}{2g}+\frac{L_{s_v}}{g}\cdot\frac{A}{a_s}\cdot\omega^2\,.\,r\right.$$

$$\left.+\frac{L_{d_v}}{g}\cdot\frac{A}{a_d}\cdot\omega^2\,.\,r+\frac{v_s^2}{2g}+\frac{v_d^2}{2g}\right) \quad ...(11.33)$$

As the lengths of pipes L_{s_v} and L_{d_v} are much smaller than L_s and L_d respectively, the corresponding accelerating heads $\frac{L_{s_v}}{g}\cdot\frac{A}{a_s}\cdot\omega^2\,.\,r$ and $\frac{L_{d_v}}{g}\cdot\frac{A}{a_d}\cdot\omega^2\,.\,r$ can be neglected. Similarly the terms $\frac{v_s^2}{2g}$ and $\frac{v_d^2}{2g}$ are also neglected.

$\therefore$ Theoretical HP required by the pump fitted with air vessels

$$=\frac{\text{Work done per second}}{550}$$

$$=\frac{w\,.\,Q}{550}\cdot\left(H_s+H_d+\frac{4f\,.\,L_s}{d_s}\cdot\frac{v_s^2}{2g}+\frac{4f\,.\,L_d}{d_d}\cdot\frac{v_d^2}{2g}\right) \quad ...(11.33a)$$

Problem 11.8 In a double acting reciprocating pump the plunger diameter is 12 inches (or 304·8 *mm*) and delivery pipe is 6 inches (or 152·4 *mm*) in diameter. The lengths of delivery and suspension pipes are 1,500 ft (or 457 *m*) and 30 ft (or 9·15 *m*) respectively. An air vessel has been fitted near the pump cylinder on the delivery side in order that the velocity of flow in the delivery pipe is constant. The height of the pump above the level of water in the suction sump is 10 ft (or 3·048 *m*). The plunger moves with simple harmonic motion and makes 60 strokes per minute. If the stroke is 16 inches (or 406 *mm*) and the water is raised through a net head of 250 ft (or 76·3 *m*), find the diameter of the suction pipe in order to avoid the separation at the beginning of the suction stroke. The separation commences to appear when the pressure in the cylinder falls to 8 ft (or 2·44 *m*) of water absolute. Compute the HP of the pump neglecting all losses except due to friction in the delivery pipe. Take $f=0·01$.

Solution

Double acting pump $D=12$ in.$=1$ ft (or 304·8 *mm*)

$d_d=6$ in.$=0·5$ ft (or 152 4 *mm*) $L_d=1,500$ ft (or 457 *m*)

$L_s=30$ ft (or 9·15 *m*) $H_s=10$ ft (or 3·048 *m*)

$S=16$ in.$=\frac{16}{12}$ ft (or 406 *mm*) $r=\frac{S}{2}=\frac{8}{12}$ ft (or 203 *mm*)

$N=60$ strokes/min$=30$ rpm $H_{vap}=8$ ft (or 2·44 *m*)

$H=250$ ft (or 76·3 *m*) $f=0·01$

Using equation 11.9

$$H_{atm}=H_s+H_{f_s}+H_{a_s}+H_{vap} \quad \text{...(neglecting valve losses)}$$

or $$34=10+\frac{4f.L_s}{d_s}.\frac{v_s^2}{2g}+\frac{L_s}{g}.\omega^2.r.\frac{A}{a_s}.\cos\theta+8$$

$$\left[\text{or } 10·36=3·048+\frac{4f.L_s}{d_s}.\frac{v_s^2}{2g}+\frac{L_s}{g}.\omega^2.r.\frac{A}{a_s}.\cos\theta+2·44\right]$$

At the beginning of suction stroke—

Here the velocity v_s is zero. $0=0$, $\therefore$ $\cos\theta=1$.

or $$34=10+0+\frac{L_s}{g}.\omega^2.r.\frac{A}{a_s}+8$$

$$\left[10·36=3·048+0+\frac{L_s}{g}.\omega^2.r.\frac{A}{a_s}+2·44\right]$$

or $$\frac{L_s}{g}.\omega^2.r.\frac{A}{a_s}=34-18$$

$$=16$$

$$[\text{or } =10·36-5·488=4·872\ ]$$

or $$\frac{30}{32·2}\times\left(\frac{2\pi\times30}{60}\right)^2\times\frac{8}{12}\times\frac{D^2}{d_s^2}=16 \quad \text{...where } D=1 \text{ and } d_s=?$$

$$\left[\text{or } \frac{9·15}{9·81}\times\left(\frac{2\pi\times30}{60}\right)^2\times0·203\times\frac{D^2}{d_s^2}=4·872\right]$$

$$\therefore \quad d_s=\sqrt{\frac{30\times 4\times \pi^2\times 900\times 8\times 1}{16\times 32\cdot 2\times 3{,}600\times 12}}=0\cdot 618 \text{ ft}$$

$$\left[\text{or} \quad d_s=\sqrt{\frac{9\ 15\times 4\pi^2\times 900\times 0\cdot 203\times 0\cdot 3048}{4\ 872\times 9\ 81\times 3{,}600}}=0\cdot 188 \text{ m}\right]$$

or $d_s=0\ 618\times 12=7\ 42$ in. $\approx 7\frac{1}{2}$ **inches** *Answer*

[or $d_s=0\ 188\times 1{,}000=$ **188 mm**] *Answer*

The quantity of water flowing in the pump

$$Q=\frac{2\ .\ A\ .\ S\ .\ N}{60}=2\times\left(\frac{\pi}{4}\times 1^2\right)\times\frac{16}{12}\times\frac{30}{60}=1\cdot 05 \text{ cfs}$$

$$\left[\text{or} \quad Q=2\times\left(\frac{\pi}{4}\times 0\cdot 3048^2\right)\times 0\ 406\times\frac{30}{60}=0\ 0296 \text{ m}^3/\text{sec}\right]$$

The velocity in the delivery pipe is uniform—

$$\therefore \quad v_d=\frac{Q}{a_d}=\frac{1\ 05}{\frac{\pi}{4}\times\left(\frac{1}{2}\right)^2}=\frac{1\ 05}{0\ 197}=5\cdot 33 \text{ ft/sec}$$

$$\left[\text{or} \quad v_d=\frac{0\cdot 0296}{\frac{\pi}{4}\times 0\cdot 1524^2}=1\cdot 62 \text{ m/sec}\right]$$

As there is air vessel on the delivery side, there is no accelerating head H_{a_d}, but the frictional head on delivery side

$$H_{f_d}=\frac{4f\ .\ L_d}{d_d}\cdot\frac{v_d^2}{2g}$$

$$=\frac{4\times 0\ 01\times 1{,}500}{\frac{1}{2}}\times\frac{(5\cdot 33)^2}{64\cdot 4}=53\cdot 3 \text{ ft of water}$$

$$\left[\text{or} \quad =\frac{4\times 0\ 01\times 457}{0\cdot 1524}\times\frac{1\ 62^2}{2\times 9\ 81}=16\cdot 2 \text{ m of water}\right]$$

Total head delivered

$$=\text{static head}+\text{frictional head losses}+\text{accelerating heads}$$

$$=(H_s+H_d)+(H_{f_s}+H_{f_d})+(H_{a_s}+H_{a_d})$$

$$=250+(0+53\cdot 3)+(16+0)=319\ 3 \text{ ft of water}$$

$$[\text{or} \quad =76\ 3+(0+16\cdot 2)+(4\cdot 872+0)]$$

$$=97\cdot 372 \text{ m of water }]$$

$\therefore$ Horsepower required by the pump

$$\frac{w\ .\ Q\ .\ H}{550}=\frac{62\cdot 4\times 1\cdot 05\times 319\cdot 3}{550}=\mathbf{38\ HP}$$ *Answer*

[or $\quad HP=\frac{1{,}000\times 0.0296\times 97{\cdot}372}{75}=38.8$ **metric HP** *Answer*]

11.20 Work Saved by Air Vessel in Overcoming Pipe Friction—When a pump is equipped with air vessels, the accelerating heads on both the suction and delivery sides are reduced, which results in the saving of some energy. The length of suction pipe being small the work saved by the air vessel, on this side of the pump, in overcoming the frictional head will also be small, and can be neglected. However, the length of the delivery pipe is large, therefore the work saved is considerable, and it is determined as below.

Considering a single acting pump, the work done per second in overcoming pipe friction on delivery side *without* air vessel

$$=w\,.\,Q\,.\,(\tfrac{2}{3}\,H_{f_d})$$

(As the motion is simple harmonic, the effective friction head would be $\frac{2}{3}$ of the total friction head because the curve representing it, is a parabola).

$$=w\,.\,Q\,.\,\frac{2}{3}\,.\,\frac{4f\,.\,L_d}{d_d\,.\,2g}\left(\frac{A}{a_d}\,.\,\omega\,.\,r\right)^2 \qquad \text{...(11.34)}$$

Work done per second in overcoming the pipe friction on delivery side *with* air vessel

$$=w\,.\,Q\,.\,H_{f_d}=w\,.\,Q\,.\,\frac{4f\,.\,L_d}{d_d}\,.\,\frac{v_d^2}{2g}$$

The velocity v_d in the pipe beyond the air vessel is uniform, and is

$$v_d=\frac{A\,.\,S\,.\,N}{60\,.\,a_d}=\frac{A}{a_d}\,.\,2r\,.\,\frac{\omega}{2\pi}$$

Substituting v_d in the above equation, work done per second in overcoming the pipe friction on delivery side *with* air vessel

$$=w\,.\,Q\,.\,\frac{4f\,.\,L_d}{d_d\,.\,2g}\,.\,\left(\frac{A}{a_d}\,.\,2r\,.\,\frac{\omega}{2\pi}\right)^2 \qquad \text{...(11.35)}$$

∴ Work saved by equipping the pump with an air vessel on delivery side

$$=w\,.\,Q\,.\,\frac{4f\,.\,L_d}{d_d\,.\,2g}\,.\,\left(\frac{A}{a_d}\,.\,\omega\,.\,r\right)^2.\left(\frac{2}{3}-\frac{1}{\pi^2}\right) \qquad \text{...(11.36)}$$

Percentage of work saved per second

$$\frac{\left(\frac{2}{3}-\frac{1}{\pi^2}\right)}{}\times 100=\mathbf{84{\cdot}8\%}$$

Similarly it may be shown that the same percentage of work can be saved by fitting another vessel on the suction side in the case of a single acting pump.

Double Acting Pump—Work done per second in overcoming friction in delivery pipe *without* air vessel

$$=w\,.\,Q\,.\,\frac{2}{3}\,.\,\frac{4f\,.\,L_d}{d_d\,.\,2g}\,.\,\left(\frac{A}{a_d}\,.\,\omega\,.\,r\right)^2 \qquad \text{...(11.37)}$$

(same as in case of single acting pump)

Work done per second in overcoming friction in delivery pipe *with* air vessel

$$=w \, . \, Q \, . \, \frac{4f \, . \, L_d}{d_d} \, . \, \frac{v_d^2}{2g}$$

$$=w \quad Q \, . \, \frac{4f \, . \, L_d}{d_d \, . \, 2g} \, . \left(\frac{A}{a_d} \, . \, \omega \, . \, r \, . \, \frac{2}{\pi}\right)^2 \qquad \ldots(11.38)$$

$$\left[\text{as } v_d=\frac{2A \, . \, S \, . \, N}{60 \, . \, a_d}\right]$$

$\therefore$ Percentage of work saved per second

$$\frac{\frac{2}{3}-\frac{4}{\pi^2}}{\frac{2}{3}} \times 103=39{\cdot}2\%$$

Problem 11.9 A single acting piston pump is equipped with an air vessel on the delivery side. The piston moves with a simple harmonic motion. The diameter and stroke of the piston are 1 ft and 2 ft respectively. The delivery pipe is 7 in. in diameter and 200 ft long. Determine the HP saved in overcoming friction in the delivery pipe by the air vessel. The pump runs at 120 rpm. Take $f=0{\cdot}01$.

Solution

$D=1$ ft	$S=2$ ft	$d_d=7$ in.
$L_d=200$ ft	$N=120$ rpm	$f=0{\cdot}01$

Work done per second in overcoming the pipe friction on delivery side without air vessel

$$=w \, . \, Q \, . \, \frac{2}{3} \, . \, H_{f_d}=w \, . \, Q \, . \, \frac{2}{3} \times \frac{4f \, . \, L_d}{d_d \, . \, 2g}\left(\frac{A}{a_d} \, . \, \omega \, . \, r\right)^2$$

(Velocity of water in the delivery pipe without air vessel is proportional to that of the piston)

Work done in overcoming friction in the delivery pipe *with* air vessel

$$=w \, . \, Q \, . \, H_{f_d}=w \, . \, Q \, . \, \frac{4f \, . \, L_d}{d_d} \, . \, \frac{v_d^2}{2g}$$

With air vessel the velocity of water in the delivery pipe is taken to be uniform and equal to $\dfrac{Q}{a_d}=\dfrac{A \, . \, S \, . \, N}{a_d \, . \, 60}$

$$=\frac{A}{a_d} \, . \, 2r \, . \, \frac{\omega}{2\pi}=\frac{A}{a_d} \, . \, \frac{r \, . \, \omega}{\pi}$$

$\therefore$ Work done against friction with air vessel

$$=w \, . \, Q \, . \, \frac{4f \, . \, L_d}{d_d \, . \, 2g} \, . \left(\frac{A}{a_d} \, . \, \frac{r \, . \, \omega}{\pi}\right)^2$$

$\therefore$ Work saved in overcoming friction in the delivery pipe by fitting the air vessel

$$= w \cdot Q \cdot \frac{4f \cdot L_d}{d_d \cdot 2g} \cdot \left(\frac{A}{a_d} \cdot \omega \cdot r\right)^2 \cdot \left(\frac{2}{3} - \frac{1}{\pi^2}\right) \text{ ft lb/sec}$$

$$= 62{\cdot}4 \times \left(\frac{\pi}{4} \times 1^2 \times 2 \times \frac{120}{60}\right) \times \frac{4 \times 0{\cdot}01 \times 200}{\frac{7}{12} \times 64{\cdot}4}$$

$$\times \left(\frac{12^2}{7^2} \times \frac{2\pi \times 120}{60} \times \frac{2}{2}\right)^2 \times \left(\frac{2}{3} - \frac{1}{\pi^2}\right)$$

$$= 56{,}680 \times (0{\cdot}66 - 0{\cdot}1017) = 32{,}300 \text{ ft lb/sec}$$

$\therefore$ HP saved by fitting the air vessel

$$= \frac{32{,}300}{550} = \mathbf{58{\cdot}7 \text{ HP}} \quad \textit{Answer}$$

11.21 Other Types of Reciprocating Pumps—

a) **Direct Acting Steam Pump :** When steam power is available, pistons of the pump and the engine can be directly connected to each other (Fig 11.21). By this arrangement, crankshaft and flywheel can be eliminated. This pump is mainly used for feeding water to boilers. It may be single or double acting depending upon the pressure required.

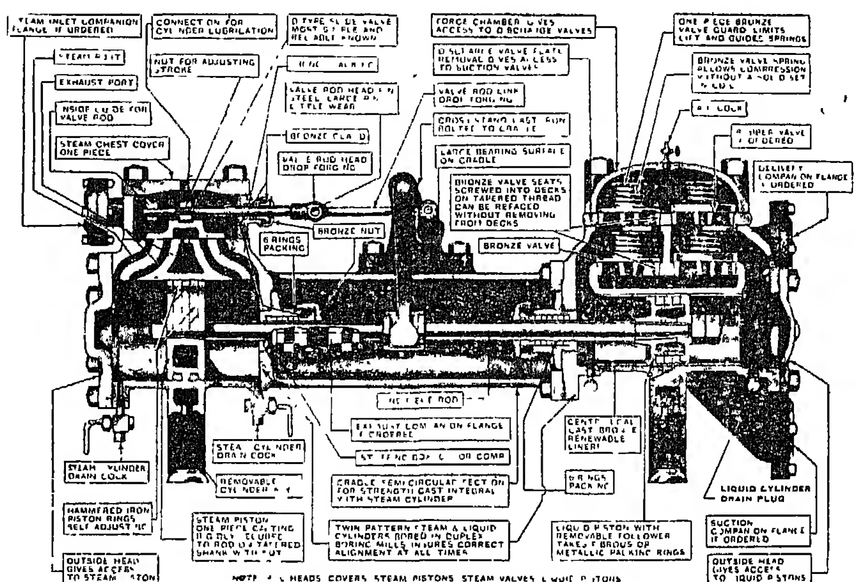


Fig 11.21 Horizontal Duplex Piston Pump, Manufactured by Worthington Simpson Ltd.

Pump Duty—The "duty" of the reciprocating pump driven by a steam engine is the No. of ft lb of work given out by it for every 1,000,000 BTU supplied to the engine by the boiler.

$\therefore$ Pump Duty

$$= \frac{w \cdot Q \cdot H \times 1{,}000{,}000}{\text{Wt. of steam used/sec} \times \text{Total heat of 1 lb of steam supplied}} \qquad \ldots(11.39)$$

This definition is applied only in FPS units.

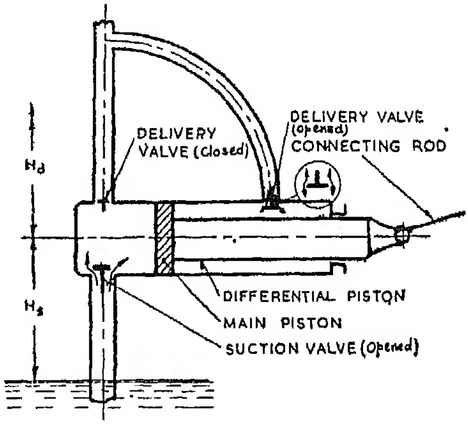


Fig 11.22 Differential Pump

b) **Differential Pump :** Single acting piston pump is equipped with a pipe bend connecting the piston rod-end of the cylinder to the delivery pipe (Fig 11.22). Cross-sectional area of piston-rod is generally made half that of the piston itself. The thick piston-rod and the actual piston are known as differential and main pistons respectively.

When the piston moves forward in the delivery stroke only half of the water in the cylinder is delivered, the rest being transferred to the other side of the piston rod. This water is delivered when the piston moves backward in the suction stroke. Thus delivery takes place at a more uniform rate.

11.22 Design of Valves for Reciprocating Pumps—The main dimension of the valve to be designed is its lift. The opening resistance of the valve should also be determined which is very important in case of suction valve, because if the opening resistance is more, the suction valve may not open.

Let Piston area $= a_p$

$\therefore$ Piston velocity $= \omega \cdot r \cdot \sin\theta = v_p \cdot \sin\theta$

Water pushed forward by the piston $= a_p \cdot v_p \cdot \sin\theta$

This water comes out of the opening shown in Fig 11.23. The valve being raised, the water discharges out.

Let area of opening $= a_1 = \frac{\pi}{4} d_1^2$

Velocity of water in the opening $= v_1$

$\therefore$ Discharge through the opening $= a_1 \cdot v_1$

and discharge through the valve $= C_d \cdot \pi \cdot d \cdot h \cdot v$

where $C_d =$ Co-efficient of discharge

$d =$ diameter of valve

$h =$ valve lift or valve displacement (See Fig. 11.25)

$v =$ velocity of water discharging through the valve.

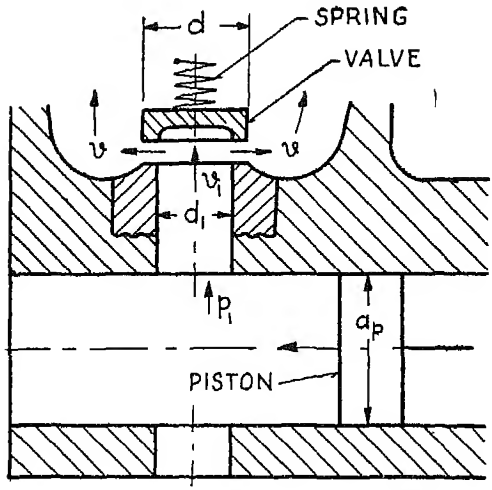


Fig 11 23 Valve of Reciprocating Pump, Being Opened

Water pushed forward by the piston=water coming out of opening=water discharging out of the valve.

or $a_p \cdot v_p \cdot \sin\theta = a_1 \cdot v_1 = C_d \cdot \pi \cdot d \cdot h \cdot v$ .. (11.40)

or $$h = \frac{a_p \cdot v_p \cdot \sin\theta}{C_d \cdot \pi \cdot d \cdot v}$$ ...(11.41)

Put $\dfrac{a_p \cdot v_p}{C_d \cdot \pi \cdot d \cdot v} = k$

Then valve displacement $h = k \cdot \sin\theta$ ...(11.42)

Valve velocity$=\dfrac{dh}{dt}=k \cdot \omega \cdot \cos\theta$ ...(11.43)

and valve acceleration$=\dfrac{d^2h}{dt^2}=-k \cdot \omega^2 \cdot \sin\theta$ ...(11.44)

Tabulating the values of displacement, velocity and acceleration of valve and piston—

	Displacement	Velocity	Acceleration
Valve	$k \cdot \sin\theta$	$k \cdot \omega \cdot \cos\theta$	$-k \cdot \omega^2 \cdot \sin\theta$
Piston	$r\,(1-\cos\theta)$	$r \cdot \omega \cdot \sin\theta$	$r \cdot \omega^2 \cos\theta$

It is seen (See Fig 11.24) that velocities of valve and piston differ by a phase angle of 90°, *i.e.*,

when $\theta=0$, velocity of valve is max

and velocity of piston is zero.

In practice whole of the water pushed forward by the piston is not delivered through the valve when the valve is moving upward as there will always be some space beneath it. The water delivered by the piston will fill up this space first and the remaining water goes out of the valve periphery.

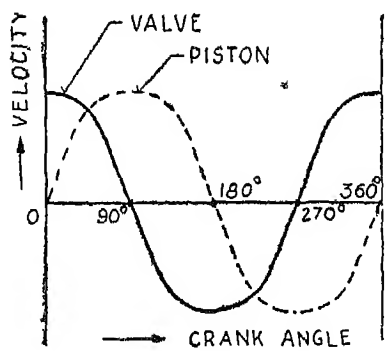


Fig 11 24 Velocity of Valve and Piston vs Crank Angle

Let v' = valve velocity,

then quantity of water beneath the valve

$$=a \,.\, v'=\frac{\pi}{4}\, d^2 \,.\, v'$$

Hence $a_p \,.\, v_p \sin\theta=a \,.\, v'+C_d \,.\, \pi \,.\, d \,.\, h \,.\, v$ .. (11.45)

This equation holds good when the valve is being lifted. When the valve is being lowered, the discharge equation will be

$$C_d \,.\, \pi \,.\, d \,.\, h \,.\, v=a_p \,.\, v_p \,.\, \sin\theta+a \,.\, v' \qquad \text{...(11.46)}$$

Both the discharge equations can be written as—

$$C_d \,.\, \pi \,.\, d \,.\, h \,.\, v=a_p \,.\, v_p \,.\, \sin\theta \pm a \,.\, v' \qquad \text{...(11.47)}$$

$+ve$ sign is for the valve being raised,

and $-ve$ sign is for the valve being lowered.

This is known as *Westphal's Law*.

or $$h=\frac{a_p \,.\, v_p \,.\, \sin\theta}{C_d \,.\, \pi \,.\, d \,.\, v} \pm \frac{a \,.\, v'}{C_d \,.\, \pi \,.\, d \,.\, v} \qquad \text{...(11.48)}$$

Now $$v'=\frac{dh}{dt}=k \,.\, \omega \,.\, \cos\theta \qquad \text{.. (See Eqn 11.43)}$$

$$\therefore \quad h=\frac{a_p \,.\, v_p \,.\, \sin\theta}{C_d \,.\, \pi \,.\, d \,.\, v} \pm \frac{a \,.\, k \,.\, \omega \,.\, \cos\theta}{C_d \,.\, \pi \,.\, d \,.\, v}$$

$$=\frac{a_p \,.\, \omega \,.\, r \sin\theta}{Cd \,.\, \pi \,.\, d \,.\, v} \pm \frac{a \,.\, \left(\frac{a_p \,.\, \omega \,.\, r}{C_d \,.\, \pi \,.\, d \,.\, v}\right) \,.\, \omega \,.\, \cos\theta}{C_d \,.\, \pi \,.\, d \,.\, v}$$

$$=\frac{a_p \,.\, \omega \,.\, r}{C_d \,.\, \pi \,.\, d \,.\, v}\left(\sin\theta \pm \frac{a \,.\, \omega \,.\, \cos\theta}{C_d \,.\, \pi \,.\, d \,.\, v}\right) \qquad \text{...(11.48a)}$$

and $$v=\frac{dh}{dt}=\frac{a_p \,.\, \omega^2 \,.\, r}{C_d \,.\, \pi \,.\, d \,.\, v}\left(\cos\theta \mp \frac{a \,.\, \omega \,.\, \sin\theta}{C_d \,.\, \pi \,.\, d \,.\, v}\right) \qquad \text{.. (11.49)}$$

From Eqn 11.48*a*, it is clear that when $\theta=0$, h is not equal to zero. Let this value of displacement be h_0

then $$h_0=\frac{a_p \,.\, \omega \,.\, r}{C_d \,.\, \pi \,.\, d \,.\, v}\left(0 \pm \frac{a \,.\, \omega}{C_d \,.\, \pi \,.\, d \,.\, v}\right)$$

$$=\frac{a_p \,.\, a \,.\, \omega^2 \,.\, r}{(C_d \,.\, \pi \,.\, d \,.\, v)^2} \qquad \text{...(11.50)}$$

Thus this amount of valve remains open, when the piston is at its dead centre. Now the piston starts its suction stroke, with which the delivery valve is sucked back on its seat. This gives rise to a phenomenon known as *Pounding of Valve*, which should never be allowed to occur. In order to avoid pounding of valve, the valve must be closed before it is sucked by the piston on the return stroke. It is possible by providing the valve with a spring. The stiffness of the spring depends upon the pump speed. The greater the speed of the pump, the stiffer the spring will be.

For the **suction valve,** which opens by the atmospheric pressure the stiffness of the spring should not be such that it may not open.

Practical Data :

$$h_0 = \frac{1}{250}\, d$$

$$N \,.\, h_{max} \approx 16$$

where d = diameter of valve

N = pump speed in rpm

h_{max} = maximum valve lift

Referring to Fig 11.23

$$\frac{\pi}{4}\, d_1^2 \,.\, v_1 = C_d \,.\, \pi \,.\, d \,.\, h \,.\, v$$

Let $d_1 = d$

then $h = \frac{d}{4} \times \frac{v_1}{v}$ (approx)

Valve Seats (See Fig 11.25) are either flat or conical. The flat seats are preferred. Disadvantages of conical seats are as follow—

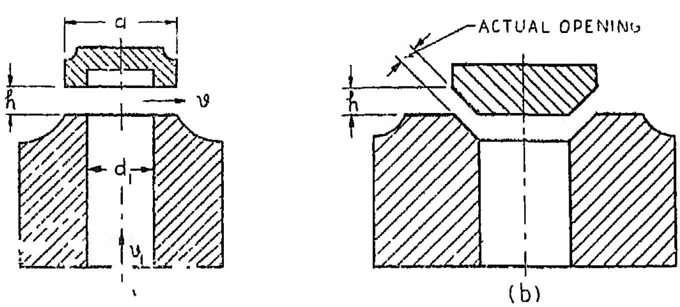


Fig 11.25 Types of Valve Seats

a) Flat Valve Seat *b*) Conical Valve Seat

1) The actual opening is less than lift

2) The valve guide must be loose, because it is difficult to attain the coincidence of axes of the valve and the seat.

Opening Resistance of Valve—Before the valve begins to open, some resistance is encountered. This is known as opening resistance of valve. On the top of delivery valve, the pressure acting is p lb/sq in (or

kg/cm²) which is due to delivery head H_d. In addition to this pressure, there is spring load 5 lb (or 2.27 kg). Let the pressure below the delivery valve be p_1 (See Fig 11.23)

Then force trying to open the valve $= a_1 \cdot p_1$

Opposing force $= a \cdot p + s + m \cdot f_1 + w'$

where $m =$ mass of valve $= \dfrac{w}{g}$

$w =$ weight of valve

$w' =$ weight of valve in water

$f_1 =$ acceleration of valve

Force trying to open the valve = opposing force

i.e. $a_1 \cdot p_1 = a \cdot p + s + m f_1 + w'$

or $a_1 \cdot p_1 - a_1 \cdot p = a \cdot p - a_1 \cdot p + s + m f_1 + w'$

(substracting $a_1 \cdot p$ from both sides)

or $a_1 \cdot (p_1 - p) = (a - a_1)\, p + s + m \cdot f_1 + w'$

or $$p_1 - p = \frac{a - a_1}{a_1}\, p + \frac{s + w'}{a_1} + \frac{m f_1}{a_1}$$

or $$\frac{p_1 - p}{w} = \frac{a - a_1}{a_1} \cdot \frac{p}{w} + \frac{s + w'}{a_1 \cdot w} + \frac{m \cdot f_1}{a_1 \cdot w} \qquad \ldots(11.51)$$

$\dfrac{p_1 - p}{w} =$ opening resistance of valve, which is very important to calculate in case of suction valve. Ordinarily it should not be more than 8 ft (or 2.44 m) of water absolute, otherwise the valve may not open.

$\dfrac{s + w'}{a_1 \cdot w} = 1{\cdot}3$ to 4 ft (or 0·4 to 1·2 m) of water

Problem 11.10 A single acting, single cylinder reciprocating pump delivers water to a height of 100 ft. The crank, having a length of 6 in. makes 40 rpm. The diameter of the pump cylinder is 6 in. The delivery valve has a diameter of $4\frac{1}{2}$ in. and the opening in the cylinder to the delivery valve has a diameter of 4 in. Assume simple harmonic motion for the piston. Determine the opening resistance of the delivery valve.

Take $\dfrac{s + w'}{a_1 \cdot w} = 2{\cdot}5$ ft of water

Solution

$H_d = 100 \text{ ft} = \dfrac{p}{w}$, $\quad r = 0{\cdot}5$ ft

$N = 40$ rpm $\quad D = 6$ in.

$d = 4{\cdot}5$ in. $\quad d_1 = 4$ in.

Maximum acceleration of piston, $f_p=\omega^2 . r . \left(1+\frac{1}{n}\right)$

(assuming n as infinity)

$$f_p=\omega^2 . r=\left(\frac{2\pi\times 40}{60}\right)^2\times 0\cdot 5=8\cdot 75 \text{ ft/sec}^2$$

Acceleration of water through the delivery opening

$$f_1=\frac{a_p}{a_1} f_p=\frac{D^2}{d_1^2} . f_p=\frac{6^2}{4^2}\times 8\cdot 75=19\ 7 \text{ ft/sec}^2$$

$$a_1=\frac{\pi}{4}d_1^2=\frac{\pi}{4}\times\left(\frac{4}{12}\right)^2=0\ 0872 \text{ sq ft}$$

Now $\quad \dfrac{s+w'}{a_1 . w}=2\cdot 5$ ft of water

$\therefore \quad s+w'=2\ 5\times a_1 . w$

$=2\cdot 5\times 0\ 0872\times 62\cdot 4$

$=13\ 6$ lb

Out of this value, take w' less than 5 lb and $w=6$ lb

$\therefore$ Opening resistance of valve

$$\frac{p_1-p}{w}=\frac{a-a_1}{a_1}\cdot\frac{p}{w}+\frac{s+w'}{a_1 . w}+\frac{m . f_1}{a_1 . w}$$

$$=\frac{4\ 5^2-4^2}{4^2}\times 100+2\cdot 5+\frac{6}{32\ 2}\times\frac{19\cdot 7}{0\ 0872\times 62\cdot 4}$$

$=26\ 5+2\ 5+0\cdot 674$

$=29\ 674$ ft of water

$\approx$ **30 ft of water** *Answer*

UNSOLVED PROBLEMS

11.1 Define pump. Explain how the liquid is raised by a pump.

11 2 What is "suction head" and what is "delivery head" as applied to pumps ?

11 3 How would you classify the pumps ?

11.4 Explain working principle of a reciprocating pump with the help of a line sketch, naming all the main parts.

11.5 Explain the difference between single-acting and double-acting reciprocating pumps with the help of line diagrams.

11.6 What is a plunger pump ? Where would you recommend to use the same ?

11.7 Explain the working principle of bucket pump. Where is it employed ?

11.8 Define slip, negative slip and co-efficient of discharge of a reciprocating pump.

11.9 Explain with the help of delivery curves, how the resultant rate of discharge can be made uniform in different types of reciprocating pumps.

11.10 What is a three-throw pump? Explain its working.

11.11 Why is the suction height of a pump limited? On what factors does it depend?

11.12 Define "separation" in a reciprocating pump

11.13 Explain how separation of flow is caused in piston pumps. What preventive measures are usually taken to reduce the same appreciably? *(Roorkee University—1958)*

11.14 Why is the speed of a reciprocating pump lower than that of centrifugal type? On what factors does the speed of the reciprocating pump depend? *(AMIE—May 1955)*

11.15 Describe with the help of indicator diagrams how the acceleration and friction in suction and delivery pipes effect the work done by a reciprocating pump.

11.16 Explain the use of air vessels on suction and delivery pipes of a reciprocating pump. *(AMIE—Nov 1953)*

11.17 Explain the functions and working of air vessels on—

a) the suction side;

b) the delivery side of a reciprocating pump.

11.18 What is the volume of air vessels fitted on suction and on delivery pipes?

11.19 What is "resonance" in reciprocating pumps? How is it avoided?

11.20 Explain the working of a direct-acting steam pump. Where is it employed? Define its "duty".

11.21 Explain the principle of working of a differential pump Explain how the rate of discharge can be made uniform with the help of such a pump.

11.22 In a reciprocating pump the velocity of water in suction pipe during the suction stroke varies between zero and v, the displacement being SHM. Prove that the mean friction head through the stroke is $\frac{4}{3}\frac{f\,l\,v^2}{g\,D}$ where l, D, f are length, diameter and frictional co-efficient of pipe respectively. *(UPSC—Dec 1953)*

11.23 Determine the HP required to drive a double acting piston pump when piston diameter is 8 in, piston stroke 16 in. and rpm of crank 25. The suction and delivery heights are respectively 15 ft and 80 ft. Assume suitable values for different efficiencies.
(9 27 HP, assuming efficiencies as 0 6 and 0·75 for suction and delivery sides) *(Jadavpur University—1956A)*

11.24 In a bucket pump, the diameters of the bucket and the pump-rod are 12 inches and $2\frac{1}{2}$ inches respectively. The pump stroke is 3 ft. The pump works under a mean suction head of 20 ft and a mean delivery head of 40 ft of water. Taking into account the influence of the pump-rod, determine the pull on the rod during the up stroke. Also calculate the volume of water discharged during (*a*) one up stroke and (*b*) one down stroke of the pump.
[2,850 lb; (*a*) 2 25 cu ft (*b*) 0·102 cu ft]

11.25 In a single acting pump the cylinder has a diameter of 6 in. and a stroke of 12 in. The water is to be raised to a height of 60 ft when

the pump is running at 40 rpm. Determine the theoretical discharge and theoretical horsepower.

If the actual discharge of the pump is 47 gpm, find the co-efficient of discharge and percentage slip.

(48·9 gpm, 0 89 HP, 0·962, 3·88%)

11.26 A single acting reciprocating pump has a cylinder bore of 6 in. and stroke of 9 inches. The suction pipe is $4\frac{1}{2}$ in. in diameter and 60 ft long. Assuming no air vessel is fitted on the suction side of pump, find the maximum permissible suction lift if the pump speed is 30 rpm.

(13 75 ft) (*Delhi University—1958*)

11.27 A single acting pump having plunger diameter 8 in., stroke 12 in., is placed with its centre line 12 ft above the level of water in the suction tank. The suction pipe is 3 in. in diameter and 15 ft long. If separation occurs when the absolute pressure head is 8 ft of water, find the maximum speed of the pump to avoid separation at the commencement of the suction stroke. Assume normal barometric pressure and simple harmonic motion of the plunger.

(27·8 rpm) (*AMI Mech E—April, 1955*)

11.28 The diameter of a plunger and stroke of a reciprocating pump are one ft and two ft respectively. On the suction side the pump is connected to a pipe 9 in. in diameter and 80 ft long. The suction lift is 14 ft. Determine the maximum speed at which it can operate without separation occuring at the beginning of the stroke. Assume the motion as SHM and take the effective height of the barometer as 28 ft of water. (12·36 rpm)

11.29 A single acting horizontal reciprocating plunger pump has a suction pipe $2\frac{1}{2}$ in. diameter and 18 ft long. The sump is 7 ft below the cylinder axis, and the pump plunger is 6 in. diameter and of 10 in. stroke. If the pump is working at 25 rpm, find the absolute pressure head in lb/sq in. at the beginning and at the end of the suction stroke. If separation occurs at 11 lb/sq in. below atmospheric pressure state whether the separation would occur at the beginning or at the end of the suction stroke. Take $f=0{\cdot}01$.

(7·7 lb/sq in. absolute ; 15·7 lb/sq in., no separation at all)

(*Rajputana University—1957*)

11.30 A single-acting piston pump with no air chamber has a piston diameter of 9 in., stroke 18 in. and a connecting rod 3 ft long. The suction head is 15 ft, diameter of suction pipe 6 in. and its length is 32 ft. If the opening resistance of the valve is equivalent to 4 ft of water and the pump is employed to pump water at 77°*F*, what would be the maximum permissble speed without separation ? The vapour pressure at 77°*F* may be taken as 0·5 psi.

(24·6 rpm) (*Roorkee University—1958*)

11.31 It is desired to obtain a discharge of 45 liters per sec from a two cylinder plunger pump, the delivery running through a pipe 25 cm diameter and 150 *m* long. The constant static head in the flow is 96 *m*. No air vessel is provided. If the length of the stroke is twice the plunger diameter, what should be the diameter?

What would be the maximum pressure in the pipe neglecting the elasticity of water and pipe walls? The pump is to run at 42 rpm. (27·35 cm ; 202·85 m of water) (*Bombay University—1957*)

11.32 A plunger is fitted in a vertical pipe full of water. Its lower end is submerged in a suction sump. The plunger is drawn up with an acceleration of 6 ft/sec^2. Find out the maximum height above the level in the sump at which plunger can work without separation. Separation occurs at a pressure of 8 ft of water absolute. Barometer height of water may be taken equal to 33·75 ft. (21·4 ft) (*Madras University—1956*)

11.33 Explain the term separation or cavitation as applied to reciprocating pumps.

The diameter of a plunger of reciprocating pump is 8 in. and stroke 12 in. Pump is to be driven with SHM at 60 rpm and it draws water from a sump 12 ft below pump centre line. Find the least dia of the suction pipe which is 15 ft long in order to prevent separation at this speed.

Assume normal barometric pressure and that separation commences when the absolute head becomes 8 ft of water.

(6½ in) (*UPSC—1954*)

11.34 How is negative slip caused in reciprocating pump? A double acting reciprocating pump lifts water through a net head of 250 ft. The centre of the cylinder is placed at 10 ft above the level of water in the sump and the length of suction pipe is 30 ft. The delivery pipe is 6 in. dia, 1,500 ft long and is fitted with an air vessel of sufficient capacity, near the pump to keep the velocity of flow in the pipe constant. The pump plunger is 12 in. dia and moves with SHM making 60 strokes per min of length 16 in. Determine the dia of the suction pipe so that no separation takes place at the beginning of the suction assuming it to occur at pressure head of 8 ft absolute of water. Neglecting all losses except due to friction in delivery pipe, determine the HP required to work the pump. f=0 01. (7½ in., 36 HP)

11.35 A single acting reciprocating pump has a plunger of diameter 10 in. and stroke 18 in., the delivery pipe is to be 4 in. diameter and water is lifted to a tank whose level is 50 ft above the pump and 100 ft horizontally from it. If sepatation takes place at 8 ft of water absolute, find the speed of the pump in rpm at which separation would occur in the delivery pipe if no air vessel was fitted for the cases :—

a) If the pipe was vertical from pump and then horizontal upto the tank.

b) If the pipe ran horizontally from the pump and then vertically to tank.

Assume normal barometric pressure and SHM of the plunger.

[(*a*) 12·77 rpm, (*b*) 17·8 rpm] (*AMI Mech E—Oct 1954*)

11.36 A double-acting single cylinder reciprocating pump of 7½ in. bore and 15 in. stroke runs at 36 double strokes per minute, suction head 12 ft and discharge head 100 ft. The length of the suction pipe is 30 ft and of the discharge pipe is 200 ft and the diameter of each pipe is 4 inches. Large air vessels are provided 10 ft away

from the pump on the suction side and 20 ft away on the discharge side, both measured along the pipe lines. $f = 0{\cdot}008$. Neglecting entrance and exit losses for the pipes, estimate for the beginning of the stroke—

a) Heads in the two ends of the cylinder,

b) Load on the piston rod neglecting the size of the piston rod and assuming simple harmonic motion.

[(*a*) 11·042 ft of water absolute ; 161·28 ft of water absolute, (*b*) 2,875 lb]
(Punjab University—1958A)

11.37 In a double-acting reciprocating pump the suction lift is 12 ft, length of suction 20 ft, diameter of suction pipe 4 in., diameter of pump plunger 6 in., length of stroke 18 in., head lost in friction in suction pipe is 0·025 $\frac{}{2g\,d}$ A very large air vessel is fitted on the suction side close to the pump. Assuming simple harmonic motion, height of water barometer 32 ft and separation 6 ft, find the maximum safe speed of the pump.
(168·5 rpm) *(Punjab University—1959A)*

11.38 A single acting reciprocating pump has a plunger 15 in. diameter and stroke 2 ft. The delivery pipe is 6 in. diameter and 300 ft long, the frictional co-efficient being 0·008. Find the HP lost in friction in the delivery pipe when the pump runs at 50 rpm ;

a) when a large air vessel is fitted near the pump outlet,

b) assuming simple harmonic motion of the plunger and no air vessel.

c) Find the HP saved by installing the air vessel.
[(*a*) 7·52 HP ; (*b*) 49·2 HP ; (*c*) 41·68 HP]
(AMI Mech E—Oct 1955)

11.39 Determine the crank angle at which the valve of piston pump actually closes. Dimensions of the pump and valve are as follows—

Piston diameter 6 in., stroke 12 in., valve diameter 4 in., valve lift (max.) $\frac{1}{2}$ in., rpm of crank driving piston is 30. Assume contraction co-efficient of the water stream coming out of the valve slit as 0·64 and valve seat flat.

Prove any formula used.

(Jadavpur University—1958)

11.40 Determine the size of an air chamber for a double-acting piston pump of piston diameter 6 inches and stroke 12 inches when piston is driven from a crank making 35 rpm. The pressure variation should not exceed 3 percent. Test if resonance takes place or not when the delivery pipe is 100 ft long and 4 inches diameter, the delivery head being 60 ft.

(Jadavpur University—1956)

11.41 A differential pump running at 40 rpm has to deliver 6,000 gpm. Determine the diameters of the main and the differential pistons if the stroke is twice the diameter of the main piston.

What size of air chamber will be needed on the suction and on the delivery sides if the fluctuations of pressure are to be within 3% ? ($7\frac{3}{5}$ in. ; $3\frac{13}{16}$ in. ; 7·36 cu ft) (*Jadavpur University—1955*)

11.42 What factors tend to limit the speed of a reciprocating pump ?

The following data refer to a reciprocating double acting force pump :

Diameter of plunger	= 101·6 *mm*
Delivery	= 3,640 litres/hr
Suction pipe	= 50·8 *mm* and 9·15 *m* long
Suction head	= 5·5 *m*

Determine the rpm of the pump if the cavitation occurs at an absolute pressure of 2·44 *m* of water.

(32 rpm) (*AICTE—1958* ; Converted to metric units)

11.43 A plunger is fitted in a vertical pipe which is full of water, and whose lower end is submerged in a suction tank. It is moved upwards with an acceleration of 1·524 m/sec^2. If air is liberated from the water when the absolute pressure falls below 1·22 *m* of water, and if the barometric height is 9·75 *m* of water, what is the maximum height above the level in the suction tank at which the plunger can operate without cavitation ?

(7·35 *m*) (*Punjab University—1959 S* ; Converted to metric units)

11.44 A three throw pump having rams 304·8 *mm* diameter by 609·6 *mm* stroke is required to lift 5,000 lit/min against a static head of 116 *m*. The friction loss in the suction pipe is estimated at 1·22 *m* and in the delivery pipe at 17·1 *m*. The pipe velocity is 0·915 *m*/sec. The overall efficiency of the pump is 90% and the slip is 2%.

Calculate the speed at which the pump should run and the metric HP required to drive it.

(38·2 rpm ; 165·5 metric HP) (*Punjab University—1960 S* ; Converted to metric units)

11.45 A single cylinder, single acting reciprocating pump of 228·6 *mm* and 381 *mm* stroke runs at 18 rpm under suction and delivery heads of 3·048 *m* and 1·524 *m* respectively. The lengths of suction and delivery pipes are 15·25 *m* and 304·8 *m* respectively. Find the diameter of suction pipe if the diameter of delivery pipe is 152·4 *mm*. There is no air vessel in the suction pipe. Assume the minimum pressure in the suction pipe as 7·94 *m* of water below atmosphere. Find the theoretical HP required to drive the pump. Take $f = 0·008$.

(106 *mm*; 0·51 metric HP) (*BHU—1960* ; Converted to metric units)

CHAPTER 12

CENTRIFUGAL PUMPS

Theory of Centrifugal Pump

12.1 Comparsion with Reciprocating Pumps—During the last 35 years the centrifugal pump has been rapidly superseding the reciprocating and other types of pumping equipment. Today it is the most popular pump for most of the jobs. The advantages of centrifugal pump over the reciprocating type can be summarised as here-under :

Centrifugal Pump	*Reciprocating Pump*
i) Smooth and even flow.	Pulsating flow.
ii) Low initial cost.	Initial cost as high as four times that of centrifugal pump.
iii) Compact, occupies less floor space. Vertical type requires even lesser space.	Occupies 6 to 8 times the space required for horizontal centrifugal pump.
iv) Gross weight is small.	Gross weight is considerable.
v) Installation is easy.	Installation is more difficult than centrifugal pump.
vi) Efficiency of low head pumps is high.	Efficiency of low head pumps may be as low as 40% chiefly because of relatively higher energy losses *viz.* frictional, valve losses etc.
vii) Construction is simplified by elimination of many parts such as non-return valves, glands, air vessel etc. Therefore less number of spare parts are required.	Complicated construction. Therefore a number of spare parts is necessary.
viii) Low maintenance cost. Periodical check up is sufficient.	Maintenance charges are high because parts like valves require constant attention.
ix) High speed. Can be coupled directly through flanged coupling to electric motors or steam turbines.	Low speed due to separation difficulties. Belt drive indispensable.
x) Uniform torque.	Torque not uniform.
xi) Easy handling of highly viscous fluids such as oils, muddy and sewage water, paper pulp, sugar mollasses, chemicals etc.	Valves and glands cause trouble when required to transmit viscous fluids.

The only advantage of the reciprocating pump over the centrifugal type seems to be that the former can build up very high pressures sometimes up to 10,000 lb/sq in. (or 700 K_g/c^2m).

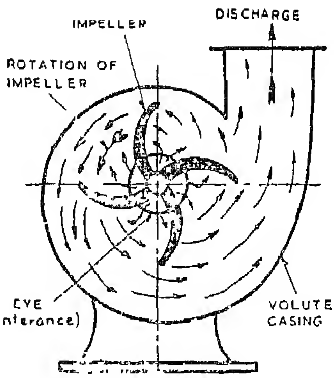


Fig 12.1 Centrifugal Pump Principle

12 2 Definitions—The centrifugal pump is a contrivance to raise liquids from a lower to a higher level by creating the required pressure with the help of centrifugal action. In general it can be defined as a machine which increases the pressure energy of a fluid, as a pump may not be used to lift water at all, but just to boost the pressure in a pipe line. Whirling motion is imparted to the liquid by means of backward curved blades mounted on a wheel known as the *impeller*. As already explained in Art 2.3, liquid enters the impeller at its centre technically known as the *eye* of the pump and discharges into the casing surrounding the impeller (See Fig 12.1). The pressure head developed by centrifugal action is entirely due to the velocity imparted to the liquid by the rotating impeller and not due to any displacement or impact.

The pump is usually named after the type of its casing (volute or diffusion). Diffusion casing is equipped with vanes and its design is adopted from the Francis turbine. The pump provided with it, is therefore called a *turbine pump.* Vertical turbine pumps which are particularly suited for pumping water from deep wells are often called *deepwell pumps.*

12.3 Principle and Operation—The basic principle on which a centrifugal pump functions has been explained earlier (cf. Art 2.3). The first step in the operation of a pump is *priming* that is, the suction pipe and casing are filled with water so that no airpocket is left. Now the revolution of the pump impeller inside a casing full of water produces a forced vortex which is responsible for imparting a centrifugal head to the water. Rotation of impeller effects a reduction of pressure at the centre. This causes the water in the suction pipe to rush into the eye. The speed of the pump should be high enough to produce centrifugal head sufficient to initiate discharge against the delivery head.

Mechanical action of the pump is to impart a velocity to the water. A water particle with a given velocity will rise to the same vertical height through which any particle should fall freely under gravity in order to attain the same velocity starting from rest. The required relation therefore is :

$$v=\sqrt{2gH}$$

or $$H=\frac{v^2}{2g}$$

Thus if the outlet velocity of water in a pump is v, the pump can theoretically deliver against a head of $\frac{v^2}{2g}$.

12.4 Classification of Centrifugal Pumps—Centrifugal pumps possess the following characteristic features on the basis of which they can be classified.

i) Working head,
ii) Type of casing,
iii) Number of impellers per shaft,
iv) Relative direction of flow through impeller,
v) Number of entrances to the impeller,
vi) Disposition of shaft,
vii) Liquid handled,
viii) Specific speed and
ix) Non-dimensional factor K_s.

First is a commercial classification of pumps from the point of view of their utility but (*ii*) to (*vii*) are all practical considerations each governing an important constructional feature of the pump. The last two are purely theoretical aspects and provide probably the soundest basis for absolute classification.

12.5 Working Head—It is the head at which water is delivered by the pump.

According to the range of working head, pumps may be divided broadly in three categories.

a) **Low Lift Centrifugal Pumps** are meant to work against heads upto 50 ft (or 15 *m*). Impeller is surrounded by a volute and there are no guide vanes. The shaft is generally horizontal and water may enter the impeller from one or both sides depending upon the quantity of water to be delivered.

b) **Medium Lift Centrifugal Pumps** are used to build up heads as high as 130 ft (or 40 *m*). They are generally provided with guide vanes. Water may enter from one or both sides depending on the quantity to be pumped.

c) **High Lift Centrifugal Pumps** are employed to deliver liquids at heads above 130 ft (or 40 *m*). High lift pumps are generally multi-stage pumps because a single impeller cannot easily build up such a high pressure. They may be horizontal or vertical, the latter being used in deep wells.

12.6 Type of Casing—Pump casing should be so designed as to minimise the loss of kinetic head through eddy-formation etc. Efficiency of the pump largely depends on the type of casing. In general, the casings are of two types and the pump is named after the casing it uses.

a) **Volute Pump**—(See Fig 12.2, 12.3 and 12.4) : It has a volute casing into which the impeller discharges water at a high velocity. Volute is of a spiral form and the cross-sectional area of the moving stream gradually increases from the tongue towards the delivery pipe. The cross-sectional area at any

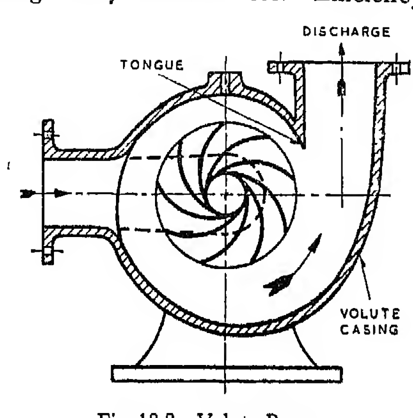


Fig 12.2 Volute Pump

point is therefore proportional to the quantity of water flowing across that section and therefore the mean velocity remains constant, the stream-lines may be assumed to be continuous Thus the losses of kinetic head which would occur if simply a circular casing were employed are avoided

The functions of a volute casing can be summarised as follows .

i) To collect water from the periphery of the impeller and to transmit it to the delivery pipe at a constant velocity. As the flow progresses from the impeller towards the opening into the delivery pipe, more and more water is added to the stream. In order that the velocity of water in the casing may not increase, the cross-sectional area of the casing is gradually increased, so that the extra area can accommodate the water added at each point of the casing.

1 Casing
2 Suction cover and branch
9 Bearing cover driving end
10 Bearing cover gland end
11 Stuffing Box
13 Impeller
14 Casing sealing rings
24 Screwed plug for priming
25 Drain plug
26 Plug for pressure gauge
27 Plug for vacuum gauge
31 Impeller locking nut
39 Pump shaft
42 Ball type thrust and journal bearing
46 Drip pipe
74 Locking washer
91 Gland Packing
166 Bearing Pedestal
328 Felt ring
335 End cover
A Oil Filling and air release plug
B Oil Level Indicator
F Oil drain

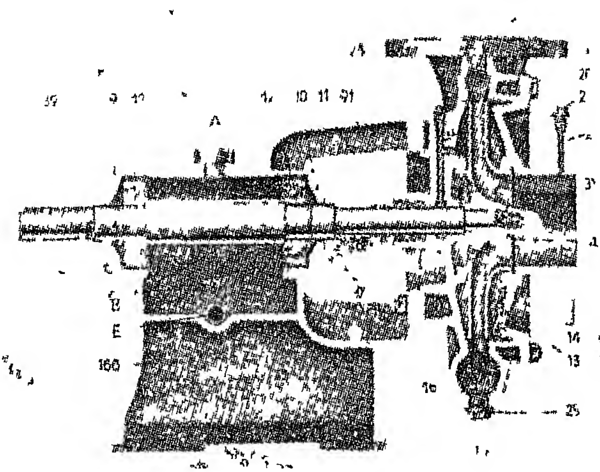


Fig 12 3 Single Stage Centrifugal Volute Pump with Ball Bearings Manufactured by K B Bremen (West Germany)

ii) To eliminate the loss of head, by making the casing of spiral or volute form. As the velocity of water leaving the impeller equals the velocity of flow in the volute, loss of head due to change of velocity can be eliminated.

iii) To increase evidently the efficiency of the pump by eliminating the loss of head due to change in velocity of flow in the volute.

A subsequent improvement of this design is the provision of an annular space between the volute and the impeller (See Fig 12 2, 12.3 and 12.4). The space which acts as a diffusor is known as *Vortex* or *Whirlpool chamber.* A free vortex is formed and as the water moves radially away from centre, velocity of whirl decreases thus building up pressure at the cost of velocity.

Almost all volute pumps are single stage type with the horizontal shafts irrespective of the shape of impeller.

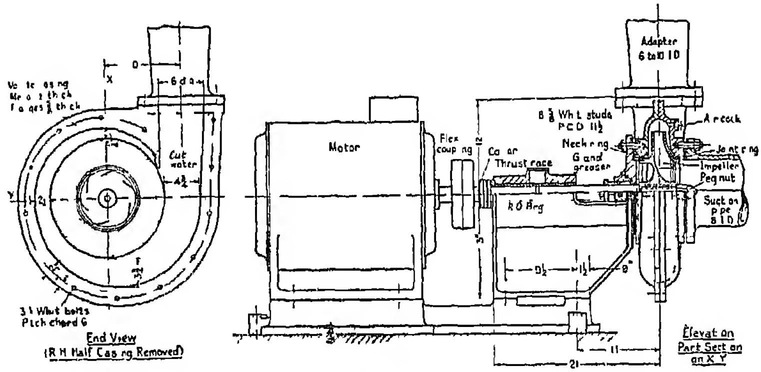


Fig 12 4 Section through a Single Stage Centrifugal Volute Pump

b) **Turbine Pump or Diffusion Pump** (See Fig 12 5)—Impeller is surrounded by a guide wheel consisting of a number of stationary vanes of diffusors providing outlets with cross-section gradually enlarging towards the periphery. Water emerging from the impeller flows past the guide vanes and as the section across flow increases velocity falls and pressure is built up. Angle of guide vanes at the entrance should coincide with the direction of absolute velocity of water at impeller outlet.

This arrangement is employed in all multi-stage pumps. Single stage pumps, however, have less expensive volute casing

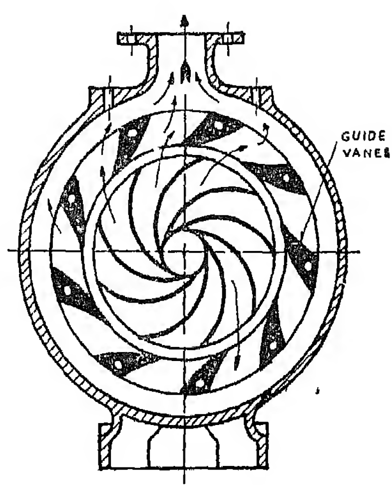


Fig 12 5 Turbine Pump or Diffusion Pump

Diffusion pumps may be either horizontal or vertical shaft type. The vertical type occupies very little space and is suitable for installation in deep wells. It is often called a deepwell pump. They may also be used in narrow wells and mines etc.

12.7 Number of Impellers—

a) **Single Stage Centrifugal Pump**—It has one impeller keyed to the shaft (Fig 12 6*a*). This is generally horizontal but can be vertical also. It is usually a low lift pump

b) **Multi-stage Centrifugal Pump**—It has two or more impellers keyed to a single shaft (Fig 12.6*b*) and enclosed in the same casing. Pressure is built up in steps. The impellers are surrounded by guide vanes and the water is led through a by-pass channel from the outlet

of one stage to the entrance of the next until it is finally discharged into a wide chamber from where it is pushed on to the delivery pipe.

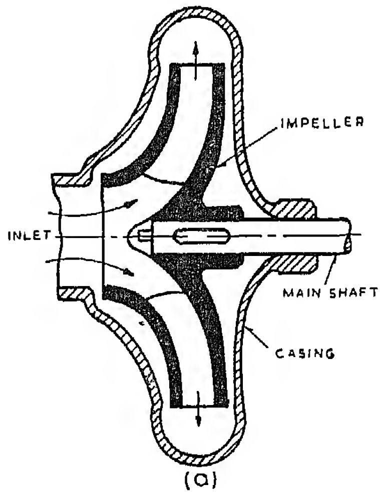


Fig 12 6 (*a*) Single Stage Centrifugal Pump

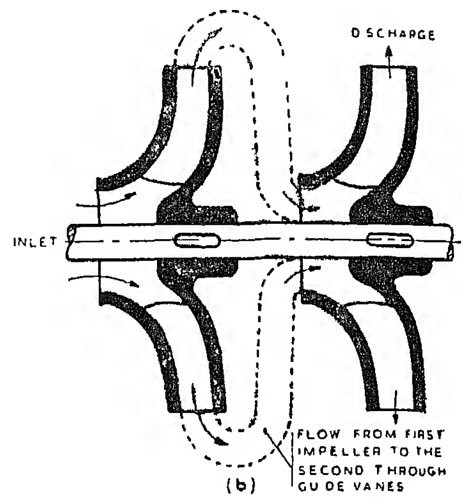


Fig 12 6 (*b*) Multi-stage (Two Stage) Centrifugal Pump

These pumps are used essentially for high working heads and the number of stages depends on the head required. The author has tested pumps having as many as fourteen stages but ordinarily not more than ten are employed. Until a few years ago, the arrangement used to generate high delivery-heads was to operate a number of single stage pumps in series but the modern multi-stage pump has completely replaced these unsatisfactory arrangements.

12 8 Relative Direction of Flow through Impeller (See Fig 12.7)—Theoretically any reaction turbine can also be used as a pump and therefore the classification of pumps from the point of view of direction of flow is similar to that of turbines (See Art 5.13). But there are no inward flow pumps primarily because of difficulty in starting them

a) **Radial Flow Pump**—(See Fig 12.7*a*) Ordinarily all centrifugal pumps are manufactured with radial flow impellers.

b) **Mixed Flow Pump**—As will be seen later (cf Art 13.5) the mixed flow impeller is just a modification of radial flow type enabling it to pump a large quantity of water. Flow through the impeller is a combination of radial and axial flows and the impeller resembles the propeller of a ship. Some mixed flow impellers look like a screw and are known as *screw* impellers (Fig 12.7*b*). In older designs a large quantity of water was delivered by running several pumps in parallel but this arrangement is now obsolete. Mixed flow pumps are mostly used for irrigation purposes where a large quantity of water at a low head is required.

c) **Axial Flow Pump**—Though axial flow pump is a roto-dynamic pump it is hardly justifiable to call it a centrifugal pump because centri-

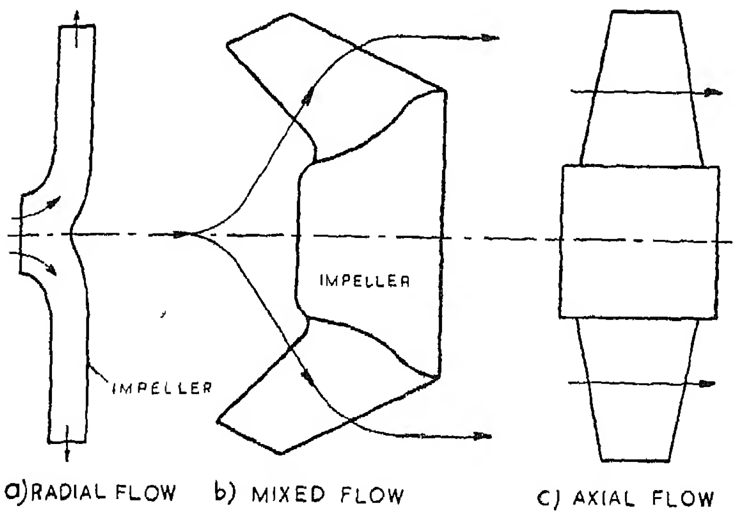


Fig 12 7 Classification of Centrifugal Pump Considering the Direction of Flow through the Impeller

fugal force is not called into play for the generation of pressure. Pressure is developed by flow of liquid over blades of aerofoil section just as the wings of an aeroplane produce the lift. The action is just the converse of a propeller turbine. If a Kaplan turbine runner (See Fig 7.19) is used as an impeller, the pump may be called a *Kaplan Pump*. It will have adjustable blades.

Axial flow pumps are designed to deliver very large quantities of water at comparatively low heads. They are ideally suited for irrigation purposes.

12 9 Number of Entrances to the Impeller—The pump may be single entry or double entry type. In the single entry or *single suction* pump water is admitted from a suction pipe on one side of the impeller (See Fig 12.6*a*).

Double entry pumps or *double suction* pumps (See Fig 12 8) admit water from both sides of the impeller. It is suitable for pumping large quantities of liquid. The arrangement has the added advantage that axial thrust is neutralised.

12 10 Disposition of Shaft—The shaft may be disposed horizontally or vertically. Generally centrifugal pumps are designed with horizontal shafts. Vertical disposition of shaft effects an economy in space occupied and is therefore suitable for deepwells and mines etc. They are also used for irrigation purposes.

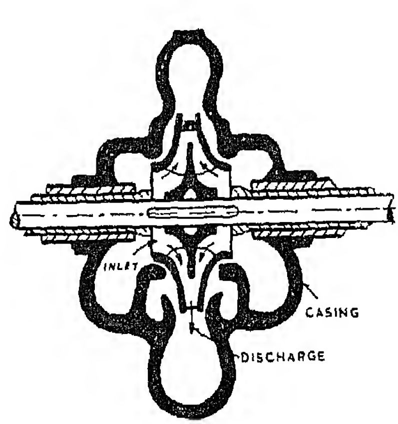
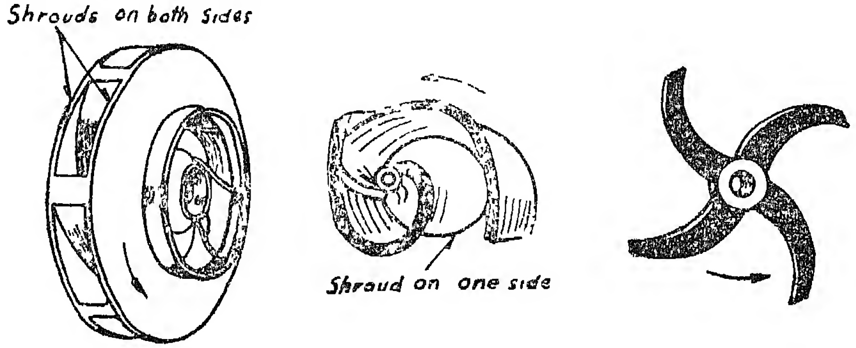


Fig 12 8 Double Suction Pump

12.11 Liquid Handled—Depending on the type and viscosity of liquid to be pumped, the pump may have a closed or open impeller (See Fig 12.9). Each of these types may have a ferrous, non-ferrous or stone coated impeller to resist chemical attack of liquid being pumped.

a) **Closed Impeller Pump** (See Fig 12.9*a*)—An ordinary centrifugal pump is equipped with a closed impeller in which the vanes are covered with shrouds on both sides. This type is meant to handle non-viscous liquids such as ordinary water, hot water, hot oil and chemicals like acids etc Material of the impeller should be selected according to the chemical properties of liquid used. For hot water at temperatures exceeding 150°C, cast steel impeller is recommended. Non-ferrous impellers are used for chemicals which are liable to corrode a ferrous surface. For pumping acids, the impeller and all inside surfaces in contact with the liquid should be coated with stone.



a) Closed Type Impeller *b*) Semi-Open Type Impeller *c*) Open Type Impeller

Fig 12.9 Different Types of Impellers

b) **Semi-Open Impeller Pump or Non-Clog Pump** (See Fig 12.9*b*)—The impeller is provided with shroud on one side only. This pump is used for viscous liquids such as sewage water, paper pulp, sugar molasses etc. In order to minimise the chances of impeller getting clogged, the number of vanes is reduced and their height is increased.

Sewage pumps are made without any protuberances around which rags could wrap and catch. They are always of the single suction type, as with the double suction pumps the shaft extends through the eyes of the impeller, thereby forming an easy place for rags to catch and wrap, and thus clogging the pump.

The non-clog pumps for paper stock and other similar material have open type impellers with entrance blades especially designed to prevent separation of stock and water.

Choice of material for manufacture of impeller is influenced by the chemical nature of the liquid to be handled. Non-clog pumps must be heavily constructed mechanically in order to give satisfactory service. The number of blades for impeller is small, using not more than two.

c) **Open Impeller Pump** (See Fig 12.9*c*)—The impeller is not provided with any shroud. Such pumps are used in dredgers and elsewhere for handing mixtures of water, sand, pebbles and clay, in which the solid

contents may be as high as one part in four. The impeller has very rough duty to perform. It is generally made of forged-steel. Its life depends upon the material handled, may be 40 to 50 hours or in some cases from 500 to 1,000 hours.

12.12 Specific Speed—As in the case of turbines, specific speed is a sound basis for a technical classification of centrifugal pumps. Specific speed is the only characteristic index or distinguishing feature of a pump when several impellers can be used for the same head and capacity. The performance and dimensional proportions of pumps having the same specific speed will be the same even though their outside diameters and actual operating speeds may be different.

Specific speed is defined as the speed of a geometrically similar pump when delivering one cusec (or one m^3/sec) against a head of one foot (or one metre). In practice, gpm (or m^3/sec) is taken as the unit of discharge.

Then specific speed

$$N_s = \frac{N \cdot \sqrt{Q}}{H^{\frac{3}{4}}} \quad \text{.. (See Eqn 3.40)}$$

where N=Actual speed of pump in rpm

Q=Quantity flowing in gpm (imperial)

H=Delivery head (total or manometric) in ft

If Q is given in cusecs,

$$N_s = \frac{N \cdot \sqrt{Q_{cusecs} \times \frac{62 \cdot 4}{10} \times 60}}{H^{\frac{3}{4}}} = \frac{19.35\, N \cdot \sqrt{Q_{cusecs}}}{H^{\frac{3}{4}}}$$

The specific speed in metric units is $N_s = \dfrac{N \cdot \sqrt{Q}}{H^{\frac{3}{4}}}$

where Q=Discharge in m^3/sec

and H=Delivery (total or manometric) head in m

then $$N_s \text{ (metric)} = \frac{N_s \text{ (British)}}{47 \cdot 25} \quad \text{...(12.1)}$$

The values of Q and H to be substituted in the equation for the purpose of calculating the specific speed are those corresponding to the maximum *efficiency* of the pump at its normal working speed.

It may be noted that the value of H to be used in the equation for N_s of a multi-stage pump is obtained by dividing the actual head developed by the number of stages. Similarly the value of Q for a double suction pump is taken as half the actual value for the purpose of this calculation.

12.13 Non-dimensional Factor K_s—The non-dimensional factor K_s (mentioned in Art 5.16) for the classification of water turbine can be used for this purpose also.

$$K_s = \frac{Q \cdot N^2}{v^3} \quad \text{where } Q = \text{Quantity discharged in cusecs (or } m^3/\text{sec)}$$

N = rpm

$v = \sqrt{2g\, H_{mano}}$ in ft/sec (or m/sec)

H_{mano} = total head the pump has to work against, in ft (or m)

Relation between N_s and K_s for Centrifugal Pumps ;

a) **FPS Units**—

$$N_s = \frac{19{\cdot}35 \cdot N \cdot \sqrt{Q}}{H^{\frac{3}{4}}} = \frac{19{\cdot}35 \cdot N \cdot \sqrt{Q}}{\left(\frac{v^2}{2g}\right)^{\frac{3}{4}}}$$

$$= 19{\cdot}35 \times (2g)^{\frac{3}{4}} \cdot \frac{N \cdot \sqrt{Q}}{v^{\frac{3}{2}}}$$

$$= (8{\cdot}02 \times 2{\cdot}83) \times 19{\cdot}35 \cdot \frac{N \cdot \sqrt{Q}}{v^{\frac{3}{2}}}$$

$$= \frac{439 \cdot N \cdot \sqrt{Q}}{v^{\frac{3}{2}}}$$

or $$N_s^2 = \frac{(439)^2 \cdot N^2 \cdot Q}{v^3} = (439)^2 \cdot K_s$$

or $$K_s = \left(\frac{N_s}{439}\right)^2$$

b) **Metric Units**—

$$N_s = \frac{N \cdot \sqrt{Q}}{H^{\frac{3}{4}}} = \frac{N \cdot \sqrt{Q}}{\left(\frac{v^2}{2g}\right)^{\frac{3}{4}}}$$

$$= (2g)^{\frac{3}{4}} \cdot \frac{N \cdot \sqrt{Q}}{v^{\frac{3}{2}}} = (2 \times 9{\cdot}81)^{\frac{3}{4}} \cdot \frac{N \cdot \sqrt{Q}}{v^{\frac{3}{2}}}$$

or $$N_s^2 = \frac{(2 \times 9{\cdot}81)^{\frac{3}{2}} \cdot N^2 \cdot Q}{v^3} = (2 \times 9{\cdot}81)^{\frac{3}{2}} \cdot K_s$$

or $$K_s = \left\{\frac{N_s}{(2 \times 9{\cdot}81)^{\frac{3}{4}}}\right\}^2 = \left(\frac{N_s}{9{\cdot}32}\right)^2$$

Representing Classification of Centrifugal Pumps in Tabular Form :

TABLE 12.1

	Type of Impeller	N_s (British)	N_s (Metric)	K_s
a	Slow runner or radial runner	500 to 1,800	10·5 to 38	1 3 to 16·8
b	Normal runner or mixed flow type (Francis runner)	1,800 to 3,900	38 to 82	16·8 to 79
c	Fast runner or mixed flow type (screw runner)	3,900 to 7,750	82 to 164	79 to 310
d	Very fast runner or axial runner	5,200 to 23,650	110 to 500	140 to 2,910

TABLE 12.2

N_s	1,000	...	...	15,000 FPS units
	20	...	...	300 Metric units
H	250	...	...	25 ft
	80	...	...	8 m
Type of casing	diffusion			volute

Problem 12.1 A six stage centrifugal pump delivers 1,600 gpm against a net pressure rise of 750 lb/sq in. Determine its specific speed if it rotates at 1,450 rpm. What type of impeller would be selected for the pump ?

Solution

No. of stages = 6 $\quad Q = 1{,}600$ gpm

$p = 750$ lb/sq in. $\quad N = 1{,}450$ rpm

$$H = \frac{p}{w} = \frac{750 \times 144}{62 \cdot 4} = 1{,}730 \text{ ft of water}$$

$$\text{Head developed per impeller} = \frac{1{,}730}{6} = 289 \text{ ft of water}$$

$\therefore$ Specific speed of the pump $N_s = \dfrac{N \times \sqrt{Q}}{H^{\frac{3}{4}}}$

$$N_s = \frac{1{,}450 \times \sqrt{1{,}600}}{289^{\frac{3}{4}}} = \mathbf{827} \quad \textit{Answer}$$

A radial impeller will suit this specific speed.

Problem 12.2 Find the specific speed of a double suction centrifugal pump delivering 20,000 gpm (or 1 52 m^3/sec) against a net head of 50 ft (15 22 m), when running at 725 rpm. State the type of impeller to be employed for this pump.

Solution

Q=20,000 gpm (or 1·52 m^3/sec)
H=50 ft (or 15·22 m)
N=725 rpm

Considering only one half of the impeller, Q is 10,000 gpm (or 0·76 m^3/sec).

Specific speed $N_s = \dfrac{N \times \sqrt{Q}}{H^{\frac{3}{4}}}$

$$= \frac{725 \times \sqrt{\frac{20{,}000}{2}}}{50^{\frac{3}{4}}} \quad \left[\text{or} \quad \frac{725 \times \sqrt{0{\cdot}76}}{15{\cdot}22^{\frac{3}{4}}} \right]$$

$= \mathbf{3{,}860}$ (or $\mathbf{88{\cdot}2}$) *Answer*

The type of the impeller to be used is mixed flow Francis type.

12 14 Evolution of Axial Flow Impeller from the Radial Type—It is interesting to note that the recent development of axial flow impellers has been considerably facilitated with the knowledge of specific speed. The study of specific speed is of great academic interest and it is a valuable aid in the design of pumps.

Table 12.1 as well as Fig 12.10 show the development of impellers with regard to their specific speeds and non-dimensional factors. This can be compared with "Evolution of Kaplan Turbine Runner" discussed in Art 7, 14 and Fig 7.12.

12.15 Layout Accessories and Starting of Centrifugal Pump—A typical layout of a centrifugal pumping installation is shown in Fig 12.11. The main accessories are—

a) Strainer and foot valve,
b) Suction pipe with fittings,
c) Delivery valve or regulating valve,
d) Delivery pipe with fittings.

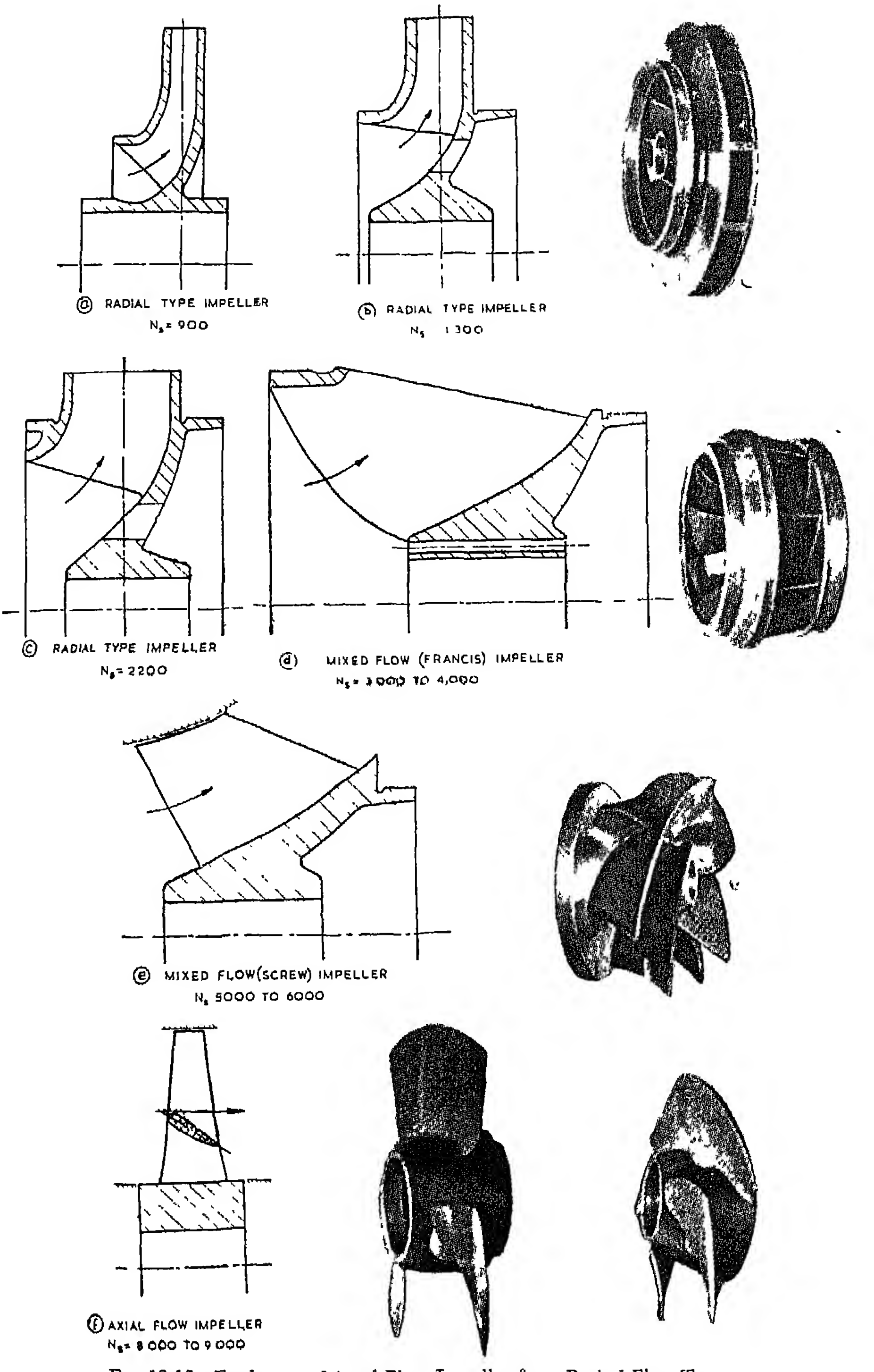


Fig 12.10 Evolution of Axial Flow Impeller from Radial Flow Type
[N_s is given in FPS units ; 47.25 N_s (metric) $= N_s$ (FPS)]

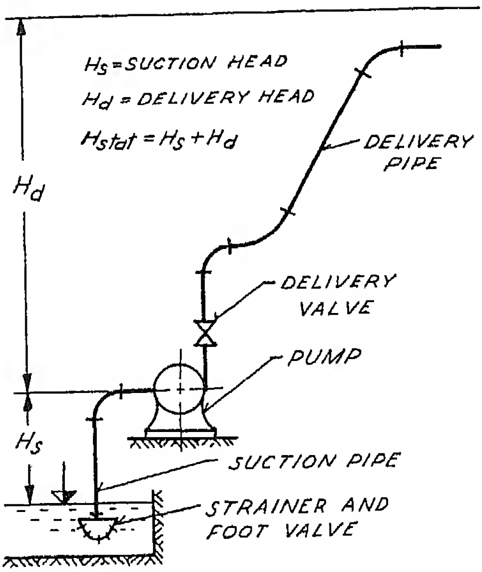


Fig 12.11 Layout of a Pump

a) **Strainer and Foot Valve** is fitted at the end of the suction pipe and is submerged in water in such a way that it is only a few inches or cm above the bottom of water to be pumped. The water from the sump or well first enters the strainer which is meant to keep the floating bodies, such as leaves, wooden pieces and other kind of rubbish, away from the pump. In the absence of strainer the foreign material will pass through the pump and choke it, thus hindering its working. With the strainer is cast a foot valve of non-return or one way type, opening upwards. The water will pass through the foot valve upwards and it will not allow the water to move downwards.

b) **Suction Pipe with Fittings**—The suction pipe is laid in such a way that it rises all its way to the pump *upwards*. In case the suction pipe first rises and then drops, thus making a loop, the air will collect in that portion and will hinder the satisfactory working of the pump. Great care must be taken to ensure that the pipe fittings are all air-tight. The fittings on the suction side may consist of different bends, but no regulating valve. The details of suction lift are given in Art 12.39. The diameter of suction pipe is slightly less than that of delivery pipe (See Fig 12.23).

c) **Delivery Valve or Regulating Valve**—As the water comes out of the pump, it must pass through a regulating valve. The regulating valve is of sluice type. It must be closed when the pump is being started and has built up its pressure. If the delivery valve is closed and the pump is working and delivery water at its full pressure, the delivery pipe or pump will not burst, as the impeller will just be churning the water in the casing. The delivery valve is also essential to vary the discharge at the time of needs. It should be closed before the pump has stopped working, otherwise the full delivery pressure will be transmitted to the suction pipe which may be harmful.

d) **Delivery Pipe and Fittings**—The water is delivered to a place or reservoir through the delivery pipe having a number of fittings mentioned in Table 12.3. The pipe diameter depends upon the discharge to be handled by the pump and can be calculated with the help of velocity of flow. Fig 12.23 gives the curve representing pipe diameters vs discharge, drawn from experimental data.

Starting of Centrifugal Pump—As given in Art 12.38 'Priming of Pumps', it is necessary that before the pump is started, its casing together with impeller, and the suction pipe must be filled with water

in order to remove the air, gas or vapour in that region. The pump will not generate its pressure, if there is any air left on the suction side. Whenever there is any difficulty in the satisfactory running of a pump, it is mainly due to pressure of air in the piping or pump. The removal of air by filling the pump with water is known as *priming*. Hence priming is necessary to start the pump.

The correct layout of pumping installation is very important for the satisfactory and successful working of the centrifugal pump. The layout should be such that no air pocket is formed in the piping. Great care must be taken on the suction side as explained above. The details of defects and remedies of centrifugal pump are given in Art 12.44.

12.16 Head of a Pump—The term "head" of a pump stands for the following two heads :

i) Static Head

ii) Manometric, Total, Gross or Effective Head.

i) **Static Head**—It is the sum of the suction and delivery heads (See Fig 12.11). Suction head is the vertical height of the centre line of the pump shaft above the surface of a available liquid *i.e.* the lower surface from which it is being raised. Delivery head is the vertical height measured from the centre line of main pump shaft to where the liquid is delivered. The static head is also known as *geodetic head*.

ii) **Manometric, Total, Gross or Effective Head**—This is the actual head against which the pump has to work. It is equal to the static head plus all the head losses occurring in flow before, through and after the impeller.

The above heads are independent of the density of the liquid being raised. A centrifugal pump running at a particular speed will raise water, oil or mercury to the same height. But the power required will be different. Also, the pressure generated in lb/sq in. (or kg/cm^2) will be different in each case. It is, therefore, convenient to express the head in feet (or m) of liquid column.

If the pump raises liquid from a source which is already subjected to some pressure, then the equivalent of this gauge pressure in feet (or m) of liquid column should be subtracted to get the real total head On the other hand if a pump takes liquid from vacuum, the equivalent of vacuum in feet (or m) should be added to get the actual total head.

If H_s and H_d be suction and delivery heads respectively, the static head,

$$H_{stat}=H_s+H_d \qquad \text{...(12.2)}$$

If H_L be the total loss of head, the manometric head,

$$H_{mano}=H_{stat}+H_L \qquad \text{...(12.3)}$$

If h be the gauge pressure of available liquid,

$$H_{real}=H_{apparent}-h$$

12.17 Power—The horsepower required to drive the pump shaft is given by

$$P=\frac{w \cdot Q \cdot H_{mano}}{550 \cdot \eta_{overall}} \text{ HP} \left[\text{or} \quad =\frac{w \cdot Q \cdot H_{mano}}{75\ \eta_{overall}} \text{ metric HP}\right] \qquad \text{...(12.4)}$$

where w=specific weight of liquid raised in lb/cu ft (or kg/m^3),
Q=discharge in cusecs (or m^3/sec),
H_{mano}=manometric or gross head in ft (or m),
$\eta_{overall}$=overall efficiency of the pump.

12.18 Efficiency—Overall Efficiency is the ratio of the power supplied by the pump to the power delivered to the pump shaft.

$$\eta_{overall}=\frac{\text{Output}}{\text{Input}}=\frac{w \,.\, Q \,.\, H_{mano}}{550 \,.\, (\text{BHP} - \text{HP lost in coupling mechanism})} \quad ...(12.5)$$

where BHP is the brake horse power of the electric motor or prime-mover which drives the pump.

The static efficiency of a pump also deserves mention because it accounts for all hydraulic losses in the pipelines.

Then $$\eta_{stat}=\frac{w \,.\, Q \,.\, H_{stat}}{550 \,.\, (\text{BHP})} \quad ...(12.6)$$

[Substitute 75 for 550 to convert to metric units in the above Eqns 12·5 and 12·6]

12 19 Loss of Head in Pipe Lines and Pipe Fittings—All head-losses from the foot valve up to the outlet of the delivery pipe are given by the following equation

$$H_L=H_{L_s}+H_{L_d}+\Sigma H_{L_f}+\frac{v_e^2}{2g} \quad ...(12.7)$$

[All losses of head are measured in feet (or m) of liquid column]

where H_L =total loss of head in the pipes and fittings,

H_{L_s} =frictional loss in suction pipe

$$=\frac{4 f L_s}{d_s}\,.\,\frac{v_s^2}{2g}$$

H_{L_d} =frictional loss in delivery pipe

$$=\frac{4 f L_d}{d_d}\,.\,\frac{v_d^2}{2g}$$

ΣH_{L_f} =sum of head losses in pipe fittings

$\frac{v_e^2}{2g}$ =loss of kinetic head due to velocity of water at exit

Pipe fittings referred to above include the strainer, the foot valve, all pipe bends, all regulating and non-return valves and all changes of cross-section.

12.20 Determination of Loss of Head in Pipe Lines and Pipe Fittings—Such losses may be classified as follows :

a) Loss of head in pipes,

b) Loss due to enlargement of cross-section of pipe,

c) Loss due to contraction of cross-section of pipe,

d) Loss due to bends in pipe,

e) Loss due to pipe fittings such as valves, joints, elbows etc.

It is, in general, not easy to determine these losses analytically. Therefore, empirical relations obtained from experiments are used. Often, it is convenient to represent experimental results graphically.

a) *Loss of head in pipes* is given by the equation $\frac{4fl}{d} \cdot \frac{v^2}{2g}$, where f, the frictional factor, is a function of Reynolds' number R_e. In order to

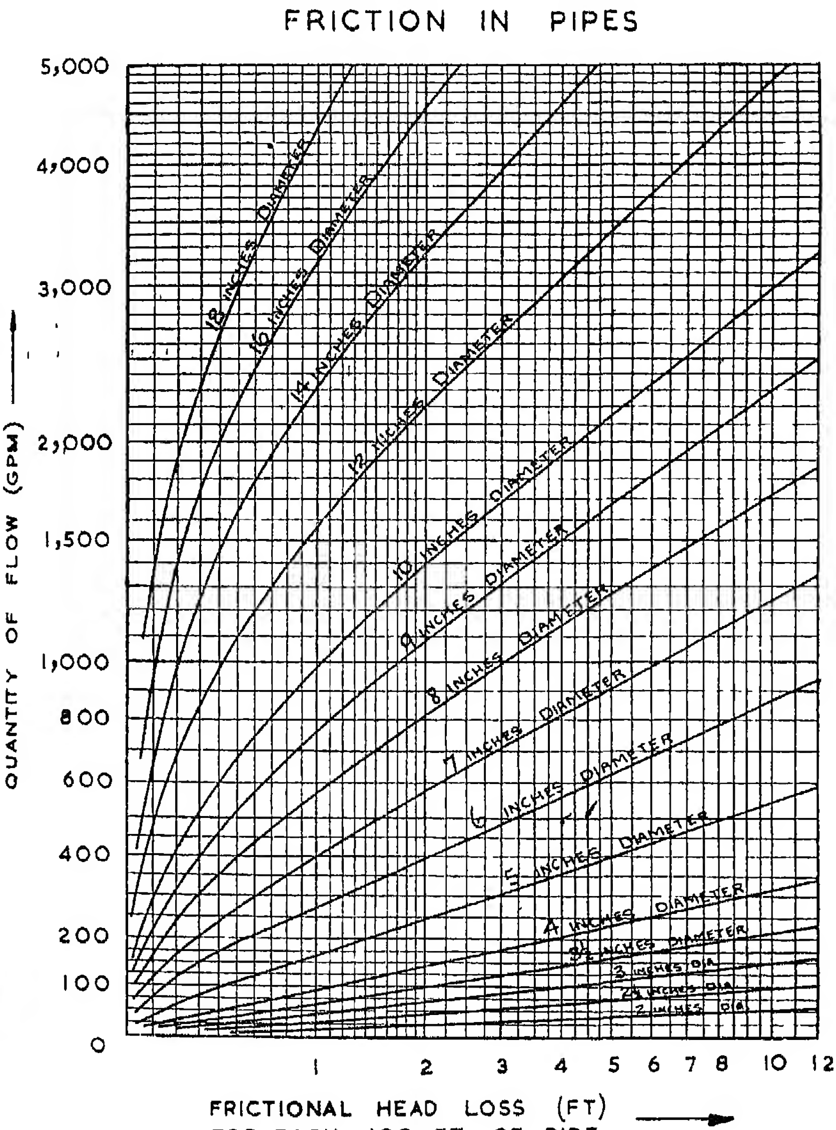


Fig 12.12 Loss of Head in Pipes due to Friction

avoid long calculations every time, loss of head in pipelines is generally given by tables or graphs which are prepared from the experimental results. Fig 12 12 shows the frictional head loss in ft for each 100 ft (or 30 48 m) of pipe length, drawn against discharge in gallons per minute for different pipe diameters in inches.

b) Loss of head due to a sudden enlargement of the cross-section of pipe is generally given by the expression $\frac{(v_1-v_2)^2}{2g}$ which was analytically derived by Carnot on certain assumptions. Here v_1 and v_2 are the velocities of flow through the pipe before and after enlargement respectively.

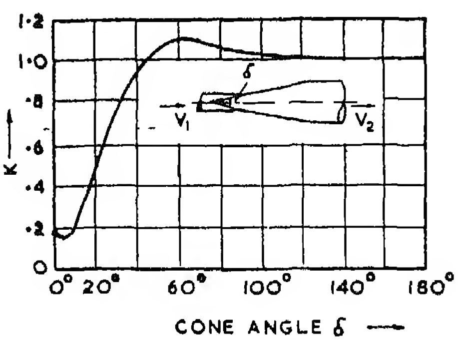


Fig 12.13 Loss of Head due to Gradual Enlargement

$k=f(\delta)$

(*Experiments by A. H. Gibson*)

Loss of head due to gradual enlargement of cross-section is given by

$$H_L=\frac{k(v_1-v_2)^2}{2g} \quad \text{.. (12.8)}$$

The factor k depends on the cone angle and a curve of k against the cone angle, is shown in Fig 12.13.

c) Loss of head due to sudden contraction is given by

$$H_L=k\cdot\frac{v_2^2}{2g} \quad \text{...(12.9)}$$

where v_2 is the velocity after contraction.

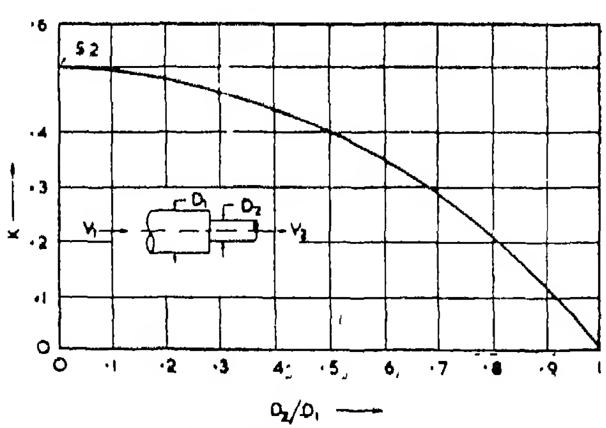


Fig 12.14 Losses of Head due to Sudden Contraction

$$k=f\left(\frac{D_1}{D_2}\right)$$

The value of co-efficient k can be determined from the curve (Fig 12.14) showing

$$k=f\left(\frac{D_1}{D_2}\right)$$

d) *Loss of head due to a 90°-bend* is given by

$$H_L = k \cdot \frac{v^2}{2g} \qquad \ldots(12.10)$$

The co-efficient k depends on the ratio $\varphi \left(=\frac{D_m}{D}\right)$ and Reynolds' Number R_e. The relationship is graphically represented in Fig 12.15.

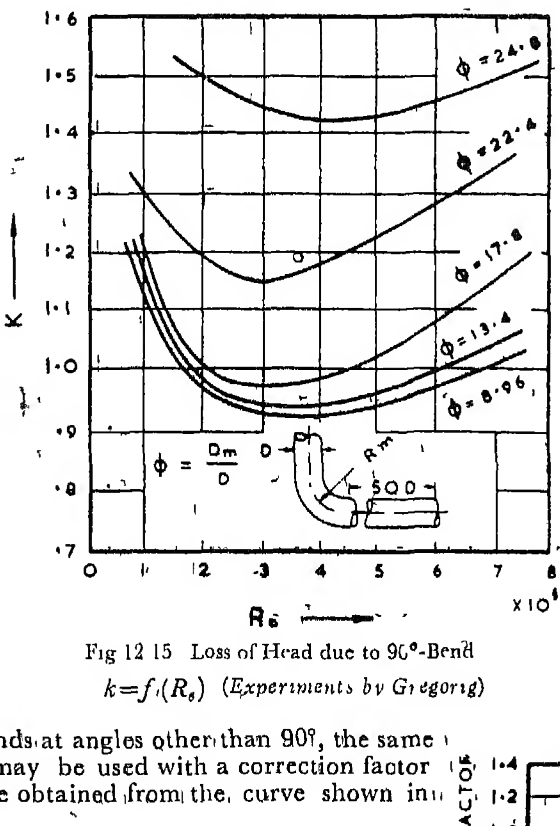


Fig 12 15 Loss of Head due to 90°-Bend
$k=f(R_e)$ (*Experiments by Gregorig*)

For bends at angles other than 90°, the same expression may be used with a correction factor which can be obtained from the curve shown in Fig 12.16.

e) *Loss of head in pipe fittings* can be expressed as a fraction of the total kinetic head, thus

$$H_L = k \cdot \frac{v^2}{2g} \qquad \ldots(12.11)$$

The value of the factor k is obtained from the experimental results.

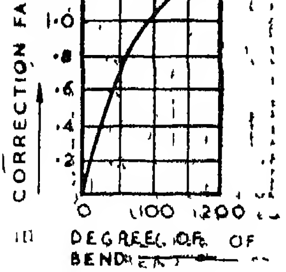


Fig 12.16 Degree of Bend

Table 12.3 shows frictional head loss of different kinds of pipe fittings given in terms of length of straight pipe having the same diameter as that of the pipe fitting.

TABLE 12.3

Frictional Head Loss of Valves and Pipe-Fittings

Length of Straight Pipe giving Equivalent Resistance to Flow

Size of Pipe in inches	90° Standard Elbow	90° Medium Radius Elbow	90° Long Radius Elbow	45° Elbow	Tee	Return Bend	Gate Valve Open	Globe Valve Open	Angle Valve Open	Swing Check
$\frac{1}{2}$	1·5	1·4	1·1	0·77	3·4	3·8	0·35	16	8·4	3·3
$\frac{3}{4}$	2·2	1·8	1·4	1·0	4·5	5·0	0·47	22	12	5·0
1	2·7	2·3	1·7	1·3	5·8	6·1	0·6	27	15	6·7
$1\frac{1}{4}$	3·7	3·0	2·4	1·6	7·8	8·5	0·8	37	18	8·3
$1\frac{1}{2}$	4·3	3·6	2·8	2·0	9·0	10	0·95	44	22	10
2	5·5	4·6	3·5	2·5	11	13	1·2	57	28	13
$2\frac{1}{2}$	6·5	5·4	4·2	3·0	14	15	1·4	66	33	17
3	8·1	6·8	5·1	3·8	17	18	1·7	85	42	20
$3\frac{1}{2}$	9·5	8·0	6·0	4·4	19	21	2·0	99	50	23
4	11	9·1	7·0	5·0	22	24	2·3	110	58	27
$4\frac{1}{2}$	12	10	7·9	5·6	24	27	2·6	130	61	30
5	14	12	8·9	6·1	27	31	2·9	140	70	33
6	16	14	11	7·7	33	37	3·5	160	83	40
8	21	18	14	10	43	49	4·5	220	110	53
10	26	22	17	13	56	61	5·7	290	140	67
12	32	26	20	15	66	73	6·7	340	170	80
14	36	31	23	17	76	85	8	390	190	93
16	42	35	27	19	87	100	9	430	220	107
18	46	40	30	21	100	110	10·2	500	250	120
20	52	43	34	23	110	120	12	560	280	134
22	58	50	37	25	130	140	13	610	310	147
24	63	53	40	28	140	150	14	680	340	160
30	79	68	50	35	165	190	17	860	420	200
36	94	79	60	43	200	220	20	1000	500	240
42	120	95	72	50	240	260	28	1200	600	280
48	135	110	82	58	275	300	26	1400	680	320

(From "Engineering Data on Flow of Fluid in Pipes"—Crane Co.)

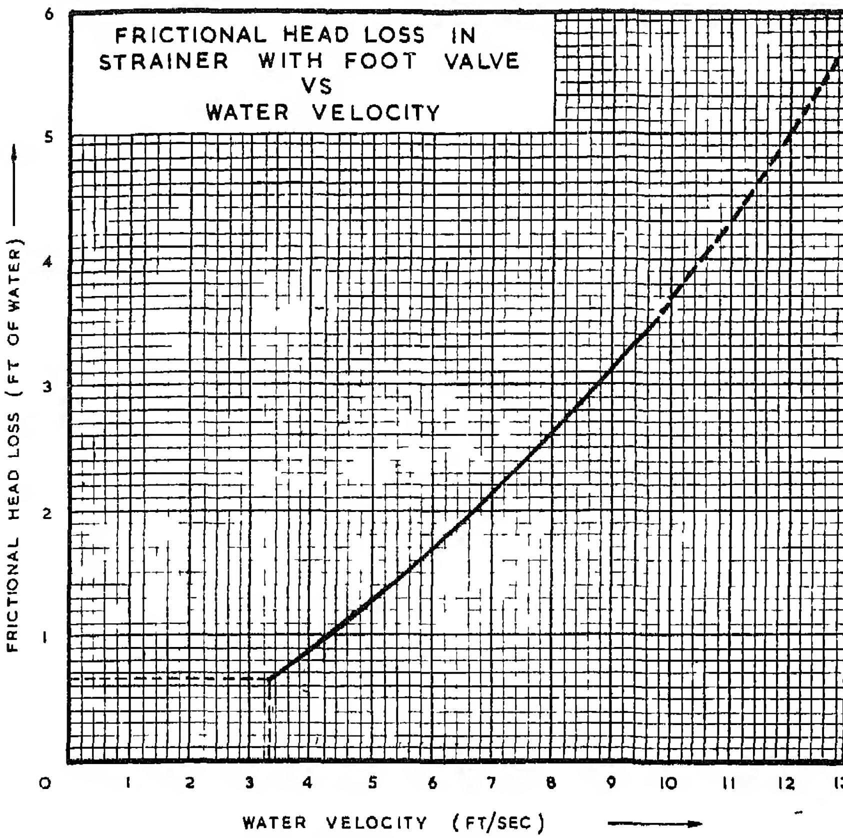


Fig 12 17 Frictional Head Loss in Strainer with Foot Valve vs Water Velocity

Problem 12.3 A centrifugal pump is installed to supply water from a reservoir to a tank which is at a vertical height of 40 ft above it. The diameter of the suction pipe is 12 in. and it is 20 ft long. At the end of the suction pipe which extends under the water level in the reservoir, are fitted a foot valve and strainer. There is one 90°-bend on the side of suction. The discharge pipe is 10 in. diameter and 450 ft long. It is fitted with a gate valve and two medium radius 90°-elbows. Find the HP of the pump to discharge 3 cusecs of water considering all losses of head in pipe lines and pipe fittings. The overall efficiency of pump is 70%. Find also the size of driving motor.

Assume—

Viscosity of water $\mu = 23\ 89 \times 10^{-6}$ slug/ft sec

Specific weight of water $w = 62\ 4$ lb/cu ft

Loss of head due to friction in pipes $H_f = \frac{4fL}{d} \cdot \frac{v^2}{2g}$

$$4f = 0{\cdot}0032 + \frac{0{\cdot}221}{R_e^{0{\cdot}237}}$$

where R_e is the Reynolds' number.

Solution

Lift $=40$ ft $\quad Q=3$ cusecs

$d_s=12$ in. $=1$ ft $\quad \eta_{overall}=70\%$

$L_s=20$ ft $\quad \mu=23{\cdot}89\times10^{-6}$ slug/ft sec

$d_d=\frac{10}{12}$ ft $\quad$ Density $\rho=\frac{w}{g}=\frac{62{\cdot}4}{32{\cdot}2}$

$L_d=450$ ft $\quad =1{\cdot}938$ lb ft^{-4} sec^2

Kinematic viscosity $\nu=\frac{\mu}{\rho}=\frac{23{\cdot}89\times10^{-6}}{1{\cdot}938}=12{\cdot}33\times10^{-6}$ ft^2 sec^{-1}

Discharge $Q=3\ cfs=\frac{\pi}{4}\,d_s^2\,.\,v_s=\frac{\pi}{4}\,d_d^2\,.\,v_d$

Velocity of water in suction pipe

$$v_s=\frac{Q}{\frac{\pi}{4}d_s^2}=\frac{3}{\frac{\pi}{4}\times1}=3{\cdot}82 \text{ ft/sec}$$

Velocity of water in delivery pipe

$$v_d=\frac{Q}{\frac{\pi}{4}d_d^2}=\frac{3}{\frac{\pi}{4}\times\left(\frac{10}{12}\right)^2}=5{\cdot}5 \text{ ft/sec}$$

$\therefore$ R_e for suction pipe $=\frac{v_s\,.\,d_s}{\nu}=\frac{3{\cdot}82\times1}{12{\cdot}83\times10^{-6}}$

$=0{\cdot}31\times10^6=310{,}000$

R_e for delivery pipe $=\frac{v_d\,.\,d_d}{\nu}$

$$=\frac{5{\cdot}5\times\frac{10}{12}}{12{\cdot}33\times10^{-6}}=372{,}000$$

Frictional factor $4f=0{\cdot}0032+\frac{0{\cdot}221}{R_e^{0{\cdot}237}}$

$4f$ for suction pipe $=0{\cdot}0032+\frac{0{\cdot}221}{310{,}000^{0{\cdot}237}}=0{\cdot}0143$

$4f$ for discharge pipe $=0{\cdot}0032+\frac{0{\cdot}221}{372{,}000^{0{\cdot}237}}=0{\cdot}0138$

$$H_{f_s}=\frac{4f_s\,.\,L_s}{d_s}\,.\,\frac{v_s^2}{2g}=\frac{0{\cdot}0143\times20}{1}\times\frac{3{\cdot}82^2}{64{\cdot}4}$$

$$=0{\cdot}065 \text{ ft of water}$$

$$H_{f_d}=\frac{4f_d\,.\,L_d}{d_d}\,.\,\frac{v_d^2}{2g}=\frac{0{\cdot}0138\times450}{\frac{10}{12}}\times\frac{5{\cdot}5^2}{64{\cdot}4}=3{\cdot}5 \text{ ft of water}$$

Loss of head in the pipe fittings is obtained from the curves given in Fig 12.13 to 12.17 and in Table 12.3.

a) Foot valve and the strainer on suction side $=0\ 764$ ft,
b) 90°-bend on the suction side $=0\ 115$ ft,
c) Gate valve on the delivery side $=0\ 85$ ft,
d) Two 90°-medium radius elbows $=0{\cdot}24$ ft,

Total loss $=1{\cdot}969$ ft of water

$$\text{Total losses}=H_{L_s}+H_{L_d}+\Sigma\ H_{L_f}+\frac{v_s^2}{2g}$$
$$=0{\cdot}065+3{\cdot}5+1{\cdot}969+\frac{5{\cdot}5^2}{64{\cdot}4}$$
$$=0{\cdot}065+3{\cdot}5+1{\cdot}969+0{\cdot}469$$
$$=6{\cdot}003 \text{ ft of water.}$$

$$\text{Total head } H_{mano}=H_s+H_d+H_L$$
$$=H_{st}+H_L$$
$$=40+6{\cdot}003$$
$$\approx 46 \text{ ft of water}$$

$$\therefore \text{ HP of pump}=\frac{w\ .\ Q\ .\ H_{mano}}{550\ .\ \eta_{overall}}=\frac{62{\cdot}4\times3\times46}{550\times0{\cdot}7}$$
$$=\mathbf{22\ 35\ HP} \quad \textit{Answer}$$

Capacity of driving motor $=22{\cdot}35+10\%$ of $22{\cdot}35$
$$=\mathbf{25\ HP} \quad \textit{Answer}$$

Problem 12 4 A centrifugal pump draws water at ordinary temperature through a 50 ft (or 15·22 *m*) long 6 inches (or 152 4 *mm*) diameter G. I. suction pipe which is fitted with one 90°—bend. The maximum capacity of the pump is 1,000 gpm (or 75·7 lit/sec). Determine the static suction head. Assume the maximum vacuum as 18 ft (or 5·5 *m*) of water. Loss in the bend may be taken to be 0·25 ft (or 0·0762 *m*) of water. $f=0{\cdot}006$.

Solution

$L_s=50$ ft (or 15 22 *m*) $\quad H_{vac}=18$ ft (or 5 5 *m*)
$d_s=0{\cdot}5$ ft (or 152·2 *mm*) $\quad f=0{\cdot}006$

$$Q=1{,}000\text{ gpm}=\frac{1{,}000\times10}{60\times62{\cdot}4}=2{\cdot}67\text{ cfs}$$
(or 75·7 lit/sec $=0\ 0757$ m^3/sec)

$$v_s=\frac{Q}{a_s}=\frac{Q}{\frac{\pi}{4}d_s^2}=\frac{2\ 67}{0{\cdot}785\times0{\cdot}25}=13{\cdot}6\text{ ft/sec}$$

$$\left(\text{or}\quad v_s=\frac{0\ 0757}{0{\cdot}785\times0{\cdot}1522^2}=4{\cdot}15\ m/\text{sec}\right)$$

$$H_{f_s}=\frac{4f\ .\ L_s}{d_s}\cdot\frac{v_s^2}{2g}=\frac{4\times0{\cdot}006\times50\times13\ 6^2}{0{\cdot}5\times2\times32{\cdot}2}=6\ 9\text{ ft of water}$$

$$\left[\text{or } H_{f_s}=\frac{4\times0{\cdot}006\times15{\cdot}22\times4{\cdot}15^2}{0{\cdot}1522\times2\times9{\cdot}81}=2{\cdot}11\ m\text{ of water}\right]$$

Loss of head due to 90°-bend=0·25 ft (or 0·0762 m)

$\therefore$ Total loss of head $H_{f\ total}=6{\cdot}9+0{\cdot}25=7{\cdot}15$ ft of water

(or $2{\cdot}11+0{\cdot}0762=2{\cdot}1862$ m of water)

$$\text{Kinetic head loss}=\frac{v_s^2}{2g}=\frac{13{\cdot}6^2}{64{\cdot}4}=2{\cdot}87 \text{ ft of water}$$

$$\left(\text{or } = \frac{4{\cdot}15^2}{2\times 9{\cdot}81}=0{\cdot}877\ m \text{ of water}\right)$$

Now $H_{vac}=H_s+H_{f_s}+\dfrac{v_s^2}{2g}$

or $18=H_s+7{\cdot}15+2{\cdot}87$ (or $5{\cdot}5=H_s+2{\cdot}1862+0{\cdot}877$)

$H_s=18-10{\cdot}02$ (or $H_s=5{\cdot}5-3{\cdot}0632$)

$=$ **8 ft** *Answer* (or $H_s\approx$ **2·44 m** *Answer*)

THEORY OF CENTRIFUGAL PUMP

12.21 Fundamental Equation of Centrifugal Pump—Let points on the liquid's path at suction inlet, impeller inlet, impeller outlet and casing outlet be denoted by i, 1, 2 and d respectively (See Fig 12.18). The equation of flow between any two consecutive points can be obtained by applying the Bernoulli's Theorem.

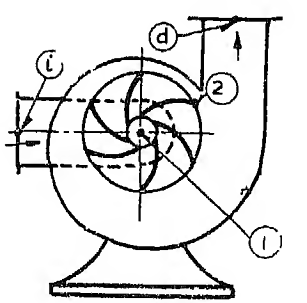


Fig 12.18 Liquid Flow Through a Centrifugal Pump

a) Thus for flow from (i) to (1) $i.e.$, through the stationary suction pipe, since v_i and v_1 represent absolute velocities of water

$$\frac{v_1^2}{2g}+\frac{p_1}{w}+z_1=\frac{v_i^2}{2g}+\frac{p_i}{w}+z_i-H_{L_{(i-1)}} \qquad \dots(\text{I})$$

b) From (1) to (2) $i.e.$, through the moveable impeller, since w_1 and w_2 represent relative velocities of water, the equation of flow is

$$\frac{w_2^2}{2g}-\frac{u_2^2}{2g}+\frac{p_2}{w}+z_2=\frac{w_1^2}{2g}-\frac{u_1^2}{2g}+\frac{p_1}{w}+z_1-H_{L_{(1-2)}} \qquad \dots(\text{II})$$

(See Appendix 4)

c) Flow from (2) to (d), $i.e.$ through the stationary casing inside which the motion of water is absolute, is given by

$$\frac{v_d^2}{2g}+\frac{p_d}{w}+z_d=\frac{v_2^2}{2g}+\frac{p_2}{w}+z_2-H_{L_{(2\text{-}d)}} \qquad \dots(\text{III})$$

Now, adding the equations (I), (II) and (III) and re-arranging

$$\frac{v_2^2-v_1^2}{2g}+\frac{w_1^2-w_2^2}{2g}+\frac{u_2^2-u_1^2}{2g}$$

$$=\left\{\left(\frac{v_d^2}{2g}+\frac{p_d}{w}+z_d\right)-\left(\frac{v_i^2}{2g}+\frac{p_i}{w}+z_i\right)\right\}+\left(H_{L_{i-1}}+H_{L_{1-2}}+H_{L_{2-d}}\right)$$

The first term on the right hand side is, by definition, the gross manometric head of the pump :

$$H_{mano}=\left(\frac{v_d^2}{2g}+\frac{p_d}{w}+z_d\right)-\left(\frac{v_i^2}{2g}+\frac{p_i}{w}+z_i\right)$$

And the second term stands for the total pump losses due to the fluid resistance, inside the pump only *i.e.*

$$\triangle H_{mano}=H_{L_{i-1}}+H_{L_{1-2}}+H_{L_{2-d}}$$

$$\therefore\quad \frac{v_2^2-v_1^2}{2g}+\frac{w_1^2-w_2^2}{2g}+\frac{u_2^2-u_1^2}{2g}=H_{mano}+\triangle H_{mano}$$

This is known as the *fundamental equation of centrifugal pump.*

Considering the losses of head in the pump, its efficiency,

$$\eta_{mano}=\frac{H_{mano}}{H_{mano}+\triangle H_{mano}} \qquad \text{...(12.12)}$$

is known as *Manometric Efficiency.*

$$\text{or}\quad H_{mano}+\triangle H_{mano}=\frac{H_{mano}}{\eta_{mano}}$$

whence,

$$\frac{v_2^2-v_1^2}{2g}+\frac{w_1^2-w_2^2}{2g}+\frac{u_2^2-u_1^2}{2g}=\frac{H_{mano}}{\eta_{mano}} \qquad \text{.. (12.13)}$$

The above equation can be simplified by substituting for w_1 and w_2 from the velocity triangles at inlet and outlet (See Fig 12.19)·

Thus :

$$w_1^2=u_1^2+v_1^2-2u_1\,v_1\cos\alpha_1$$

$$\text{and}\quad w_2^2=u_2^2+v_2^2-2u_2\,v_2\cos\alpha_2$$

$$\therefore\quad w_1^2-w_2^2=u_1^2-u_2^2+v_1^2-v_2^2$$
$$-2u_1\,v_1\cos\alpha_1+2u_2\,v_2\cos\alpha_2$$

Substituting this in the fundamental equation,

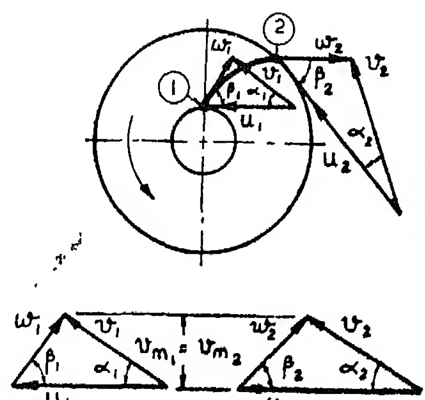


Fig 12.19 Velocity Triangles of a Centrifugal Pump

$$\frac{H_{mano}}{\eta_{mano}}=\frac{v_2^2-v_1^2}{2g}+\frac{u_2^2-u_1^2}{2g}$$
$$+\frac{u_1^2-u_2^2+v_1^2-v_2^2-2u_1\,v_1\cos\alpha_1+2u_2\,v_2\cos\alpha_2}{2g}$$
$$=\frac{2u_2\,v_2\cos\alpha_2-2u_1\,v_1\cos\alpha_1}{2g}$$

$$\text{or}\quad \frac{H_{mano}}{\eta_{mano}}=\frac{u_2\,v_2\cos\alpha_2-u_1\,v_1\cos\alpha_1}{g} \qquad \text{...(12.13}a\text{)}$$

Generally $\alpha_1=90°$ and $\cos \alpha_1=0$, then, neglecting prerotation*,

$$\text{or} \quad \frac{H_{mano}}{\eta_{mano}}=\frac{u_2\, v_2 \cos \alpha_2}{g}=\frac{u_2 \,.\, v_{u_2}}{g} \qquad \text{.. (12 14)}$$

$$\text{or } H_{mano}=\eta_{mano}\times\frac{\text{Peripheral speed at outlet} \times \text{velocity of whirl at outlet}}{g}$$

This is the form in which the fundamental equation is used in practice.

12 22 Work Done and Manometric Efficiency—

Work done/sec by the pump impeller is given by Eqn 1.34 (*a*) *i e*

$$P=\frac{w \,.\, Q}{g}(u_1 \,.\, v_{u_1}-u_2 \,.\, v_{u_2})$$

The suffices (1) and (2) used in this equation will hold true if point 1 denotes the pressure side and point 2 denotes the suction side of the pump However if point 1 denotes inlet and point 2 the outlet of the pump impeller then the above equation will be written as

$$P=\frac{w \,.\, Q}{g}(u_2 \,.\, u_{u_2}-u_1 \,.\, v_{u_1}) \qquad \text{.. (12.15)}$$

$$\text{or} \quad \text{Work done/sec per lb (or kg) of water}=\frac{u_2 \,.\, v_{u_2}-u_1 \,.\, v_{u_1}}{g}$$

Since $\alpha_1=90°$, *i.e.* radial entrance,

$\therefore$ $\cos \alpha_1=0$.

$$\therefore \quad \text{Work done/sec per lb (or kg) of water}=\frac{u_2 \,.\, v_{u_2}}{g} \qquad \text{.. (12.16)}$$

(See also Eqn 1.37)

This is the energy supplied to the fluid by the impeller per lb (or kg) per sec. But the energy supplied to the fluid by the impeller is equal to the head generated, provided there is no loss inside the pump.

i.e. Head generated by the pump = Difference between the total enery of fluid at inlet and outlet of the pump.
= Manometric head (H_{mano})

$$\text{Thus} \quad \frac{u_2 \,.\, v_{u_2}}{g}=H_{mano}$$

.. ...(if there is no internal loss of the pump)

In practice there are always some head losses inside the pump as described in Art 12.21.

*The water approaching the impeller eye should acquire prerotation in the direction of impeller rotation. Prerotation in impellers corresponds to running in the direction of the train's motion before boarding a moving train.

$$\therefore \quad \frac{u_2 \cdot v_{u_2}}{g} = H_{mano} + \text{losses}$$

$$= H_{mano} + \triangle H_{mano}$$

$$\text{Manometric Efficiency} = \frac{H_{mano}}{H_{mano} + \triangle H_{mano}} = \frac{H_{mano}}{\dfrac{u_2 \cdot v_{u_2}}{g}}$$

i. e. $\text{Manometric Efficiency} = \dfrac{\text{manometric head}}{\text{energy given to the fluid/lb (or kg)/sec}}$

or
$$\eta_{mano} = \frac{H_{mano}}{\dfrac{u_2 \cdot v_{u_2}}{g}}$$

but
$$H_{mano} = H_{static} + H_f + \frac{v_d^2}{2g}$$

(Considering also the exit velocity head, which is generally neglected).

$$\therefore \quad \eta_{mano} = \frac{H_{static} + H_f + \dfrac{v_d^2}{2g}}{\dfrac{u_2 \cdot v_{u_2}}{g}} \quad \ldots(12.17)$$

12 23 Pressure Rise in Pump Impeller and Manometric Head—Applying Bernoulli's Theorem between inlet and outlet edges of impeller (See Fig 12.19).

Energy at inlet=energy at outlet—work done by impeller

$$\frac{p_1}{w} + \frac{v_1^2}{2g} = \frac{p_2}{w} + \frac{v_2^2}{2g} - \frac{u_2 \cdot v_{u_2}}{g}$$

$\therefore$ Pressure rise between outlet and inlet edges of impeller

$$\frac{p_2 - p_1}{w} = \frac{v_1^2}{2g} - \frac{v_2^2}{2g} + \frac{u_2 \cdot v_{u_2}}{g}$$

Now $\quad v_{u_2} = u_2 - v_{m_2} \cot \beta_2$

(See outlet velocity triangle Fig 12.19)

and
$$v_2^2 = v_{m_2}^2 + v_{u_2}^2$$
$$= v_{m_2}^2 + (u_2 - v_{m_2} \cot \beta_2)^2$$
$$= v_{m_2}^2 (1 + \cot^2 \beta_2) + u_2^2 - 2u_2 \cdot v_{m_2} \cot \beta_2$$

$$\therefore \quad \frac{p_2 - p_1}{w} = \frac{v_1^2}{2g} - \frac{1}{2g}\left\{ v_{m_2}^2 (1 + \cot^2 \beta_2) + u_2^2 - 2u_2 \cdot v_{m_2} \cot \beta_2 \right\} + \frac{u_2}{g}(u_2 - v_{m_2} \cot \beta_2)$$

but $1 + \cot^2 \beta_2 = \operatorname{cosec}^2 \beta_2$

$$\therefore \quad \frac{p_2 - p_1}{w} = \frac{1}{2g} \left(v_1^2 - v_{m_2}^2 \operatorname{cosec}^2 \beta_2 - u_2^2 + 2u_2 \cdot v_{m_2} \cot \beta_2 + 2u_2^2 - 2u_2 \cdot v_{m_2} \cot \beta_2\right)$$

or Pressure rise $= \frac{1}{2g} \left(v_1^2 + u_2^2 - v_{m_2}^2 \operatorname{cosec}^2 \beta_2\right)$ ...(12.18)

Manometric head $\quad H_{mano} = \eta_{mano} \cdot \frac{u_2 \cdot v_{u_2}}{g}$ (See Eqn 12.14)

$$= \eta_{mano} \cdot \frac{u_2 (u_2 - v_{m_2} \cot \beta_2)}{g} \quad ...(12.19)$$

Problem 12.5 A centrifugal fan has to deliver 150 ft³/sec, when running at 750 rpm. The diameter of the impeller at inlet is 21 in. and at outlet is 30 in. It may be assumed that the air enters radially with a speed of 50 ft/sec. The vanes are set backwards at outlet at 70° to the tangent, and the width at outlet is 4 in. The volute casing gives a 30% recovery of the outlet velocity head. The losses in the impeller may be taken as equivalent to 25% of the outlet velocity head. Blade thickness effects may be neglected. Determine the manometric efficiency and the pressure at the discharge.

(Punjab University—1958 Supp)

Solution

$Q = 150$ ft³/sec $\qquad N = 750$ rpm

$D_1 = 21$ in. $\qquad D_2 = 30$ in.

$v_1 = 50$ ft/sec $\qquad \beta_2 = 70°$

$B_2 = 4$ in.

Peripheral velocity at inlet $\quad u_1 = \frac{\pi D_1 N}{60}$

$$= \pi \times \left(\frac{21}{12}\right) \times \frac{750}{60} = 68{\cdot}7 \text{ ft/sec}$$

Peripheral velocity at outlet $u_2 = \frac{\pi D_2 N}{60}$

$$= \pi \times \frac{30}{12} \times \frac{750}{60} = 98{\cdot}2 \text{ ft/sec}$$

Discharge at outlet $\quad Q =$ velocity of flow at outlet $\times$ area of flow at outlet

or $\quad 150 = v_{m_2} \cdot \pi D_2 B_2$

$$\therefore \quad v_{m_2} = \frac{150}{\pi D_2 B_2} = \frac{150}{\pi \times \frac{30}{12} \times \frac{4}{12}}$$

$$= 57{\cdot}3 \text{ ft/sec}$$

From outlet velocity triangle (See Fig 12.19)

$$\tan \beta_2 = \frac{v_{m_2}}{u_2 - v_{u_2}}$$

or $\tan 70° = \dfrac{57{\cdot}3}{98{\cdot}2 - v_{u_2}}$

$$98{\cdot}2 - v_{u_2} = \frac{57{\cdot}3}{2{\cdot}747} = 20{\cdot}8$$

$\therefore$ $v_{u_2} = 98{\cdot}2 - 20{\cdot}8 = 77{\cdot}4$ ft/sec

$\therefore$ Absolute velocity at exit $v_2 = \sqrt{v_{m_2}^2 + v_{u_2}^2}$

$$= \sqrt{57{\cdot}3^2 + 77{\cdot}4^2} = \sqrt{3{,}280 + 6{,}000}$$
$$= \sqrt{9{,}280} = 96{\cdot}3 \text{ ft/sec}$$

Applying Bernoulli's theorem at inlet and outlet of impeller—

$$\frac{p_1}{w} + \frac{v_1^2}{2g} = \frac{p_2}{w} + \frac{v_2^2}{2g} - \frac{v_{u_2} \cdot u_2}{g}$$

(Assuming $\alpha_1 = 90°$)

or $v_{u_1} = 0$

or $\dfrac{p_2}{w} - \dfrac{p_1}{w} = \dfrac{v_1^2 - v_2^2}{2g} + \dfrac{v_{u_2} \cdot u_2}{g}$

$$= \frac{50^2 - 96{\cdot}3^2}{64{\cdot}4} + \frac{77{\cdot}4 \times 98{\cdot}3}{32{\cdot}2}$$
$$= \frac{2{,}500 - 9{,}270}{64{\cdot}4} + 236$$
$$= -105 + 236$$
$$= +\mathbf{131 \text{ ft of air}}$$

Velocity head at outlet $= \dfrac{v_2^2}{2g} = \dfrac{96{\cdot}3^2}{64{\cdot}4} = 144$ ft of air

Head recovered in volute casing $= 30\%$ of $\dfrac{v_2^2}{2g}$

$$= 0{\cdot}3 \times 144 = 43{\cdot}2 \text{ ft of air}$$

Head lost in impeller $= 25\%$ of $\dfrac{v_2^2}{2g}$

$$= 0{\cdot}25 \times 144 = 36 \text{ ft of air}$$

Net pressure rise $= \dfrac{p_2 - p_1}{w} +$ head recovered in casing—head lost in impeller

$= 131 + 43{\cdot}2 - 36 =$ **138·2 ft of air**

$$\eta_{mano} = \frac{\text{Net pressure rise}}{\text{Head delivered per lb of air}} = \frac{138{\cdot}2}{v_{u_2} \cdot \dfrac{u_2}{g}}$$

$$= \frac{138{\cdot}2}{236} = \mathbf{0{\cdot}585} \quad \textit{Answer}$$

12.24 Minimum Starting Speed of Centrifugal Pump—When the pump is started, there will not be any flow of water until the pressure

rise in the impeller is large enough to overcome the gross or manometric head.

Centrifugal head or pressure head caused by the centrifugal force on rotating water when the impeller is rotating, but there is *no flow*

$$=\frac{u_2^2-u_1^2}{2g} \qquad \text{(See Eqn 2 2)}$$

Flow will commence only if—

$$\frac{u_2^2-u_1^2}{2g} \geqq H_{mano}$$

or
$$\frac{u_2^2-u_1^2}{2g} \geqq \eta_{mano} \cdot \frac{u_2 \cdot v_{u_2}}{g} \qquad \text{.. (12 20)}$$

$$\text{Theoretically } \frac{u_2^2-u_1^2}{2g} \geqq \frac{u_2 \cdot v_{u_2}}{g} \qquad \text{. (12 20}a)$$

(See Solved Problem 2 5).

12.25 Efficiencies of Centrifugal Pump—

a) **Overall Efficiency**—As already discussed in Art 12.18, the overall efficiency of a pump is

$$\eta_{overall}=\frac{\text{Fluid or water horsepower output}}{\text{horsepower input to pump shaft}}$$

$$=\frac{\text{WHP}}{\text{SHP}} \qquad \text{. (12 21)}$$

This is also known as *gross efficiency*.

The shaft horsepower (SHP) of a centrifugal pump is required to supply the following powers, i e

$$P_{shaft}=P_{input\ to\ impeller}+P_{leakage}+P_{mech\ loss} \qquad \text{.. (12.22)}$$

where P_{shaft}=SHP=(BHP of driving unit)—(HP lost in coupling)

$P_{input\ to\ impeller}$=energy given to the impeller per lb (or kg) per sec

$$=\frac{u_2 \cdot v_{u_2}}{g} \text{ per lb (or kg) of water}$$

$$=\text{WHP}+P_{hyd} \qquad \text{. (12.23)}$$

$$\text{WHP}=\frac{w \cdot Q \cdot H_{mano}}{550}$$

$$P_{hyd}=\frac{w \cdot Q \cdot (\triangle H_{mano})}{550}$$

=Power required to overcome head losses due to

i) Circulatory or secondary flow

ii) Frictions of volute and impeller

iii) Turbulence.

$P_{leakage}$=Power required to overcome leakage

This is the power required to pump through the impeller the additional amount of water which leaks and is not delivered. The leakage water is that which slips back from the pressure side to the suction side of impeller. It may also include the water which is used for balancing purposes. (See Fig 12.26).

$P_{mech\ loss}$=Power required to overcome *all* mechanical losses, which are—

i) Disc friction loss (See also Art 7 28*d*)

ii) Bearings and glands losses

b) **Mechanical Efficiency** is the ratio of the power delivered by the impeller to the fluid, to the horsepower input to the pump shaft, *i e.*

$$\eta_{mech}=\frac{w\left(\frac{Q+\Delta Q}{550}\right)\cdot\frac{u_2\cdot v_{u_2}}{g}}{\text{SHP}} \quad \ldots(12\ 24)$$

$$=\frac{(SHP)-P_{mech\ loss}}{SHP} \quad \ldots(12.24a)$$

c) **Volumetric Efficiency**—

$$\eta_Q=\frac{Q}{Q+\Delta Q} \quad .\ (12\ 25)$$

where Q=Discharge utilised
ΔQ=Amount of leakage

d) **Manometric Efficiency**—As already discussed

$$\eta_{mano}=\frac{\text{actual measured head}}{\text{head imparted to fluid by impeller}}$$

$$=\frac{H_{mano}}{\frac{u_2\cdot v_{u_2}}{g}}=\frac{w\left(\frac{Q+\Delta Q}{550}\right)\cdot H_{mano}}{\text{SHP}-P_{mech\ loss}} \quad .\ (12\ 26)$$

This is also known as *hydraulic efficiency.*

From the above Eqns 12.5 to 12 6

$$\eta_{overall}=\eta_{mech}\cdot\eta_Q\cdot\eta_{mano} \quad .\ (12.27)$$

12.26 Virtual, Ideal and Manometric Heads of Centrifugal Pump—The fundamental equation (Eqn 12.14) of a centrifugal pump is derived as—

$$\frac{H_{mano}}{\eta_{mano}}=\frac{u_2\cdot v_{u_2}}{g}$$

where H_{mano}=actual measured head

and $\frac{u_2\cdot v_{u_2}}{g}$=theoretical head or better known as **virtual** head (H_{vir})

i e. $$H_{vir}=\frac{u_2\cdot v_{u_2}}{g} \quad ..\ (12.28)$$

The virtual head is obtained if there is no hydraulic loss (See P_{lva} in Art 12.25 above), *i.e.*, the flow is frictionless and non-turbulent, and the impeller has infinite number of blades.

Influence of Number of Blades—The virtual head (See Eqn 12.14 and 12.28) is developed considering infinite number of blades in the impeller through which the water has to pass. In practice the impeller must have only a finite number of blades, in which case the velocity of whirl (v_{u_2}) is reduced owing to the secondary or circulatory flow effects in the impeller. The new value of velocity of whirl is given by

$$v'_{u_2} = v_{u_2} - \Delta v_{u_2}$$

The head developed, taking v'_{u_2} into consideration, will be known as *ideal or Euler head* H_i

$$\text{Thus} \quad H_i = \frac{u_2 \cdot v'_{u_2}}{g} = \frac{u_2(v_{u_2} - \Delta v_{u_2})}{g} \qquad \ldots (12.29)$$

The ideal head $H_i = \varepsilon\, H_{vir}$ where ε is a Greek letter and known as *Correction Factor* for finite number of blades in an impeller.

$$\varepsilon = \frac{H_i}{H_{vir}} = \frac{v_{u_2} - \Delta v_{u_2}}{v_{u_2}} = 1 - \frac{\Delta v_{u_2}}{v_{u_2}} \qquad \ldots (12.30)$$

The following table showing the relation between ε and z the number of blades, is due to Pfleiderer.

TABLE 12.4

Correction Factor ε vs z

z	1	2	4	6	10	20	∞
ε	0·25	0·4	0·572	0·666	0·77	0·87	1·0

It should be seen that the reduction in head from H_{vir} to H_i does not represent a loss but a discrepancy, for secondary flow effects, which is not taken into account in the basic assumptions.

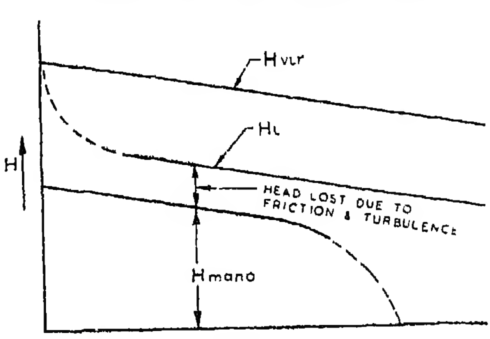


Fig 12.20 Virtual, Ideal and Manometric Heads

Manometric head (H_{mano}) or the actual head developed by the pump is still smaller than ideal head H_i, because there are friction losses in suction passages, interior of impeller and discharge volute. Over and above there is head loss due to turbulence. Fig 12.20 shows the head curves drawn against discharge.

12.27 Overall Head Co-efficient or Speed Ratio—From the fundamental equation 12.13*a* derived above,

$$\frac{1}{\eta_{mano}}=\frac{u_2 \cdot v_2 \cos \alpha_2 - u_1 \cdot v_1 \cos \alpha_1}{g \cdot H_{mano}}$$

$$=\frac{2(u_2\, v_{u_2} - u_1 \cdot v_{u_1})}{2g\, H_{mano}}$$

Introducing non-dimensional velocity co-efficients*

$$K_{u_2}=\frac{u_2}{\sqrt{2g\, H_{mano}}},\quad K_{v_{u_2}}=\frac{v_{u_2}}{\sqrt{2g\, H_{mano}}} \text{ etc,}$$

$$\frac{1}{\eta_{mano}}=2(K_{u_2} \cdot K_{v_{u_2}} - K_{u_1} \cdot K_{v_{u_1}})$$

or $$\eta_{mano}=\frac{1}{2(K_{u_2} \cdot K_{v_{u_2}} - K_{u_1} \cdot K_{v_{u_1}})} \qquad \ldots(12.31)$$

If $\alpha_1=90°$, $K_{v_{u_1}}=0$

and $$\eta_{mano}=\frac{1}{2\, K_{u_2} \cdot K_{v_{u_2}}} \qquad \ldots(12.32)$$

Since $\eta_{mano}<1$, the product $K_{u_2} \cdot K_{v_{u_2}} > \frac{1}{2}$.

If $K_{u_2}=1$ (See Table 12.5), then $K_{v_{u_2}} \approx 0{\cdot}5$.

TABLE 12.5

Practical Data for Speed Ratio K_{u_2}

	Small Pumps	Large Pumps
Pumps without guide vanes	$K_{u_2}=1{\cdot}06$	$K_{u_2}=0{\cdot}95$
Pump with guide vanes	$K_{u_2}=1{\cdot}01$	$K_{u_2}=0{\cdot}93$

Problem 12.6 A diffusion type centrifugal pump has a suction lift of 5 ft and the delivery tank is 45 ft above the pump. The velocity of water in the delivery pipe is 5 ft/sec. The radial velocity of flow through the wheel is 10 ft/sec and the tangent to the vane at exit from the wheel makes an angle of 120° with the direction of motion. Assuming that the water enters radially and neglecting friction and other losses, find

*The factor K_{u_2} is known as the "speed ratio" as in the case of turbines. See Eqn 3.28a.

a) the velocity of wheel at exit,
b) the pressure head at exit from the wheel,
c) the velocity head at exit from the wheel
and *d*) the desirable direction of the fixed guide vanes.

(*AMIE—April 1949*)

Solution

$H_s = 5$ ft	$\beta_2 = 180° - 120° = 60°$
$H_d = 45$ ft	$\alpha_1 = 90°$ ($\because$ water enters radially)
$v_d = 5$ ft/sec	$v_{m_2} = v_{m_1} = 10$ ft/sec

$$H_{mano} = H_{stat} = H_s + H_d = 5 + 45 = 50 \text{ ft}$$

(neglecting all frictional losses)

or $\quad H = H_{mano} = 50$ ft

$$\tan \beta_2 = \frac{v_{m_2}}{u_2 - v_{u_2}} \quad \text{(See Fig 12.19)}$$

or $$\tan 60° = \frac{10}{u_2 - v_{u_2}}$$

$$\therefore \quad (u_2 - v_{u_2}) = \frac{10}{1{\cdot}732} = 5{\cdot}78 \text{ ft/sec}$$

Work done per lb of water $= \dfrac{v_{u_2} \cdot u_2}{g} = H$

(Assuming the efficiency to be 100% *i.e.* neglecting all losses)

or $\quad v_{u_2} \cdot u_2 = 50 \times 32{\cdot}2 = 1{,}610$

Substituting for v_{u_2}

$$(u_2 - 5{\cdot}78)\, u_2 = 1{,}610 \quad \text{or} \quad u_2^2 - 5{\cdot}78\, u_2 - 1{,}610 = 0$$

whence $$u_2 = 43{\cdot}09 \text{ ft/sec}$$

$$v_{u_2} = \frac{1{,}610}{u_2} = \frac{1{,}610}{43{\cdot}09} = 37{\cdot}4 \text{ ft/sec}$$

and $$v_2 = \sqrt{v_{m_2}^2 + v_{u_2}^2} = \sqrt{100 + 37{\cdot}4^2}$$

$$= \mathbf{38{\cdot}7 \text{ ft/sec}} \qquad \textit{Answer}$$

$\therefore$ Velocity head at exit $= \dfrac{v_2^2}{2g} = \dfrac{38{\cdot}7^2}{64{\cdot}4} = \mathbf{23{\cdot}15 \text{ ft}}$ *Answer*

Now $\quad \dfrac{v_2^2}{2g} + \dfrac{p_2}{w} = H_d$

$\therefore$ Pressure head at the exit of the wheel $\dfrac{p_2}{w} = H_d - \dfrac{v_2^2}{2g}$

$$= 45 - 23{\cdot}15$$

$$= \mathbf{21{\cdot}85 \text{ ft}} \qquad \textit{Answer}$$

Desirable direction of fixed guide vane is α_2

where $\quad v_2 \cos \alpha_2 = v_{u_2}$

$$\therefore \qquad \cos \alpha_2 = \frac{v_{u_2}}{v_2} = \frac{37\ 4}{38\ 7} = 0{\cdot}965$$

$$\text{or} \qquad \alpha_2 = \mathbf{15^\circ - 12'} \qquad \textit{Answer}$$

Problem 12.7 A centrifugal pump lifts water under a static head of 119 ft (or 36·3 *m*) of water of which 13 ft (or 3 965 *m*) is suction lift. Suction and delivery pipes are both 6 inches (or 152·4 *mm*) in diameter. The head loss in suction pipe is 6 8 ft (or 1·826 *m*) and in the delivery pipe 23·2 ft (or 7 08 *m*). The impeller is 15 inches (or 381 *mm*) in diameter and 1 inch (or 25 4 *mm*) wide at mouth and revolves at 1,200 rpm. Its exit blade angle is 35°. If the manometric efficiency of the pump is 82%, determine the discharge and the pressure at the suction and delivery branches of the pump.

(AMIE—Nov 1948)

Solution

$H_{stat} = 119$ ft (or 36·3 *m*)	$D_2 = 15$ in. (or 381 *mm*)
$H_s = 13$ ft (or 3·965 *m*)	$b_2 = 1$ in. (or 25·4 *mm*)
$H_d = 119 - 13 = 106$ ft (or 32·335 *m*)	$N = 1{,}200$ rpm
$H_{L_s} = 6{\cdot}8$ ft (or 1·826 *m*)	$\eta_{mano} = 82\%$
$H_{L_d} = 23\ 2$ ft (or 7·08 *m*)	$d_d = d_s = 6$ in. (or 152·4 *mm*)

Total head to be supplied by the pump $H_{mano} = H_{stat} + \Sigma H_L$

$$= 119 + 6{\cdot}8 + 23{\cdot}2$$
$$= 149 \text{ ft of water}$$

(or $\quad H_{mano} = 36{\cdot}3 + 1{\cdot}826 + 7{\cdot}08 = 45{\cdot}206$ *m* of water)

Peripheral velocity of wheel at outlet $u_2 = \dfrac{\pi \cdot D_2 \cdot N}{60}$

$$= \frac{\pi \times \frac{15}{12} \times 1{,}200}{60} = 78{\cdot}5 \text{ ft/sec}$$

$$\left[\quad \text{or} \quad u_2 = \frac{\pi \times 0{\cdot}381 \times 1{,}200}{60} = 23{\cdot}9 \ m/\text{sec} \quad \right]$$

Assume flow at the inlet to be radial, $\quad \therefore \quad \alpha_1 = 90^\circ$

Work done per lb of water $= \dfrac{v_{u_2} \cdot u_2}{g}$

Manometric efficiency $\qquad = \dfrac{H_{mano}}{\dfrac{v_{u_2} \cdot u_2}{g}}$

or $$0{\cdot}82=\frac{149}{\dfrac{v_{u_2}\times 78{\cdot}5}{32{\cdot}2}}$$

$$\left[\text{or}\quad 0{\cdot}82=\frac{45{\cdot}206}{\dfrac{v_{u_2}\times 23{\cdot}9}{9\ 81}}\right]$$

or $$v_{u_2}=\frac{149\times 32{\cdot}2}{0{\cdot}82\times 78\ 5}=74{\cdot}5 \text{ ft/sec}$$

$$\left(\text{or}\quad v_{u_2}=\frac{45{\cdot}206\times 9{\cdot}81}{0\ 82\times 23{\cdot}9}=22{\cdot}68\ m/\text{sec}\right)$$

Now $$\tan\beta_2=\frac{v_{m_2}}{u_2-v_{u_2}}\quad \text{(See Fig 12 19)}$$

$$\tan 35^\circ=\frac{v_{m_2}}{78{\cdot}5-74{\cdot}5}\quad \left[\text{or } \tan 35^\circ=\frac{v_{m_2}}{23{\cdot}9-22{\cdot}68}\right]$$

or $$v_{m_2}=0{\cdot}7002\times 4=2{\cdot}8 \text{ ft/sec}$$

(or $$v_{m_2}=0{\cdot}7002\times 1{\cdot}22=0{\cdot}855\ m/\text{sec})$$

and $$Q=(\pi\ .\ D_2\ .\ b_2)\ .\ v_{m_2}$$

$$=\left(\pi\times\frac{15}{12}\times\frac{1}{12}\right)\times 2{\cdot}8=\mathbf{0\ 916\ cfs}\qquad \textit{Answer}$$

[or $$Q=(\pi\times 0{\cdot}381\times 0{\cdot}0254)\times 0{\cdot}855=\mathbf{0{\cdot}026\ m^3/sec}\qquad \textit{Answer}]$$

Velocity in suction or delivery pipe $=v_d=v_s=\dfrac{Q}{a_s}$

$$=\frac{0{\cdot}916}{\frac{\pi}{4}\times(0{\cdot}5)^2}=4{\cdot}66 \text{ ft/sec}$$

$$\left[\text{or}\quad v_d=v_s=\frac{0{\cdot}026}{\frac{\pi}{4}\times 0{\cdot}1524^2}=1{\cdot}422\ m/\text{sec}\right]$$

$$\text{Velocity head}=\frac{4\ 66^2}{64{\cdot}4}=0{\cdot}337 \text{ ft of water}$$

$$\left[\text{or}\quad =\frac{1{\cdot}442^2}{2\times 9{\cdot}81}=0{\cdot}0103\ m \text{ of water}\right]$$

Total effective pressure on the delivery side $=H_d+H_{L_d}+\dfrac{v_d^2}{2g}$

$$=106+23{\cdot}2+0{\cdot}337=129{\cdot}537 \text{ ft}$$

(or $$=32{\cdot}335+7{\cdot}08+0{\cdot}0103=39{\cdot}4253\ m \text{ of water})$$

or $$=\frac{129{\cdot}537\times 62{\cdot}4}{144}=\mathbf{56{\cdot}1\ lb/sq\ in.}\qquad \textit{Answer}$$

[or $39\ 4253 \times \frac{1,000}{100 \times 100} =$ **3 94253 kg/cm²** *Answer*]

Pressure on the suction side$=H_s+H_{L_s}+\frac{v_s^2}{2g}=13+6{\cdot}8+0\ 337$

$=20{\cdot}137$ ft of water vacuum

or $34-20{\cdot}137=13{\cdot}863$ ft of water absolute

or $13{\cdot}863 \times \frac{62{\cdot}4}{144} =$ **6 lb/sq in. absolute** *Answer*

[or Pressure on the suction side$=3{\cdot}965+1{\cdot}826+0\ 0103$

$=5{\cdot}8013$ *m* of water vacuum

oı $10{\cdot}36-5\ 8013 \approx 4{\cdot}56$ *m* of water absolute

or $4{\cdot}56 \times \frac{1,000}{100 \times 100} =$ **0 456 kg/cm² absolute** *Answer*]

Problem 12.8 A radial, single stage, double suction, centrifugal pump is manufactured for the following data :

$Q=1,000$ gpm	$D_1=4$ in.
$H=100$ ft	$D_2=11\frac{1}{2}$ in.
$N=1,750$ rpm	$b_1=1$ in. per side
$\eta_{overall}=50\%$	$b_2=\frac{15}{16}$ in. in total
Leakage losses $=30$ gpm	$\alpha_1=90°$
Mechanical losses$=1{\cdot}41$ HP	$\beta_2=27°$

Contraction factor due to vane thickness$=0{\cdot}87$

Determine—

a) Inlet Vane angle β_1 ,

b) Angle at which the water leaves the wheel α_2 ,

c) Speed ratio K_{u_2} ,

d) Absolute velocity of water leaving the impeller v_2,

e) Manometric efficiency,

f) Volumetric and mechanical efficiencies.

Solution

Total quantity of water handled by the pump

$$Q_{total}=Q_{delivered}+Q_{leaked}=1,000+30$$

$$=1,030 \text{ gpm}$$

$$Q \text{ per side} = \frac{1,030}{2}=515 \text{ gpm}$$

$$=\frac{515 \times 10}{62{\cdot}4 \times 60}=1{\cdot}375 \text{ cfs}$$

a) Peripheral speed at inlet

$$u_1=\frac{\pi \cdot D_1 \cdot N}{60 \times 12}=\frac{\pi \times 4 \times 1,750}{60 \times 12}=30{\cdot}5 \text{ ft/sec}$$

Area of flow at inlet $= \pi \; D_1 \,.\, b_1 \times$ contraction factor

$$= \pi \times \frac{4}{12} \times \frac{1}{12} \times 0{\cdot}87 = 0\ 0758 \text{ sq ft}$$

$\therefore$ Velocity of flow at inlet

$$v_{m_1} = \frac{Q}{\text{area of flow}} = \frac{1{\cdot}375}{0\ 0758} = 18{\cdot}1 \text{ ft/sec}$$

Now $\qquad \alpha_1 = 90°$

$\therefore$ From inlet velocity triangle (See Fig 12 19)

$$\tan \beta_1 = \frac{v_{m_1}}{u_1} = \frac{18{\cdot}1}{30\ 5} = 0{\cdot}593$$

or $\qquad \beta_1 = \tan^{-1} 0{\cdot}593 = \mathbf{30° - 40'}$ $\qquad$ *Answer*

b) Area of flow at outlet

$$= \pi \,.\, D_2 \; b_2 \times \text{contraction factor}$$

where $\qquad b_2 = \frac{15}{16} \times \frac{1}{2} = \frac{15}{32}$ inch for one side

$$\therefore \qquad \text{Area} = \pi \times \frac{11{\cdot}5}{12} \times \frac{15}{32 \times 12} \times 0{\cdot}87$$

$$= 0\ 1023 \text{ sq ft}$$

$\therefore$ Velocity of flow at outlet

$$v_{m_2} = \frac{Q}{\text{area of flow}} = \frac{1{\cdot}375}{0{\cdot}1023} = 13{\cdot}45 \text{ ft/sec}$$

Peripheral speed at outlet

$$u_2 = \frac{\pi \,.\, D_2 \,.\, N}{60 \times 12} = \frac{\pi \times 11{\cdot}5 \times 1{,}750}{60 \times 12} = 87{\cdot}8 \text{ ft/sec}$$

Now $\qquad \beta_2 = 27°$

$\therefore$ From outlet velocity triangle (See Fig 12.19)

$$\tan \beta_2 = \frac{v_{m_2}}{u_2 - v_{u_2}}$$

or $$\tan 27° = \frac{13\ 45}{87\ 8 - v_{u_2}}$$

or $$v_{u_2} = 61{\cdot}4 \text{ ft/sec}$$

Further, $$\tan \alpha_2 = \frac{v_{m_2}}{v_{u_2}} = \frac{13{\cdot}45}{61{\cdot}4} = 0\ 219$$

$\therefore \qquad \alpha_2 = \tan^{-1} 0{\cdot}219 = \mathbf{12° - 21'}$ $\qquad$ *Answer*

c) Speed ratio $K_{u_2} = \dfrac{u_2}{\sqrt{2g\, H_{mano}}} = \dfrac{87{\cdot}8}{\sqrt{64{\cdot}4 \times 100}} = \mathbf{1{\cdot}905}$ $\qquad$ *Answer*

d) Absolute velocity of water leaving the impeller

$$v_2 = \frac{v_{u_2}}{\cos \alpha_2} = \frac{61{\cdot}4}{0{\cdot}9772} = \mathbf{62\ 8\ ft/sec}$$ $\qquad$ *Answer*

e) Manometric efficiency—

$$\frac{H_{mano}}{\eta_{mano}}=\frac{u_2 \, . \, v_{u_2}}{g} \quad \text{(See Eqn 12·14)}$$

or
$$\eta_{mano}=\frac{g \, . \, H_{mano}}{u_2 \, . \, u_{u_2}}=\frac{32{\cdot}2\times 100}{87{\cdot}8\times 61{\cdot}4}=0{\cdot}598$$

or **59·8%** *Answer*

f) Volumetric efficiency $\eta_Q=\dfrac{Q}{Q_{total}}=\dfrac{1{,}000}{1{,}030}=0{\cdot}971$

or **97·1%** *Answer*

$$\text{Water HP}=\frac{w \, . \, Q \, . \, H_{mano}}{550}=62{\cdot}4\times\frac{500\times 10}{62{\cdot}4\times 60}\times\frac{100}{550}$$
$$=15{\cdot}15 \text{ HP}$$

$$\text{Shaft horsepower SHP}=\frac{\text{Water HP}}{\eta_{overall}}=\frac{15{\cdot}15}{0{\cdot}55}=27{\cdot}55 \text{ HP}$$

$$\eta_{mech}=\frac{SHP-P_{mech\ loss}}{SHP}=\frac{27{\cdot}55-1{\cdot}41}{27{\cdot}55}$$
$$=0{\cdot}948 \quad \text{or} \quad \mathbf{94{\cdot}8\%} \quad \textit{Answer}$$

12.28 Note on Fundamental Equation—A little consideration of the various factors involved in the fundamental equation would suggest that, if for a particular pump the outlet peripheral speed u_2 and the outlet relative velocity of water w_2 are kept constant, the velocity of whirl v_{u_2} would vary with angle α_2. The smaller the angle α_2 the larger would be v_{u_2}, thus increasing the total head H_{mano}

However, the best practical value of α_2 is about 20°. Further, if w_2 is higher, v_2 will also be correspondingly higher for a constant value of u_2 and the head H_{mano} will be more. But a higher w_2 also implies higher frictional loss and therefore it is not advisable to raise the head in this manner.

Practical Data :

$\alpha_1=90°$ $\qquad$ $\beta_1=25°$ to $30°$ $\qquad$ $\beta_2=30°$ to $40°$

$\dfrac{\tan\alpha_2}{\tan\beta_2}=0{\cdot}6$ $\qquad$ $v_2=0{\cdot}6$ to $0{\cdot}8$ times u_2

12.29 Breadth of Impeller—Let Q be the quantity in cusecs of (or m^3/sec) water flowing through the impeller. Then, from the equation of continuity,

$$Q=\pi \, D_1 \, . \, B_1 \, . \, v_{m_1}$$
$$=\pi \, D_2 \, . \, B_2 \, . \, v_{m_2}$$
$$=\pi \, D \, . \, B \, . \, v_m$$

where D and B are the diameter and breadth of the impeller at points indicated by the suffixes (Fig 12.21a), and v_{m_1} and v_{m_2} are the velocities of flow at inlet and outlet respectively.

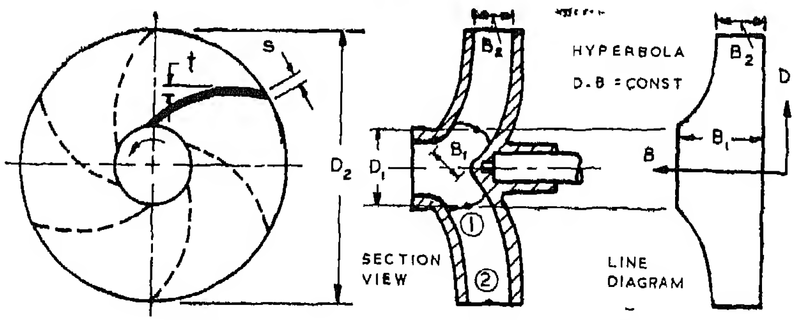


Fig 12.21 (a) Breadth and Diameter of Impeller

From the above equations in which the thickness of the vanes has not been considered, approximate breadths of the impeller at inlet and outlet can be worked out as hereunder :

$$B_1 = \frac{Q}{\pi\, D_1 \,.\, v_{m_1}} \qquad \ldots(12.33)$$

and

$$B_2 = \frac{Q}{\pi\, D_2 \,.\, v_{m_2}} \qquad \ldots(12{\cdot}34)$$

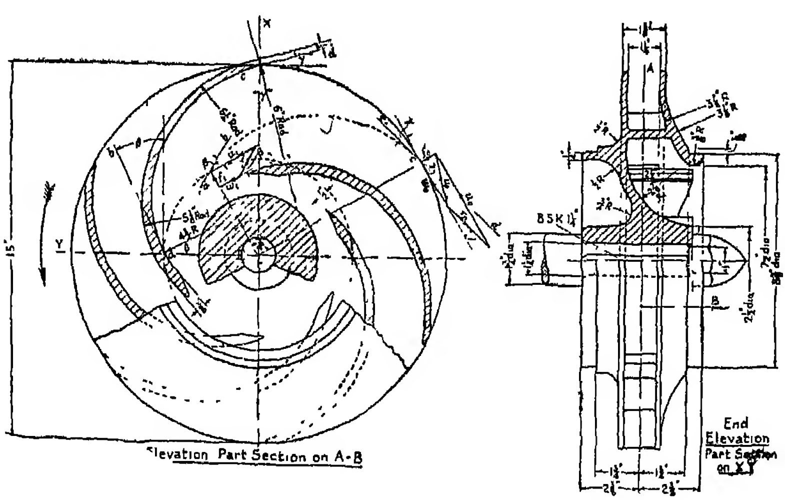


Fig 12.21 (b) Section through an Impeller of a Centrifugal Pump

The velocities of flow at inlet and outlet are generally taken to be equal

i.e., $v_{m_1} = v_{m_2}$

$\therefore \quad D_1 \,.\, B_1 = D_2 \,.\, B_2$

or $\quad D \,.\, B = \text{const} \qquad \ldots(12.35)$

A curve B vs D will, therefore, be a hyperbola (See Fig 12.21a). This is, generally, the theoretical curvature of impeller blades.

For a more accurate computation, vane thickness s should be considered and if z be the number of blades.

$$Q=(\pi\ D_1-z\ .\ s_1)\ .\ B_1\ .\ v_{m_1}$$
$$=(\pi\ D_2-z\ .\ s_2)\ .\ B_2\ .\ v_{m_2}$$
$$=(\pi\ D-z\ .\ s)\ .\ B\ .\ v_m \qquad ...(12.36)$$

Here s is the thickness of vane measured along the circle, and if t be the actual metal thickness,

$$s=\frac{t}{\sin\beta}$$

12.30 Different Shapes of Blades—The blade of an impeller may have one of the following different shapes :

	Shape	α_1	β_1	α_2	β_2	η %
i)	Blades bent backward	$\frac{\pi}{2}$	$<\frac{\pi}{2}$	$<\frac{\pi}{2}$	$<\frac{\pi}{2}$	85 to 90
ii)	Straight blades	$\frac{\pi}{2}$	$<\frac{\pi}{2}$	$<\frac{\pi}{2}$	$<\frac{\pi}{2}$	≈ 80
iii)	Blades ending radially	$\frac{\pi}{2}$	$<\frac{\pi}{2}$	$<\frac{\pi}{2}$	$\frac{\pi}{2}$	80 to 85
iv)	Blades bent forward	$\frac{\pi}{2}$	$<\frac{\pi}{2}$	$\frac{\pi}{2}$	$>\frac{\pi}{2}$	≈ 75

In order to have a high efficiency, the centrifugal pumps are generally provided with impellers having their blades bent backwards. Straight blades are used for small pumps where economy is the main consideration. Blades ending radially are better than straight blades but cannot compete effectively with the first type although they have the advantage of being cheap. Blades bent forward yield a very low head efficiency and are, therefore, not used.

12.31 Curvature and Proportions of Blades—

i) **Blades Bent Backwards** : With reference to Fig 12.22, if M_2 be the centre of curvature of the impeller blade section as shown, then from triangle $M_1 M_2 P_1$,

$$(M_1\ M_2)^2=\rho^2+R_1^2-2\rho\ R_1\cos\beta_1$$

and from $\triangle\ M_1\ M_2\ P_2$,

$$(M_1\ M_2)^2=\rho^2+R_2^2-2\rho\ R_2\cos\beta_2$$

Solving for ρ

$$R_1^2+\rho^2-2R_1\ \rho\cos\beta_1=R_2^2+\rho^2-2R_2\ \rho\cos\beta_2$$

or $\qquad R_1^2-2R_1\ \rho\cos\beta_1=R_2^2-2R_2\ \rho\cos\beta_2$

or
$$\rho=\frac{R_2^2-R_1^2}{2(R_2\cos\beta_2-R_1\cos\beta_1)} \qquad ...(12.37)$$

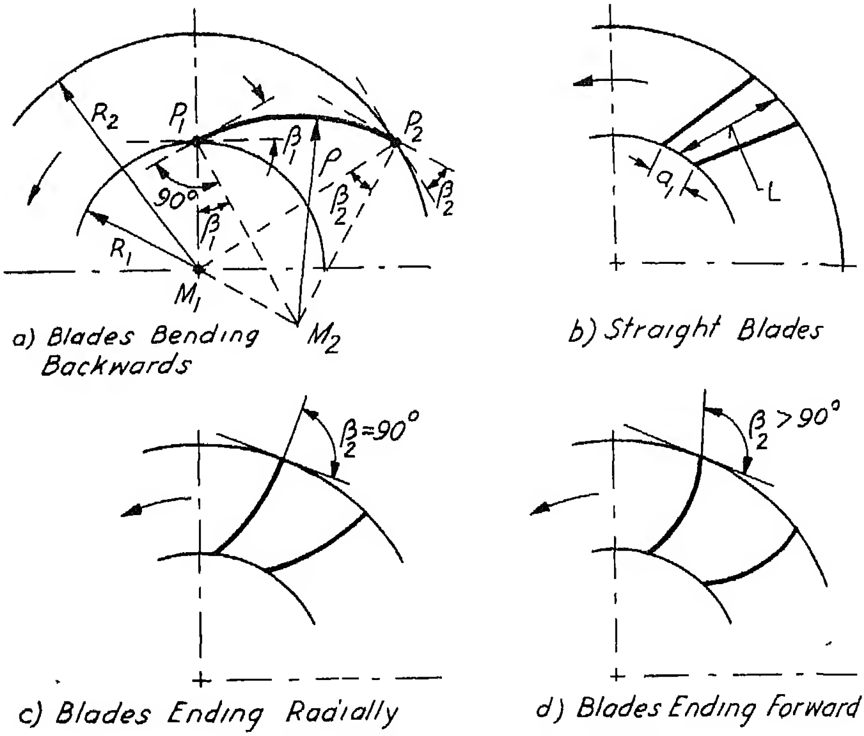


Fig 12.22 Determination of Blade Curvature

ii) **Straight Blades :**

Radius of curvature $\rho = \infty$

$\therefore \quad R_2 \cos \beta_2 - R_1 \cos \beta_1 = 0$

or
$$\frac{R_1}{R_2} = \frac{\cos \beta_2}{\cos \beta_1} = \nu \text{ (say)} \qquad \ldots(12.38)$$

Generally, the ratio ν varies from $\frac{1}{2}$ to $\frac{1}{4}$

also $\qquad L = 3\,a_1$ to $5\,a_1$ $\qquad$ (See Fig 12.22*b*)

iii) **Blades Ending Radially :**

Here $\quad \beta_2 = \frac{\pi}{2}, \qquad \therefore \quad \cos \beta_2 = 0$

and
$$\rho = \frac{R_2^2 - R_1^2}{-2R_1 \cos \beta_1} \qquad \ldots(12.39)$$

The negative sign shows reversal of curvature *i.e.*, from convex forward to concave forward.

iv) **Blades Bent Forward :**

$$\beta_2 > \frac{\pi}{2} \qquad \therefore \quad \cos \beta_2 < 0$$

and
$$\rho = \frac{R_2^2 - R_1^2}{2\,(R_2 \cos \beta_2 - R_1 \cos \beta_1)} \qquad \ldots(12.40)$$

Since $\cos \beta_2$ is negative, the two terms in the denominator are additive and ρ is small. Also, the negative sign indicates concave curvature forward.

12 32 Diameters of Impeller (Fig 12.21*a*)

i) **Outside Diameter of Impeller :**

a) **FPS Units –**

As in the case of reaction (specially Francis) turbines the outlet peripheral velocity u_2 of pump is given by :

$$u_2 = \frac{\pi \cdot D_2 \cdot N}{60 \times 12}$$ where D_2=outside dia of impeller in inches,

N=rpm of pump shaft.

If H is the total head in ft

$$u_2 = K_{u_2} \cdot \sqrt{2g\,H}$$

$$\therefore \quad D_2 = \frac{60 \times 12}{\pi \cdot N} \cdot K_{u_2} \cdot \sqrt{2g} \cdot \sqrt{H} \text{ in.}$$

$$= \frac{1{,}840 \cdot K_{u_2} \cdot \sqrt{H}}{N} \text{ in.} \qquad ...(12.41)$$

$$\text{if} \quad K_{u_2} \approx 1, \text{ then } D_2 \approx \frac{1{,}840 \cdot \sqrt{H}}{N} \text{ in.} \qquad ..(12.41a)$$

b) **Metric Units—**

$$u_2 = \frac{\pi \cdot D_2 \cdot N}{60}$$

also $u_2 = K_{u_2} \cdot \sqrt{2g\,H}$

$$\therefore \quad D_2 = \frac{60}{\pi \cdot N} \cdot K_{u_2} \cdot \sqrt{2g} \cdot \sqrt{H} \text{ metres} \qquad ...(12.41b)$$

If $K_{u_2} \approx 1$,

$$D_2 = \frac{60}{\pi} \times \sqrt{2 \times 9{\cdot}81} \cdot \frac{\sqrt{H}}{N} = \frac{84{\cdot}6\sqrt{H}}{N} \text{ metres} \qquad ...(12.41c)$$

where H is in metres.

These equations are also used to determine the head which a pump can develop if D_2 and N are known, specially, in practice, as a check for an existing pump.

ii) **Inlet Diameter :**

The inlet diameter D_1 is $\frac{2}{3}$ to $\frac{1}{3}$ of D_2 depending upon specific speed N_s or total head H_{mano}.

TABLE 12.6

Practical Values of $\frac{D_1}{D_2}$

H_{mano}				
	ft	30	150	300
	m	10	50	100
$\frac{D_1}{D_2}$		$\frac{2}{3}$	$\frac{1}{2}$	$\frac{1}{3}$

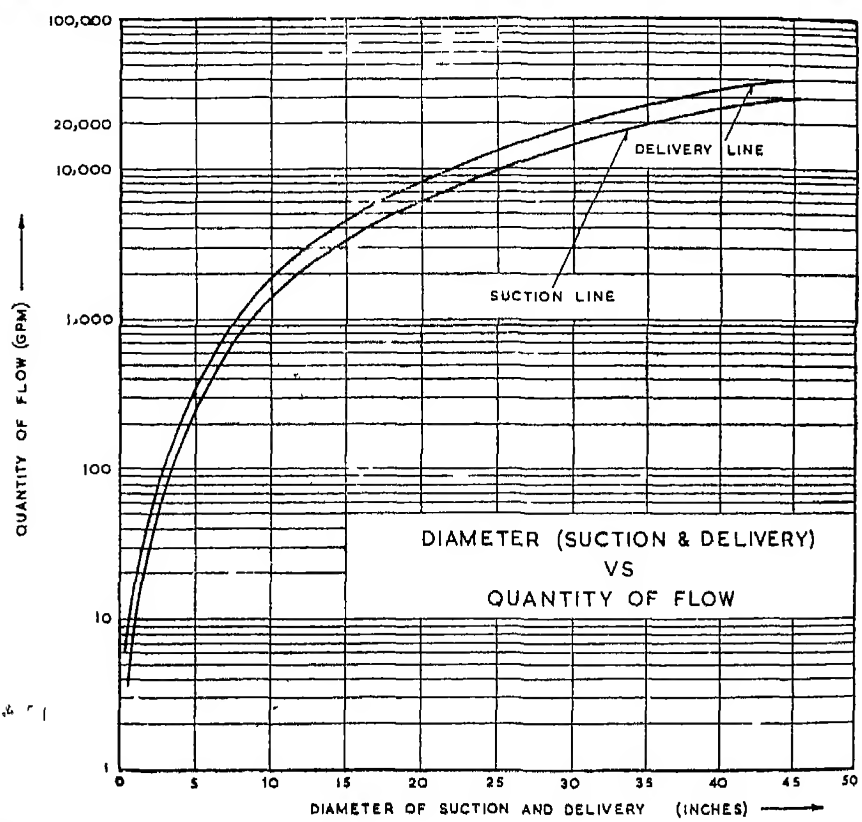


Fig 12.23 Pipe Diameter vs Discharge

12.33 Pipe Diameters—

i) **Suction Pipe :** The amount of water to be pumped is given by

$$Q=\frac{\pi}{4}\,.\,d_s^2\,.\,v_s \quad \text{where } d_s=\text{diameter of suction pipe}$$

$$v_s=\text{velocity of water in suction pipe}$$

(Generally v_s is 5 to 10 ft/sec or 1·5 to 3 m/sec)

$$\therefore \quad d_s=\sqrt{\frac{Q}{\frac{\pi}{4} \cdot v_s}} \tag{12.42}$$

Fig 12.23 shows d_s and d_d vs Q drawn from experimental data.

ii) **Delivery Pipe :**

$Q=\frac{\pi}{4} \cdot d_d^2 \cdot v_d,$ where d_d=diameter of delivery pipe

v_d=velocity of water in delivery pipe

(Generally v_d is 5 to 12 ft/sec or 1·5 to m/sec)

$$\therefore \quad d_d=\sqrt{\frac{Q}{\frac{\pi}{4} \cdot v_d}} \qquad \ldots(12.43)$$

Generally v_d is equal to or slightly higher than v_s. See also Fig 12.23 for practical values of d_d.

12.34 Pump Casing—

i) **Volute Casing :** A volute is shown in Fig 12.24. The point where the volute begins is known as theoretical tongue or cutwater. But in, practice, the actual tongue is at an angular distance ϕ_t from this point. This angle ϕ_t is known as the tongue-angle and is approximately equal to α_2. The volute is so designed that flow out of the impeller is uniform all around its periphery. If the total rate of flow be Q, then the quantity of water flowing across a section of the volute at an angle $\phi°$ from the theoretical tongue is $\frac{\phi}{360} Q$.

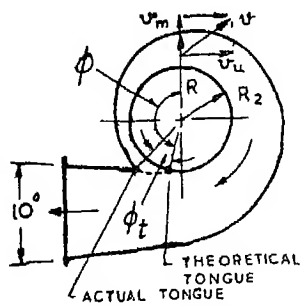


Fig 12.24 Volute Casing

The volute curve is of spiral form. The outlet of volute may be extended, as shown, in order to lower the outlet velocity of water down to a value required to carry the given discharge through the delivery pipe. However, the angle of divergence for this part should not exceed 10° so as to avoid boundary layer effect and cavitation.

Shape of Volute : According to Pfleiderer, to ensure spiral flow inside the volute,

$$R \cdot v_u=C$$

where v_u is the velocity of whirl at any radius R (See Fig 12.24) and C is a constant. The constant C can be evaluated from known values of R and v_u at outlet of impeller.

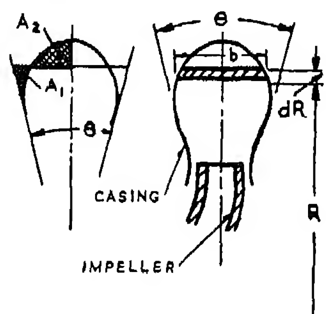


Fig 12.25 Shape of Casing

If b the breadth at a radius R at some section at an angle ϕ from the theoretical tongue (See Fig 12.24 and 12.25), then rate of flow through an elementary strip of thickness dR is

$$dQ_\phi = dA \cdot v_u$$
$$= b \cdot dR \cdot v_u$$

But $\quad v_u = \dfrac{C}{R}$ ...(See Eqn 2.8)

$\therefore \quad dQ_\phi = b \cdot dR \cdot \dfrac{C}{R}$

Now $\quad Q_\phi = \int dQ_\phi = C \int_{R_2}^{R_\phi} \dfrac{b \cdot dR}{R}$ ...(12.44)

This must be equal to $\dfrac{\phi}{360} \cdot Q$

The external side of the spiral casing may be of any shape. Generally it is circular or elliptical. Elliptical form can also be obtained be equalising areas A_1 and A_2 shown in Fig 12 25. The angle of volute θ is generally 60°.

ii) **Diffusion Casing** : As has already been explained in Art 12 6*b* this casing contains a number of vanes which provide passages for liquid to flow. The cross sectional area of any passage between two vanes must gradually enlarge in order to reduce the velocity of water emerging from the impeller (Fig 12.5) so that pressure head can be built up at the expense of kinetic energy. The angle of divergence for the enlarging section must not exceed 10°. The outlet area should be so chosen as to reduce the velocity of water to a final value just sufficient to carry the discharge through the delivery pipe. A sufficient number of vanes should be employed to ensure uniformly guided flow of water.

12.35 Axial Thrust in Centrifugal Pumps—Axial thrust is a force acting parallel to the axis of the pump shaft, caused due to the following reasons—

a) The water while passing through the impeller is rotating with a forced vortex, but that outside the shrouding (See Fig 2.8 and 2.9) is in a state of comparative rest. This causes a differential static thrust acting parallel to the axis of pump shaft and towards the impeller inlet.

b) Liquid enters the pump axially and is then deflected from its original path to a radial direction. The dynamic action of liquid causes a force to act on the pump in the direction of flow at inlet. The magnitude of this force, measured by the change in momentum per second, is

$$\frac{w \cdot Q}{g} \cdot u_1$$

To enable the pump to withstand this thrust, the following methods may be employed.

a) *For small pumps* :

i) Providing a thrust ball-bearing in the direction of axial thrust shown in Fig 12.26*a*.

ii) Inserting a cast iron ring in the casing which should fit in with a similar ring cast integral with the impeller as shown in Fig 12.26*b*.

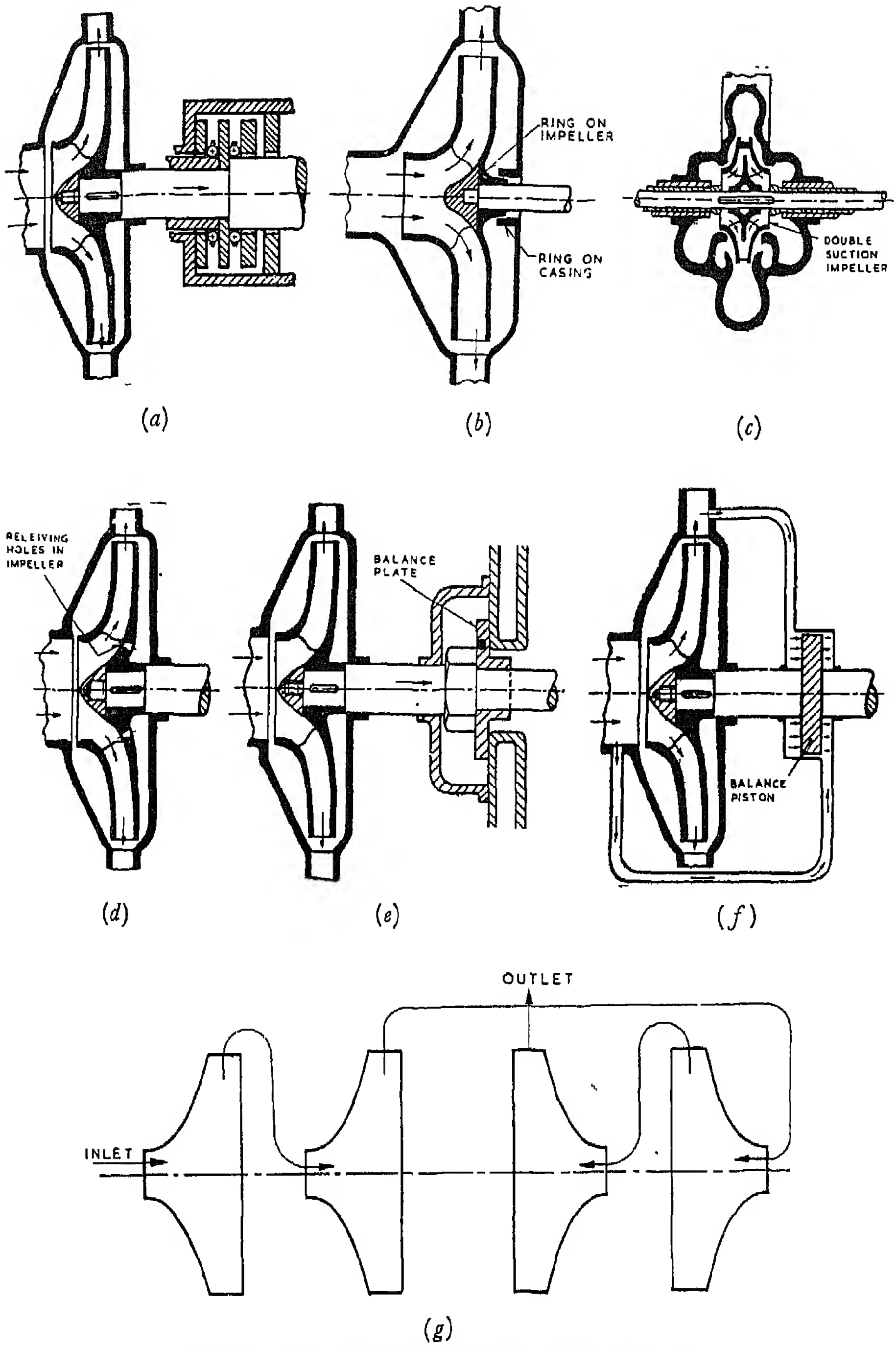


Fig 12.26 Balancing of Axial Thrust in Centrifugal Pumps

b) *For large pumps* :

Where the axial thrust is heavy.

i) Use of double suction impeller (See Fig 12.26*c*).

Suction on two sides of the impeller neutralises the thrust. But this method can be employed only for single stage pumps.

ii) Provision of relieving holes (See Fig 12.26*d*).

Relieving holes are provided in the impeller to allow suction pressure to act on both sides.

iii) A balance plate may be fitted at the end of the pump shaft (See Fig 12.26*e*).

iv) Balance Piston—This is employed for large pumps (See Fig 12.26*f*).

c) *For multi-stage pumps* :

The number of impellers are made generally even. This will facilitate to arrange the inlets of the half of the impellers in the opposite direction as shown in Fig 12.26*g*. These will balance the axial thrust produced by the other half numbers of the impellers.

12.36 Variation of Speed and Diameter—Even after a pump has been manufactured and used, its head or capacity or both may have to be changed to suit a new set of conditions. This can be accomplished by varying the speed of pump or by changing the diameter of the impeller. It should be borne in mind that this step is to be taken only when a minor change in head or capacity is required, otherwise it is recommended that a new pump be manufactured with altogether different dimensions.

a) **Effect of Variation of Speed on Discharge, Head and Power :**

Peripheral velocity $u = \dfrac{\pi \cdot D \cdot N}{60}$

$\therefore \quad u \propto N$

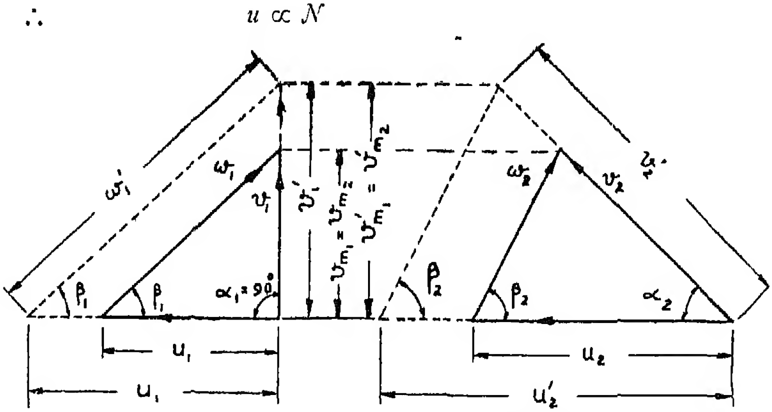


Fig 12.27 Effect of Variation of Speed on Velocity Triangles of a Centrifugal Pump

From the velocity triangles shown in Fig 12.27 it is evident that since the vane angles remain constant, if u changes to u', then

$$w \propto u$$

$$\therefore \quad w \propto N$$

But the discharge, Q = area across flow multiplied by w

$\therefore \quad Q \propto w$

i.e. $\quad Q \propto N$

Thus discharge varies directly as speed.

Further, from the fundamental equation (Eqn 12.12) of pumps it is obvious that head depends on squares of velocities u_2, w_2 and v_2. From Fig 12.27, it is seen that these quantities are proportional to speed, N.

Summing up,

$$\frac{Q}{Q'} = \frac{N}{N'} \qquad \text{.. (12.45)}$$

$$\frac{H}{H'} = \left(\frac{N}{N'}\right)^2 \qquad \text{...(12.46)}$$

and

$$\frac{\text{HP}}{\text{HP}'} = \left(\frac{N}{N'}\right)^3 \qquad \text{.. (12.47)}$$

Hydraulic losses vary more or less uniformly with the hydraulic horsepower. But mechanical losses are relatively small at higher speeds. Therefore, considering the brake horsepower, the total efficiency of the pump will increase slightly with an increase in speed.

Problem 12.9 A centrifugal pump delivers 300 gpm (or 22·7 lit/sec) of water against a head of 40 ft (or 12·2 m) and requires 6 HP (or 6 08 metric HP) when running at 1,450 rpm. Find the discharge, head and horsepower required if it has to run at 1,800 rpm.

Solution

N = 1,450 rpm $\qquad N'$ = 1,800 rpm

Q = 300 gpm (or 22·7 lit/sec) $\qquad$ HP = 6 (or 6·08 metric HP)

H = 40 ft (12·2 m)

Now,

$$\frac{Q}{Q'} = \frac{N}{N'} \qquad \therefore \quad Q' = \frac{N'}{N} \times Q = \left(\frac{1{,}800}{1{,}450}\right) \times 300$$

$$= \mathbf{372{\cdot}4\ gpm} \quad \textit{Answer}$$

$$\left[\text{or} \quad Q' = \frac{1{,}800}{1{,}450} \times 22{\cdot}7 = \mathbf{28{\cdot}2\ lit/sec} \quad \textit{Answer}\right]$$

Further,

$$\frac{H}{H'} = \left(\frac{N}{N'}\right)^2 \qquad \therefore \quad H' = \left(\frac{N'}{N}\right)^2 \times H = \left(\frac{1{,}800}{1{,}450}\right)^2 \times 40$$

$$= \mathbf{61{\cdot}6\ ft\ of\ water} \quad \textit{Answer}$$

$$\left[\text{or} \quad H' = \left(\frac{1{,}800}{1{,}450}\right)^2 \times 12{\cdot}2 = \mathbf{18\ 8\ m\ of\ water} \quad \textit{Answer}\right]$$

Also,

$$\frac{\text{HP}}{\text{HP}'} = \left(\frac{N}{N'}\right)^3 \qquad \therefore \quad \text{HP}' = \left(\frac{N'}{N}\right)^3 . \text{HP} = \left(\frac{1{,}800}{1{,}450}\right)^3 \times 6$$

$$= \mathbf{11{\cdot}5\ HP} \quad \textit{Answer}$$

[or $\mathrm{HP}'=\left(\frac{1{,}800}{1{,}450}\right)\times 6{\cdot}08=$ **11·65 metric HP** *Answer*]

b) **Effect of Alteration of Diameter on Discharge, Head and Power**—It has been mentioned earlier that the capacity of a pump can be changed by altering either its speed or the diameter of its impeller. The former is generally not possible because the speed of driving motor is fixed. Therefore, the diameter of the impeller has to be enlarged or reduced according as H or Q is to be increased or decreased. It may be effected by fixing rings to the shrouds of impeller (See Fig 12.28) to increase the outside diameter and extending the impeller blades to the required size. The effect of alteration of D is twofold.

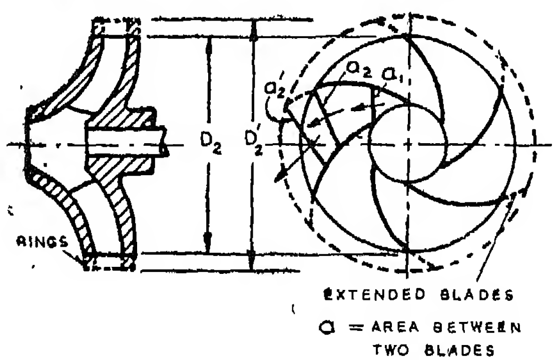


Fig 12.28 Effect of Increasing the Outlet Diameter of an Impeller

First, it changes u without changing N.

$$u \propto D$$

$$\therefore \quad w \propto D$$

Second, with reference to Fig 12.28, area across flow,

$$a \propto D$$

But $$Q = w \,.\, a$$

$$\therefore \quad Q \propto D^2$$

Also, since H depends on u^2, w^2 etc and the latter are proportional to D,

$$H \propto D^2$$

Now, power, $\mathrm{HP} \propto Q \,.\, H$

$$\therefore \quad \mathrm{HP} \propto D^4$$

Summing up

$$\frac{Q}{Q'}=\left(\frac{D}{D'}\right)^2 \qquad \ldots(12.48)$$

$$\frac{H}{H'}=\left(\frac{D}{D'}\right)^2 \qquad \ldots(12.49)$$

and $$\frac{\mathrm{HP}}{\mathrm{HP}'}=\left(\frac{D}{D'}\right)^4 \qquad \ldots(12.50)$$

The efficiency η increases slightly with size.

If, together with the diameter D the height of impeller passage and dimensions of other parts are proportionately altered,

Then $\quad a \propto D^2$

$\therefore \quad Q \propto D^3$

and $\quad HP \propto D^5$

Problem 12.10 A single stage centrifugal pump is built to give a discharge of Q when working against a manometric head of 56 ft. On test it was found that the head actually generated was 61 ft for the designed discharge Q. If it is required to reduce the original diameter of 13·5 inches without reducing the speed of the impeller, compute the reduction necessary.

(Punjab University—September 1954)

Solution

Head generated by pump $H=\dfrac{u^2}{2g}=\dfrac{\left(\dfrac{\pi\,.\,D\,.\,N}{60}\right)^2}{2g}$

or $\quad H \propto D^2$

$$\therefore \quad \frac{H}{H'}=\left(\frac{D}{D'}\right)^2$$

$H=61$ ft, $\quad H'=56$ ft

$D=13{\cdot}5$ inches

$$\therefore \quad D'=D\,.\left(\frac{H'}{H}\right)^{\frac{1}{2}}=13{\cdot}5\times\left(\frac{56}{61}\right)^{\frac{1}{2}}$$

$$=\mathbf{12{\cdot}93\ in.} \quad \textit{Answer}$$

c) **Relation between Q, H, HP, N and D**—If the diameter and other dimensions of impeller are proportionately altered and the speed is also changeable, then the general relations between these quantities can be derived by combining the results obtained above.

Hence
$$Q \propto D^3\,.\,N \qquad \text{...(12.51)}$$
$$H \propto D^2\,.\,N^2 \qquad \text{...(12.52)}$$
$$HP \propto D^5\,.\,N^3 \qquad \text{...(12.53)}$$

These relations are approximate because leakage, windage and bearing losses have been neglected.

12.37 Model Pumps—It has been mentioned in Chapter 9 that water turbines are always manufactured according to given specifications and not on mass scale. Almost all turbines are large units, so turbine models are prepared to predict the performance of the actual size machines before they are actually manufactured. In case of pumps, mostly they are manufactured on mass scale. In some cases large-sized pumps are to be installed. In such cases mass production cannot be resorted to, therefore, similar to water turbines large units of pumps are also not manufactured without making and testing their models because

it would be possible to make any alternation in design in time which would otherwise be difficult to carry out in the prototype machines.

The model should have complete geometrical similarity with the prototype, not only in pump proper but in intake and discharge conduits also The speed of the model can be determined on the assumption that the model and the prototype have the same specific speeds The same conditions of similarity as explained in Art 9 2 for turbines would apply to the pumps also. The speed and discharge for the model pump can be determined from equation 9.10 and 9.11 respectively.

i.e. $$\frac{N_m}{N_a}=\frac{D_a}{D_m}\times\sqrt{\frac{H_m}{H_a}} \qquad \text{(See Eqn 9.10)}$$

and $$\frac{Q_m}{Q_a}=\left(\frac{D_m}{D_a}\right)^2\sqrt{\frac{H_m}{H_a}} \qquad \text{(See Eqn 9 11)}$$

where suffices '*a*' and '*m*' indicate actual-size and model pump respectively.

Problem 12 11 To predict the performance of a large centrifugal pump, its model, having the following specifications, was made :

$P=24$ HP (or 24·35 metric HP), $H=25$ ft (or 7 62 m)
$N=625$ rpm

Diameter of model pump impeller is 9 times smaller than that of the prototype The prototype pump has to work against a head of 100 ft (or 30·48 m). Find its working speed and HP required to drive it. Determine also the rate of flow of both the pumps.

(*UPSC—June 1952*)

Solution

$P_m=24$ HP (or 24·35 metric HP), $H_m=25$ ft (or 7 62 m)
$N_m=625$ rpm $D_{2_a}=9\ D_{2_m}$

$H_a=100$ ft (or 30 48 m)

Scale $\frac{D_a}{D_m}=9$

a) For similar pumps, specific flow is same,

i.e., $$(Q_{11})_a=(Q_{11})_m \qquad \text{(See Eqn 9.3)}$$

or $$\frac{Q_a}{D_a^2\ .\ \sqrt{H_a}}=\frac{Q_m}{D_m^2\ .\ \sqrt{H_m}}$$

Now $$P=\frac{w\ .\ Q\ .\ H}{550}\text{ HP}\left[\text{or } P=\frac{w\ .\ Q\ .\ H}{75}\text{ metri}\right.$$

$$Q=\frac{550\ .\ P}{w\ .\ H}\text{ cfs}\left[\text{ or } Q=\frac{75\ .\ P}{w\ .\ H}\ m^3/\text{sec}\right]$$

or $$Q_m=\frac{550\times 24}{62{\cdot}4\times 25}=8{\cdot}46\text{ cusecs} \quad \text{Answer}$$

[or $$Q_m = \frac{75 \times 24{\cdot}35}{1{,}000 \times 7\ 62} = 0\ 239\ \mathbf{m^3/sec} \quad \textit{Answer}$$]

and $$Q_a = Q_m \cdot \left(\frac{D_a}{D_m}\right)^2 \cdot \sqrt{\frac{H_a}{H_m}}$$

$$= 8\ 46 \times (9)^2 \times \sqrt{\frac{100}{50}}$$

$$= \mathbf{1{,}370\ cusecs} \quad \textit{Answer}$$

[or $$Q_a = 0\ 239 \times 9^2 \times \sqrt{\frac{30\ 48}{7\ 62}} = \mathbf{38\ 7\ m^3\ sec} \quad \textit{Answer}$$]

b) For similar pumps the specific speed is same,

i.e., $$\frac{N_a \cdot \sqrt{Q_a}}{H_a^{\frac{3}{4}}} = \frac{N_m \cdot \sqrt{Q_m}}{H_m^{\frac{3}{4}}}$$ (See Eqn 9 1 and 3.40)

or $$N_a = N_m \cdot \sqrt{\frac{Q_m}{Q_a}} \cdot \left(\frac{H_a}{H_m}\right)^{\frac{3}{4}}$$

$$= 625 \times \left(\frac{8\ 46}{1{,}370}\right)^{\frac{1}{2}} \times \left(\frac{100}{25}\right)^{\frac{3}{4}}$$

$$= \mathbf{139\ rpm} \quad \textit{Answer}$$

[or $$N_a = 625 \times \left(\frac{0{\cdot}239}{38\ 7}\right)^{\frac{1}{2}} \times \left(\frac{30{\cdot}48}{7{\cdot}62}\right)^{\frac{3}{4}} = \mathbf{139\ rpm} \quad \textit{Answer}$$]

c) For similar pumps the specific power is same,

i.e., $$\frac{P_a}{D_a^2 \cdot H_a^{\frac{3}{2}}} = \frac{P_m}{D_m^2 \cdot H_m^{\frac{3}{2}}}$$ (See Eqn 9 12)

$$\therefore \quad P_a = P_m \cdot \left(\frac{D_a}{D_m}\right)^2 \left(\frac{H_a}{H_m}\right)^{\frac{3}{2}} = 24 \times 9^2 \times 4^{\frac{3}{2}}$$

$$= \mathbf{15{,}500\ HP} \quad \textit{Answer}$$

[or $$P_a = 24{\cdot}35 \times 9^2 \times 4^{\frac{3}{2}} = \mathbf{15{,}750\ metric\ HP} \quad \textit{Answer}$$]

12.38 Priming of Pumps—

Priming—Before starting a pump, its impeller and suction pipe have to be filled with water in order to remove any air, gas or vapour from the waterways of the pump. If a centrifugal pump is not primed before starting, air pockets inside the impeller may give rise to vortices and cause discontinuity of flow. The wearing rings may rub and sieze causing serious damage if the pump is allowed to run dry. It is also essential that packing be lubricated by liquid leaking past it.

Ordinarily, priming is done by pouring water through a funnel, displaced air being allowed to escape through air vents. When a pump is being primed or stopped, the delivery valve should be kept closed.

Necessity of priming is the main disadvantage of a centrifugal pump. To overcome this difficulty, self-priming devices have been invented.

Priming Devices—

a) Pouring Water—Water is poured in the pump through priming funnel. Air vent is opened to provide exit to the air. It is closed after the priming is over.

b) Priming Chamber—In small pumps a priming chamber may be used on the delivery side of the impeller. When the pump is stopped, some water is stored in the tank and this can be used to fill the impeller and suction line before restarting. Normally, the capacity of the tank is about three times the volume of the suction pipe.

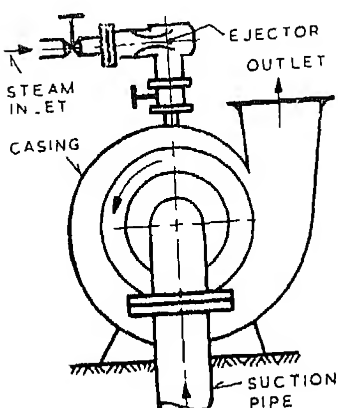


Fig 12.29 Priming of Centrifugal Pump

c) Vacuum Producing Devices—The suction line and pump are exhausted of all air so that atmospheric pressure forces the water up into the pump. An ejector using high pressure water, steam or compressed air is employed to create vacuum at the top of the casing (See Fig 12.29) so that water is sucked into the suction pipe and the impeller.

With the increased use of electric power, motor driven wet vacuum pump have become popular. Modern pumps equipped with vacuum producing devices operate on the same principle as compressors or vacuum pumps.

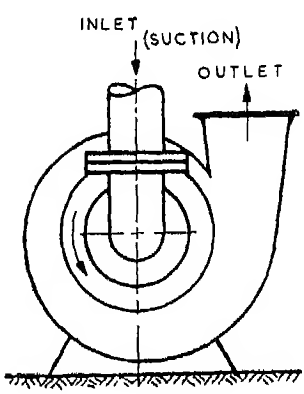


Fig 12.30 Self-Priming Device in Sihi-Pump

Self-Priming Devices—

a) Patent Device in Sihi Pumps—Both the suction pipe and the rising main are connected to the top of the pump (See Fig 12.30). Some water is always left in the pump when it comes to rest.

b) By Interposing a Special Reservoir in Suction Pipe—Each manufacturer has got its own patent, designed on this principle.

12.39 Suction Lift—Centrifugal pumps are designed for a total suction lift of 15 feet (or 4·5 m) of the liquid to be handled. So long as the suction lift is kept below this value, the output (Q) and efficiency (η) of the pump are unaffected. As soon as the maximum lift exceeds the limiting value, the output would fall abruptly. It is not only the output

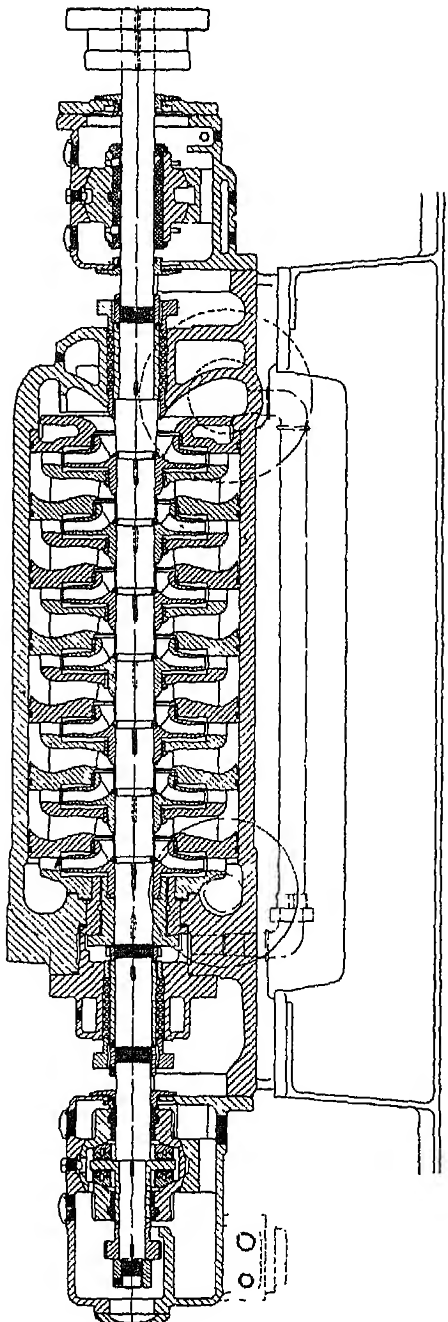


Fig 12.31 Multistage Centrifugal Pump for Boiler Feed Purposes, 7-stages (Manufactured by Ingersoll-Rand)

which is affected but, there is every possibility of having the following troubles in the working of the pump :

1. With the drop of output the efficiency will also be much less.
2. Seizing of the pump may take place.
3. The pump may become unprimed.
4. The pump is subject to cavitation and air binding, making the operation very noisy ; the wear becomes rapid.

The suction lift is limited by the following factors.

1. Viscosity : Centrifugal pump should not be used for liquids of viscosity higher than 1,500 Say-Bolt Universal Seconds if the output is over 2,000 gpm (or 150 lit/sec).
2. Temperature (See Table 7.3),
3. Total lift.
4. Losses in pipe fittings.
5. High velocity of water entering the pump.
6. Mechanical defects such as leaking of joints, thick blades of impeller and blunt or incorrect shape of blades of impeller, or that of the guides or volute because of which there is a possibility of formation of eddies.

12.40 Horsepower of Driving Motor—Centrifugal pumps are generally coupled to 3 phase electric *AC* motors. Direct current motors are recommended where duty of the pump changes periodically. Horsepower of a driving motor should exceed the horsepower calculated for the pump by the following percentages :

1. For a pump requiring up to 2 horsepower add about 50%
2. For a pump requiring from 2 to 5 horsepower add about 30%
3. For a pump requiring from 5 to 10 horsepower add about 20%
4. For a pump requiring from 10 to 20 horsepower add about 15%
5. For a pump requiring more than 20 horsepower add about 10%.

The percentages stated above are valid for pumps handling water or similar liquids. For special duty pumps, the percentages will differ according to the nature of the liquid to be pumped.

Electric motors below $\frac{1}{3}$ HP are not employed for driving pumps. For 50 cycles frequency, the speed of an induction type motor may be about 1450 or 2900 rpm.

12.41 Multi-stage Pump—If a pump is required to deliver against a very high head which cannot be built up in a single impeller, a number of impellers are connected in series, so that discharge from one goes to the inlet of the next and so on (Fig 12.6*b*). All impellers are identical and keyed to the same shaft. Theoretically, at every stage the head should be raised by the same amount. The last diffuser discharges into the delivery pipe (See Fig 12.31).

Particular attention should be paid to the design of a multi-stage pump shaft. It must not run at its critical speeds. This is particularly important in case of boiler feed-pumps.

Problem 12.12 A centrifugal pump is required to deliver 900 gpm of water at room temperature against a head of 320 ft, when running at 1450 rpm. Find the number of stages for best efficiency.

Solution

$Q = 900$ gpm $\qquad H = 320$ ft $\qquad N = 1{,}450$ rpm

$\therefore$ Specific speed, $N_s = \dfrac{N \times \sqrt{Q_{gpm}}}{H^{\frac{3}{4}}}$

For a single stage pump with the given specifications

$$N_s = \frac{1{,}450 \times \sqrt{900}}{320^{\frac{3}{4}}} = 574 \text{ units.}$$

Pumps having a specific speed less than 600 are generally not recommended. In fact, the efficiency of a pump falls considerably if the specific speed is less than 1,000. This is because the impeller becomes disproportionate, the diameter being too large relative to the width. It results in leakage and higher disc friction and fluid friction losses owing to narrow passage for the fluid. It is, therefore, advisable to use impellers of small diameters and consequently high specific speed. This will reduce the disc friction losses which vary with the cube of the radius and the centrifugal stresses which vary with the radius. The efficiency will rise and the cost will fall.

To select a suitable number of stages consider

i) a three-stage pump $N_s = \dfrac{1{,}450 \times \sqrt{900}}{\left(\dfrac{300}{3}\right)^{\frac{3}{4}}} \approx 1{,}330$ units

ii) a five-stage pump $N_s = \dfrac{1{,}450 \times \sqrt{900}}{\left(\dfrac{320}{5}\right)^{\frac{3}{4}}} \approx 1{,}920$ units

It is known from experience and the available data that a 1,000 gpm pump is most efficient and economical if $N_s \approx 2{,}000$. Therefore, five stages may be recommended.

Problem 12.13 A two stage centrifugal pump is required for a fire engine for a duty of 800 gpm at a head of 250 ft. If the overall efficiency is 75% and specific speed per stage is about 2,000, find

a) the running speed in rpm,

and *b*) the BHP of the driving engine.

If the actual manometric head developed is 65% of the theoretical head, assuming no slip, the outlet angle of the blades 30° and radial flow velocity at exit 0·15 times the tipped speed at exit, find the diameter of the impellers. Indicate the advantage of the two stage pump over the single stage for this duty.

(*UPSC Dec—1954*)

Solution

i) $\qquad Q = 800$ gpm

$\qquad H = 250$ ft $\qquad \therefore$ H per stage $= \dfrac{250}{2} = 125$ ft.

$N_s = 2{,}000$ per stage $\qquad \eta_{overall} = 0.75$

a) $$N_s = \frac{N \cdot \sqrt{Q}}{H^{\frac{3}{4}}}$$

$$\therefore \quad N = \frac{N_s \cdot H^{\frac{3}{4}}}{\sqrt{Q}} = \frac{2{,}000 \times 125^{\frac{3}{4}}}{\sqrt{800}} \quad \textbf{2,650 rpm} \quad \textit{Answer}$$

b) $$Q = 800 \text{ gpm}$$
$$= \frac{800}{6 \cdot 24 \times 60} = 2 \cdot 14 \text{ cusecs}$$

$$\therefore \text{ BHP of the driving engine} = \frac{w \cdot Q \cdot H}{550 \cdot \eta_{overall}}$$

$$= \frac{62 \cdot 4 \times 2 \cdot 14 \times 250}{550 \times 0.75} = \textbf{80 8 HP} \quad \textit{Answer}$$

ii) $H_{mano} = 0 \cdot 65\ H_{th}$

$\beta_2 = 30°$ $\qquad v_{m_2} = 0 \cdot 15\ u_2$

From outlet velocity triangle (See Fig 12 19)

$$\text{or } \tan \beta_2 = \frac{v_{m_2}}{u_2 - v_{u_2}} \qquad \text{or } \tan 30° = \frac{0 \cdot 15\ u_2}{u_2 - v_{u_2}}$$

$$\text{or } 0 \cdot 5774 = \frac{0 \cdot 15\ u_2}{u_2 - v_{u_2}} \qquad \therefore \quad \frac{u_2}{u_2 - v_{u_2}} = \frac{0 \cdot 5774}{0\ 15} = 3 \cdot 85 \qquad \ldots(1)$$

Assuming flow at entrance to be radial and $\alpha_1 = 90°$, fundamental equation of pump would be :

$$\frac{H_{mano}}{\eta_{mano}} = \frac{u_2 \cdot v_{u_2}}{g}$$

where η_{mano} the manometric efficiency of pump is 65%

$$\therefore \quad \frac{125}{0 \cdot 65} = \frac{u_2 \cdot v_{u_2}}{32 \cdot 2} \qquad \text{or} \qquad u_2 \cdot v_{u_2} = \frac{125 \times 32 \cdot 2}{0\ 65} = 6{,}192$$

$$\text{or} \qquad v_{u_2} = \frac{6{,}192}{u_2} \qquad \ldots(2)$$

Substituting for v_{u_2} in equation (1),

$$\frac{u_2}{u_2 - \dfrac{6{,}192}{u_2}} = 3 \cdot 85 \qquad \text{or} \quad u_2 = 91 \cdot 43 \text{ ft/sec}$$

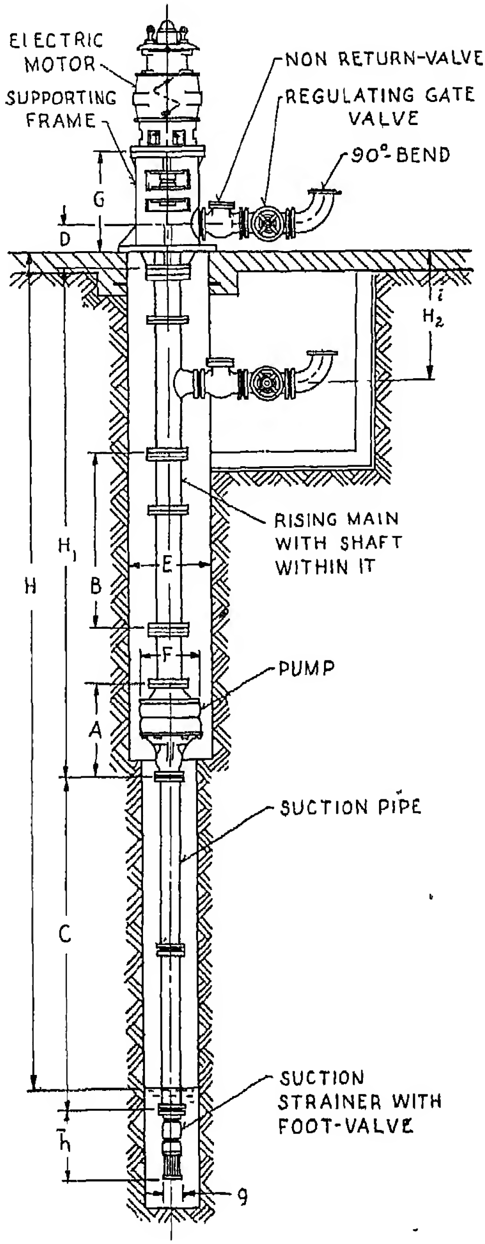
$$\text{Further} \qquad u_2 = \frac{\pi \cdot D_2 \cdot N}{60} \quad \text{or } 91 \cdot 43 = \frac{\pi \times D_2 \times 2{,}650}{60}$$

or $\qquad D_2 = 0 \cdot 66$ ft $\qquad$ or $7 \cdot 92$ in. $\approx$ **8 in.** *Answer*

$\therefore$ Diameter of each impeller is 8 in. as all impellers in a multistage pump are similar.

12 42 Deepwell Pump or Vertical Turbine Pump (Fig 12.32)—This is generally a multi-stage pump, the number of stages depending upon the head required. The impellers are assembled in a group at the lower end of the pump column and shaft. All the impellers and at least ten feet of suction pipe, with a strainer at the end, are placed below the water level. That is why such pumps are known as deep-well or borehole pumps. Water is conducted to the surface through the rising main pipe or column pipe, as it is sometimes called, which connects the impellers with the outlet. The impellers are keyed to a vertical pump shaft which is further coupled to a line shaft enclosed in a cover pipe with bronze bearings, placed at suitable intervals along the shaft, to prevent vibration and whip. The shaft is also aligned with these bearings. The bearings may be lubricated by oil, introduced at the top of the cover pipe, or by water. When there is no cover pipe, water naturally passing through the column pipe lubricates the bearings which are then made from rubber. At the upper end of the line shaft is fitted a thrust bearing which carries the weight of all impellers and the shaft, and balances the thrust on the impellers, caused by pressure head.

The pump is generally driven by a direct coupled electric motor of vertical type, placed at the top of the line shaft, usually on ground level. The motor is



A—Length of Bowl B—Length of Main Pipe C—Length of Suction Pipe D—Head above ground Level E—Inside diameter of Column Pipe F—Outside diameter of Bowl G—Height of Supporting Frame H—Head below Ground Level H_1—Water Level while Pumping H_2—Depth of Delivery connection below Ground Level $\bar{h}$—Strainer Length g—Dia of Strainer.

Fig 12.32 Deep-well or Vertical Turbine Pump

properly protected against water. Sometimes, the pump is driven by a primemover, generally a diesel engine. In such cases a belt drive is employed for small and medium sized pumps and toothed gearing for larger sizes.

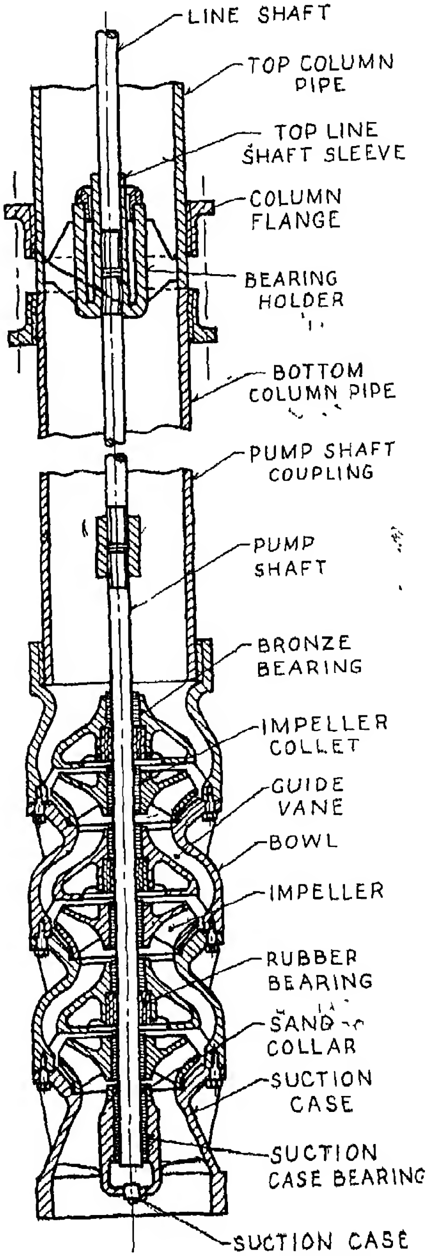


Fig 12.33 Longitudinal Section Through a Three-Stage Bore-Hole Centrifugal Pump with Shaft Guided in the Rising Main.

The impeller used in a turbine pump is closed or semi-open. Open impellers are seldom used. The impeller normally rotates with the blades bending backwards. If the direction of rotation is reversed, the discharge will fall, and the line shaft will be unscrewed at the upper end.

Bypassing in Turbine Pumps—When water leaks out from the discharge to the suction side of the impeller, the phenomenon is known as bypassing. Its effect is to lower the discharge, and hence the efficiency of the pump is reduced. Since the ordinary volute pump is horizontal, the tendency to bypass is less and it is further prevented by replaceable wear rings. Corrosion of impeller and guide vanes caused by impure water increases bypassing and friction, thereby lowering the efficiency of the pump. Impellers and guide vanes should therefore be made of corrosion resistant metals. Erosion is the wear of impeller caused by sand particles in water. Besides corrosion and erosion, cavitation (resulting from a high velocity of flow) is another phenomenon detrimental to the metal.

12.43 Special Purpose Pumps—Special purpose centrifugal pumps are designed either to handle liquids with extraordinary chemical or physical properties *e.g.* abnormal density or viscosity, or special duties *e.g.* fire-extinguishing. This last one is just an ordinary high head multi-stage pump.

The impellers of special purpose pumps have to be made of a special material or covered with a protective coating when required to handle chemicals. For instance, if a pump is meant to raise acids, the impeller and

whole inside surface of pipes, casing etc., should be coated with some acid-resisting substance *i.e.* stone.

Pumps for Liquids of Special Densities :

$$p = w \,.\, H$$

$\therefore$ Pressure generated $\propto$ Density of liquid (H constant)

Also $\quad \text{HP} = \dfrac{w \,.\, Q \,.\, H}{550} \qquad \left[\text{or } = \dfrac{w \,.\, Q \,.\, H}{75} \text{ metric HP}\right]$

$\therefore$ Power consumed $\propto$ Density of liquid (Q and H constant)

or $\quad \text{HP} = \dfrac{p \,.\, Q}{550} \qquad \left[\text{or } = \dfrac{p \,.\, Q}{75} \text{ metric HP}\right] \quad \ldots(12.54)$

This equation shows that power required to deliver a certain quantity of liquid at a given pressure is independent of the density.

Problem 12.14 A centrifugal pump handles a mixture of sand and water whose specific gravity is 1·08. The quantity of mixture to be pumped is 4,000 gpm (or 303 lit/sec) against a total head of 40 ft (or 12·2 *m*). Find the HP required by the pump if its overall efficiency is 75%. Find also the pressure in lb/sq in. (or kg/cm^2) developed by the pump.

Solution

Specific gravity of the mixture = 1·08

$\therefore$ Specific weight $= 62{\cdot}4 \times 1{\cdot}08$ lb/cu ft
(or $1{,}000 \times 1{\cdot}08$ kg/m^3)

$H = 40$ ft (or 12·2 m) $\qquad Q = 4{,}000$ gpm (or 303 lit/sec)

$\eta = 0{\cdot}75$

Power required for the pump $= \dfrac{w \,.\, Q \,.\, H}{550 \times \eta}$ HP

$$\left[\text{or } = \frac{w \,.\, Q \,.\, H}{75 \,.\, \eta} \text{ metric H P}\right]$$

$$= 1{\cdot}08 \times 62{\cdot}4 \times \frac{4{,}000 \times 10}{62{\cdot}4 \times 60} \times 40 \times \frac{1}{550 \times 0{\cdot}75}$$

$$= \mathbf{69{\cdot}75\ HP} \quad \textit{Answer}$$

$$\left[P = 1{\cdot}08 \times 1{,}000 \times \frac{303}{10^3} \times 12{\cdot}2 \times \frac{1}{75 \times 0{\cdot}75} = \mathbf{71\ metric\ HP} \quad \textit{Answer}\right]$$

Pressure developed in the pump, $p = w \,.\, H = (1{\cdot}08 \times 62{\cdot}4) \times \dfrac{40}{144}$

$$= \mathbf{18{\cdot}7\ lb/sq\ in.} \quad \textit{Answer}$$

$$\left[\text{or } \quad p = 1{\cdot}08 \times 1{,}000 \times \frac{12{\cdot}2}{100 \times 100} = \mathbf{1{\cdot}318\ kg/cm^2} \quad \textit{Answer}\right]$$

Problem 12.15 A centrifugal pump is required to handle 700 gpm of brine against a net pressure of 65 lb/sq in. Determine the head in ft of liquid and the HP required by the pump if its overall efficiency is 62%. Take the specific gravity of brine as 1·19.

Solution

$Q=700$ gpm $\quad p=65$ lb/sq in. $\quad \eta=62\%$

Sp gr$=1{\cdot}19$

$p=65\times 144$ lb/sq ft

$w=1{\cdot}19\times 62{\cdot}4$ lb/cu ft

$$H=\frac{p}{w}=\frac{65\times 144}{1{\cdot}19\times 62{\cdot}4}=\mathbf{126\ ft\ of\ brine}\quad \textit{Answer}$$

$$\text{HP required by the pump}=\frac{w\,.\,Q\,.\,H}{550\times\eta}$$

$$=\frac{700\times 10\times 1{\cdot}19}{60}\times\frac{126}{550\times 0{\cdot}62}$$

$$=\mathbf{51{\cdot}3\ HP}\quad \textit{Answer}$$

Problem 12.16 A centrifugal pump is required to handle 700 gpm of gasoline against a net pressure of 65 lb/sq in. Determine the head in ft of liquid and the HP required by the pump, if its overall efficiency is 62%. Take the specific gravity of gasoline as 0·7.

Solution

$Q=700$ gpm $\quad p=65$ lb/sq in. $\quad \eta=62\%$

Sp gr$=0{\cdot}7$

$p=65\times 144$ lb/sq ft

$w=0{\cdot}7\times 62{\cdot}4$ lb/cu ft

$$H=\frac{p}{w}=\frac{65\times 144}{0{\cdot}7\times 62{\cdot}4}=\mathbf{214{\cdot}5\ ft\ of\ gasoline}\quad \textit{Answer}$$

$$\text{HP required by the pump}=\frac{w\,.\,Q\,.\,H}{550\times\eta}$$

$$=\frac{700\times 10\times 0{\cdot}7}{60}\times\frac{214{\cdot}5}{550\times 0{\cdot}62}$$

$$=\mathbf{51{\cdot}3\ HP}\quad \textit{Answer}$$

It is evident from the two proceeding examples that the power required by the pump for a given discharge is independent of the specific weight of the liquid so long as the net pressure p generated is the same. The head in feet of liquid column, however, depends on the specific gravity.

UNSOLVED PROBLEMS

12.1 What are the advantages of a centrifugal pump over a reciprocating type ? *(Jadavpur University—1955)*

12.2 Explain with sketches the working of a single-stage centrifugal pump.

12.3 Name the heads under which centrifugal pumps are classified.

12.4 What is the difference between low lift, medium lift and high lift centrifugal pumps ?

SYMPTOMS CAUSES	The pump fails to
Pump and suction line are not sufficiently primed with the liquid handled	
Excessive suction lift	
Insufficient margin between suction lift and vapour pressure	
Liquid contains gas	
Air pocket in suction line	
Air leak in suction line	
Air leaks into pump along stuffing box	
Foot valve too small	
Foot valve partly clogged	
Foot valve and suction line not fully submerged	
Connection of water seal on suction stuffing box blocked	
Lantern ring in stuffing box incorrectly fitted	
Speed too low	
Speed too high	
Incorrect direction of rotation	
Total manometric head of the system greater thar the manometric head of the pur	
Total manometric head of the system lowe the manometric head of th	
Specific gravity of liquid handled it was originally sr	
Viscosity of liquid handled is origina'	
The pump is operatin	
Parallel the	
Line	

Bea

Axia

12.5 Show by sketches the difference between the volute and the diffusion pump.

12.6 What are the functions of a volute casing of a centrifugal pump ? *(Jadavpur University—1955)*

12.7 What is the difference between single-stage and multi-stage pumps?

12.8 What is a multi-stage centrifugal pump ? How does it function ?

12.9 What is the difference between radial, axial and mixed flow pumps ? Describe with the help of line diagrams.

12.10 What is the difference between axial flow pump and Kaplan pump ?

12.11 What is the difference between single suction and double suction pumps ? State the advantages of double suction over single suction type.

12.12 State the difference between closed, semi-closed and open impeller of a centrifugal pump.

12.13 What kind of impeller would you select for a pump required to handle paper pulp or molasses ? *(AMIE—Nov 1954)*

12.14 Define specific speed of a centrifugal pump. State the difference of specific speed if the discharge is given in gpm or cusecs and also in m^3/sec

12.15 What value of head and discharge will be used in determining the specific speed of multi-stage and double suction pumps respectively?

12.16 How does the specific speed of a turbine differ from that of a centrifugal pump ?

12.17 What is non-dimensional factor K_s in case of centrifugal pumps ? State the difference between K_s and N_s for centrifugal pumps.

12.18 State the specific speed of a radial flow, mixed flow (Francis type), mixed flow (Screw type) and axial flow centrifugal pumps.

12.19 Show by means of different sketches how the ordinary centrifugal type of pump impeller is developed to an axial flow type. Write the approximate ranges of specific speed of the different shapes of impeller you draw. *(AMIE—Nov 1954)*

12.20 Define static and manometric head of a centrifugal pump. Indicate these heads in a line diagram.

12.21 Define overall efficiency of a pump.

12.22 State the different types of head losses which occur outside the pump itself. How will you determine such losses ?

12.23 Define manometric efficiency of a centrifugal pump.

12.24 What are the different efficiencies of a centrifugal pump ?

12.25 Differentiate between virtual, ideal and manometric head. State the influence of number of blades on virtual head.

12.26 Explain with reasons the advantages of setting back the vane angle at exit in the case of a centrifugal pump. *(AMIE—Nov 1953)*

12.27 What is an axial thrust in centrifugal pumps ? State its causes. Describe a few methods to withstand such a thrust for *a*) small pumps, *b*) large pumps and *c*) multi-stage pumps.

12.28 State the effect of variation of speed on discharge, head and power.

12.29 State the effect of change of diameter of impeller on discharge, head and power on centrifugal pump.

12.30 What are model pumps? Under what circumstances they are required to be prepared?

12.31 What is meant by "Priming" of a pump? What are the self-priming arrangements employed for big pumping units?

12.32 How does the location of the centrifugal pump affects its suction head?

12.33 Why can the suction height of a pump not exceed a certain limit?
(*AMIE—May 1955*)

12.34 How would you determine the horsepower of a motor if the horsepower of a pump is given?

12.35 Draw the line diagram of a multi-stage pump giving the names of its main parts. How will you balance the axial thrust by fixing the impellers in the opposite directions?

12.36 What is a deep-well pump? What is its practical use?

12.37 Draw sectional view of a multiple-stage deep-well pump.

12.38 Prove that the discharge of a centrifugal pump is independent of specific weight of the liquid it handles, so long as the net intensity of pressure generated is the same.

12.39 A multi-stage centrifugal pump raises 1,000 gpm of water against a total pressure of 600 lb/sq in. The pump shaft rotates at 1,725 rpm. Determine the type of impeller if the pump has four stages.
(680 rpm ; radial flow type) (*Jadavpur University—1956 Suppl*)

12.40 A multi-stage pump running at 1,500 rpm delivers 16,000 gpm of water against a head of 60 ft. A similar multi-stage pump has a specific speed of 1,460 rpm, find the number of stages.
(36 Impellers in parallel) (*UPSC—Dec 1953*)

12.41 A pump is used to deliver 2,500 gpm of oil to a large storage tank through an 8 in. clean steel pipe 400 ft long. The elevation of surface of oil in the storage tank is 20 ft above the delivery end of the pump. The pipe line includes a gate valve, 3 short radius elbows, and the outlet is submerged. Sp gr 0·87 and kinematic viscosity$=4\times10^{-5}$ sq ft/sec. Find pressure in lb/sq in. at the delivery end of the pump.

Use the following values of k in $h_f=k\cdot\frac{v^2}{2g}$ for the losses in valves :—

Gate valve=0·19, short radius elbows=0·9.

Determine the value of f from the relation $4f=0.0032+\frac{0.221}{R_e^{0.237}}$

(34 psi)

12.42 In order to determine the efficiency of a centrifugal pump, the following observations were made :—

Pressure gauge reading on suction side=8 ft of water

Pressure gauge reading on delivery side=412 ft of water

Total water raised by the pump =3,000 gallons/min

Total input to the pump =580 HP

Find the efficiency of the pump.
(65·9%) (*Roorkee University—1956*)

12.43 The impeller of a centrifugal pump has an outer diameter of 14 in. and runs at 1,000 rpm. The blades are bent at 150° to the direction of motion at discharge and the radial velocity of flow is constant through the impeller at 8 ft/sec. The measured head across the pump is 71 ft and the frictional resistance of the pump is estimated to be 6·4 ft. Find out—

a) the efficiency according to velocity triangles,

b) the fraction of the kinetic energy of water leaving the impeller which is converted to pressure head in the casing.

(79·3%, 63%) (*London University—1944—Ext.*)

12.44 A centrifugal pump impeller has an outside diameter of 8 in. and rotates at 2,900 rpm. Determine the head generated if the vanes are curved backwards at 25° and the velocity of flow which is constant throughout the wheel is 10 ft/sec. Assume hydraulic efficiency as 75%.

Determine also the HP required to turn the impeller if the breadth of the wheel at outlet be $\frac{5}{8}$ in. Neglect the effect of vane thickness and mechanical friction and leakage losses.

(187·4 ft ; 31 HP) (*Jadavpur University—1956*)

12.45 A centrifugal pump having impeller diameter 38 inches delivers 17,500 gpm at a total manometric head of 212 ft when running at 750 rpm, the measured shaft HP being 1,370. The impeller vanes are backward curved and make an angle of 20° to the tangent and the effective circumferential area at outlet is 3·76 sq ft.

Assuming leakage loss through clearance rings 3% of the discharge, external losses including disc friction, bearing and gland friction 50 HP, determine—

a) the theoretical head which could be developed if the net HP given to the water flowing through the impeller was converted to head without hydraulic losses.

b) the theoretical head assuming an infinite number of impeller vanes.

c) the hydraulic efficiency and the overall efficiency.

Explain why the head obtained from (*a*) is less than that from (*b*) and state what design factors influence this ratio.

[*a*) 242 ft of water ; *b*) 345 ft of water head obtained in (*a*) is less than (*b*), because the impeller has got a number of blades in (*a*) ; *c*) $\eta_{overall}=82\%$, $\eta_{hyd}=85{\cdot}2\%$]

(*AMI Mech E—Oct 1955*)

12.46 Following information is given about a 950 rpm centrifugal pump having 2 ft outside dia.

Radial velocity (constant) = 18 ft/sec

$$H_i=\frac{1}{g}\cdot u_2\cdot v_2 \qquad ..(1)$$

$$H_a=\frac{1}{g}\cdot u_2\cdot v'_2 \qquad ...(2)$$

$$H_i=H_a(1+p) \qquad ...(3)$$

where equations (1) and (2) represent respectively the ideal and actual work on the runner vanes per pound of water v_2 and v'_2 are

the ideal and actual whirl components and u_2 the tangential velocity and p is a factor relating the difference between the actual and ideal conditions. If the energy received per lb of water is 50 ft (H) and the hydraulic efficiency $\frac{H}{H_a}=0.86$, construct the ideal and actual outlet velocity triangles and find thereby the outlet blade angle and ideal and the actual whirl components. Value of p can be taken as 0·3.

(18·9 ft/sec, actual ; 24 4 ft/sec, ideal ; 13° 30′)
(Punjab University—1957)

12.47 A new pumping station is to be installed to supply water to a town with 13,000 inhabitants. The town is situated at a height of 7,200 ft above mean sea level. The water consumption per head is 40 gallons a day. Half of the total consumption is to be supplied in 8 hours. The total static head is 200 ft, and the head losses in the suction and the delivery sides are 110 ft. Select a suitable pump for the purpose and find the HP of the driving motor for it. Assuming a suitable motor speed, determine the specific speed of the pump impeller, as well as its inlet and outlet diameters. Assume the pump and motor efficiencies.

(Water HP=51, with pump efficiency of 75% motor HP=75, 5-stage pump, N_s=1,530 with 1,450 rpm, impeller diameters 11 in. and 5½ in. with speed ratio of 1·02)
(AMIE—Nov 1954)

12.48 A single-stage centrifugal pump was designed to give a discharge against a manometric head of 80 ft with an impeller dia of 15 in. After manufacture the pump was tested and it was found that when the specified discharge was flowing through the pump, the head generated was 74 ft. Now the shaft speed cannot be altered as the pump is coupled to an induction motor having a speed of 1,450 rpm.

Suggest the method of getting the design head. If certain corrections are necessary, calculate them.

(Dia of impeller should be increased from 15 in. to 15·6 in.)
(Punjab University—1958 A)

12.49 A centrifugal pump was manufactured in order to couple directly to a 15 HP electric motor running at 1,450 rpm, delivering 500 gpm against a total head of 70 ft. However a customer likes to use diesel engine in place of motor. The speed of the engine is 1,000 rpm and the pump is to be coupled directly to the engine shaft. Find the probable discharge and the head developed by the pump. Also specify the HP of the engine you would employ.

(345 gpm, 33·3 ft, 4·92 HP, 5 HP engine)
(Jadavpur University—1955)

12.50 It is proposed to build a three-stage centrifugal pump to handle 780 gpm of water at a speed of 900 rpm and under a total manometric head of 230 ft. The vanes are to be radial at inlet and are to be curved backward at exit at an angle of 45°. Assuming a manometric efficiency of 84% and a mechanical efficiency of 75% and considering that vane thickness accounts for 8% of the circumferential area, determine the probable external diameter and

width of each impeller and the HP input necessary. The velocity of flow may be assumed constant at 10 ft/sec.

(15 in., $\frac{5}{8}$ in., 86·4 HP) (*Roorkee University—1958*)

12.51 A multi-stage boiler feed pump is required to pump 240,000 lb of water per hour against a pressure difference of 450 lb per sq inch when running at a speed of 2,900 rpm. The density of pre-heated feed water is 60 lb per cuft.

If all the impellers are identical and the specific speed per stage is not to be less than 1,000, find—

a) the least number of stages and the head h per stage.

b) the diameters of the impellers assuming the peripheral velocity$=0·96\sqrt{2gh}$

c) the shaft HP required to drive the pump, assuming an overall efficiency of 78%.

[*a*) 216 ft per stage ; *b*) 9 inches ; *c*) 168 HP]
(*Delhi University—1957*)

12.52 A centrifugal pump running at 750 rpm discharges water at a rate of 2 cusecs against a head of 50 ft. Find the discharge and the head pumped if the same pump is running at a speed of 1,500 rpm.

(4 cusecs ; 200 ft) (*Delhi University—1956*)

12.53 A centrifugal pump draws oil of specific gravity 0 78 at a temperature of 400°F from a closed tank in which the pressure is 85 lb/sq in. gauge. The vapour pressure of oil at the above temperature is 90 lb/sq in. absolute. The pump is located at 10 ft above the oil level in the tank. If the pump is installed at an elevation of mean sea level, find the available suction head. Assume the frictional losses in the suction pipe equal to 3 ft of oil.

(16 ft of oil) (*AMIE—May 1955*)

12.54 A fluid is to be pumped 1,000 ft, through a pipe 8 in. diameter. Determine the HP required to pump one ton per minute, when the kinematic viscosity is 0·02 ft sec units, and the density of fluid is 57 lb per cuft. (5·69 HP) (*Roorkee University—1955*)

12.55 A centrifugal pump runs at 750 rpm and the difference in surface levels at suction and delivery, together with the pipe friction losses, is 50 ft. The relative velocity of water at impeller exit is inclined backwards at 45° to the tangent. The velocity of flow through the impeller, and in suction and delivery pipes is 7 ft/sec. A manometric efficiency of 77% is expected, and the water enters the impeller without whirl.

Find the diameter of the impeller and the fraction of kinetic energy at discharge from the impeller which must be recovered as pressure if other losses are neglected.

If the impeller diameter at inlet is 0·4 times the diameter at outlet, and entry is through one side only, find the quantity of water the pump can discharge for these conditions, and the axial width of the impeller at inlet and exit. Find also the vane angle at inlet. If, when running at this speed with the discharge valve closed, 2 HP is required to drive, find the probable HP required at the given discharge, and the overall efficiency.

(15·19 in., 0·45, 1·409 cfs, 1·519 in., 0·61 in., 19°—24′, 12·53 HP, 65%)

12.56 The outside diameter of the vanes of a centrifugal pump is 20 in. The vanes are curved back to make an angle of 40° with the periphery. The pump has no volute and is needed for a lift of 35 ft. Calculate the proper speed and the hydraulic efficiency. By how much would the lift be increased if a whirlpool chamber of 40° were added and the pump run at the calculated speed?

Velocity of flow may be taken as $= \frac{1}{8}\sqrt{2gH}$.

(545 rpm ; 58·7% ; 18·95 ft)

12.57 A turbine pump has six impellers of 10 in. diameter with vanes curved backwards at exit at an angle of 45 deg. to the tangent. At full capacity the velocity of the water relative to the vanes as it leaves the impellers is 8 ft/sec. Calculate the total head against which the pump will deliver water when it is running at a speed of 750 rpm, assuming that 50 percent of the kinetic energy of the water is converted into pressure energy in the diffusers.

(82·50 ft of water) [*AMI Mech E (Lond)—Oct 1958*]

12.58 A centrifugal pump draws water from a sump through a vertical 152·4 mm pipe. The pump has a horizontal discharge pipe 101·6 mm diameter which is 3.23 m above water level in the sump. While pumping 35·3 lit/sec, gauges near the pump at entrance and discharge read $-0·324$ kg/cm^2 and $+$ 1·8 kg/cm^2 respectively. The discharge gauge is 0·915 m above the suction gauge. Determine the horsepower output of the pump. (11 metric HP)

12.59 A centrifugal pump impeller runs at 950 rpm. Its external and internal diameters are 508 mm and 254 mm. The vanes are set back at an angle of 35° to the outer rim. If the radial velocity of water through the impeller be maintained constant as 1·83 m/sec, find the angle of the vanes at inlet, the velocity and direction of water at outlet and the work done by the impeller per kg of water.

(8°—12′; 22·7 m/sec ; 58·5 kgm)

12.60 A single stage centrifugal pump has an impeller of 254 mm diameter which rotates at 1,800 rpm and lifts 60·5 lit/sec to 24·4 m with an efficiency of 70%. Obtain the number of stages and diameter of each impeller of a similar multistage pump to lift 75·7 lit/sec to 177 m at 1,500 rpm.

(8 stages ; 290 mm dia) (*Punjab University—1959 S*; converted to metric units)

12.61 A centrifugal pump of 4 ft (or 1·22 m) diameter runs at 200 rpm and pumps 66·5 cfs (or 1,880 lit/sec), the average lift being 20 ft (or 6·1 m). The angle which the vanes make at exit with the tangent to the impeller is 26°, and the radial velocity of flow is 8 ft/sec (or 2·44 m). Determine the useful horsepower and the efficiency. Find also the least speed to start pumping against a head of 20 ft (or 6·1 m), the inner diameter of the impeller being 2 ft (or 0·61m).

[151 HP (or 153 metric HP) ; 60·5% ; 198 rpm]
(*Punjab University—1959 A* ; converted to metric units also)

12.62 A multistage centrifugal pump is to be designed to deliver 10,000 gpm (or 757 lit/sec) of water against a manometric head of 200 ft (or 60·96 m). There are to be four equal impellers keyed to the same shaft which has a speed of 350 rpm. The vanes are curved back so that the direction of the relative velocity at discharge makes an angle of 120° with the direction of the corres-

ponding peripheral velocity, and the impeller is surrounded by guides. Assume that the water enters the vane passages in a radial direction, that the velocity of flow through the impeller is 0·27 of the peripheral velocity, and that the losses in the pump amount to one-third of the velocity head at discharge from the impeller. Find—

a) the outer dimensions of the impeller,

b) the manometric efficiency,

c) the angle of guides.

[D_2=2·6 ft (or 794 mm) B_2=3·08 in. (or 78·2 mm) ; 84 2%; 17½°]
(*London University—1942*)

12.63 In a long pipe-line carrying crude oil, it is usual to instal several pumping stations spaced at intervals along the line, rather than to rely on a single station at the pipe inlet. Why is this ? What are the advantages of multiple stations ? Illustrate your answer by sketching comparative hydraulic gradients.

A horizontal pipe-line 235 miles (or 378 km) long and 24 in. (or 610 mm) diameter is to carry 210,000 barrels per day of crude oil of C.G. 0·89. Its viscosity is such that in the formula $p=\frac{4fl}{d}\times$velocity head, the value of the frictional co-efficient f is 0·007. It is stipulated that at each pumping station, the pressure-difference generated is not to exceed 600 lb/in² (or 42·2 kg/cm²). How many identical stations would be needed, and what should be the power delivered to the oil in each one ?

Note : 1 barrel of oil=5·615 ft³ or 0·159 m³)
(*AMI Mech E—1959 Oct* ; converted to metric units also)
(5 stations ; 2,000 HP)

CHAPTER 13

SOME MORE PUMPS AND WATER LIFTING DEVICES

13.1 Propeller Pumps or Axial Flow Pumps (Fig 13.1) - As already explained in Art 12.8*c* the propeller or axial flow pumps are specially designed for large capacities and low heads, generally under forty feet or about twelve metres. This pump is often called propeller pump because its impeller somewhat resembles a marine propeller. The axial flow impeller has evolved from the radial type (See Art 12.14 and Fig 12.10). The radial flow pump (*i.e.*, the ordinary centrifugal pump) develops the liquid head by centrifugal action as distinguished from the hydrodynamic "lifting" (See Art 10.21) produced by the vanes of an axial flow or propeller pump. The specific speed of the impeller of a propeller pump is 5,200 to 23,650 FPS units (or 110 to 500 metric units) (See Table 12.1). Axial flow pumps are generally designed to operate with the shaft vertical, mainly for the sake of compactness.

Propeller or axial flow pumps are the converse of propeller or Kaplan turbines (See Chapter 7). When the pump is required to operate with a high percentage variation in head, the efficiency of fixed blade impeller is low at part and overloads. To overcome this difficulty, adjustable blade impeller has been developed. Pumps so

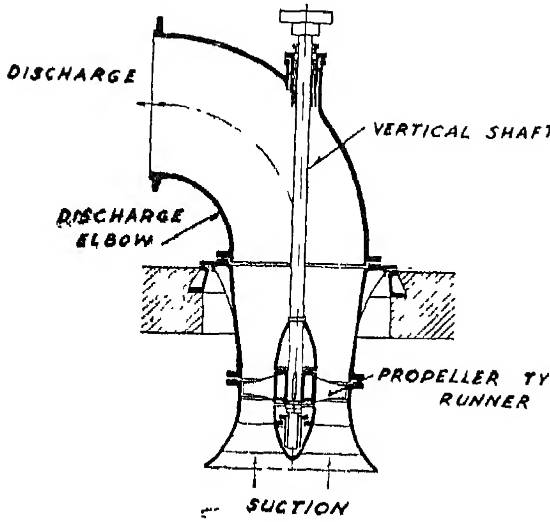


Fig 13.1 Section Through a Propeller Pump, Vertical Closed Type

Manufactured by Sulzer Bros Ltd

equipped are sometimes called *Kaplan* pumps. The discharge of a Kaplan pump can be regulated by adjusting the position of impeller blades.

Problem 13.1 Find the main dimensions of a propeller pump which is required to deliver 2,800 gallons per minute (or 212 lit/sec) of water against an average head of 9 ft (or 2·74 *m*). Assume specific speed of the pump as 9,680 FPS units (or 205 metric units), speed ratio 2·1 and flow ratio 0·45.

Solution

$Q = 2{,}800$ gpm (or 212 lit/sec) $\qquad H = 9$ ft (or 2·74 m)

$N_s = 9{,}680$ FPS units (or 205 metric units)

$K_{u_2} = 2{\cdot}1 \qquad K_{v_{m_2}} = 0{\cdot}45$

Specific speed of the pump is given by, $N_s = \dfrac{N \cdot \sqrt{Q}}{H^{\frac{3}{4}}}$

$$\text{or} \quad N = \frac{N_s \cdot H^{\frac{3}{4}}}{\sqrt{Q}} = \frac{9{,}680 \times 9^{\frac{3}{4}}}{\sqrt{2{,}800}} = 950 \text{ rpm}$$

$$\left[\text{or} \quad N = \frac{205 \times 2{\cdot}74^{\frac{3}{4}}}{\sqrt{\dfrac{212}{1{,}000}}} = \frac{205 \times 1{\cdot}655 \times 1{\cdot}288}{0{\cdot}46} = 950 \text{ rpm} \right]$$

In order to be able to couple the pump with an induction type electric motor, adopt a speed of **960 rpm** *Answer*

Peripheral velocity of impeller at outlet

$$u_2 = K_{u_2} \cdot \sqrt{2g\,H}$$

$$= 2{\cdot}1 \times \sqrt{64{\cdot}4 \times 9} = 50{\cdot}5 \text{ ft/sec}$$

$$[\text{or} \quad u_2 = 2{\cdot}1 \times \sqrt{2 \times 9{\cdot}81 \times 2{\cdot}74} = 15{\cdot}4 \; m/\text{sec}]$$

$$\text{but} \quad u_2 = \frac{\pi \cdot D_2 \cdot N}{60}$$

$\therefore$ Impeller outlet diameter $D_2 = \dfrac{60 \cdot u_2}{\pi \cdot N}$

$$= \frac{60 \times 50{\cdot}5}{\pi \times 960} = 1{\cdot}00 \text{ ft} \approx \mathbf{12 \text{ in.}}$$

$$\left[\text{or} \quad D_2 = \frac{60 \times 15{\cdot}4}{\pi \times 960} = 0{\cdot}305 \; m \quad \text{or} \quad \mathbf{305 \text{ mm}} \right]$$

Impeller inlet diameter $D_1 = D_2 =$ **12 in.** (or **305 mm**) *Answer*

Velocity of flow $\qquad v_{m_2} = K_{v_{m_2}} \cdot \sqrt{2g\,H} = 0{\cdot}45 \times \sqrt{64{\cdot}4 \times 9}$

$$= 10{\cdot}83 \text{ ft/sec}$$

[or $v_{m_2}=0{\cdot}45\times\sqrt{19{\cdot}62\times2{\cdot}74}=3{\cdot}305$ m/sec]

$$Q=\frac{2{,}800\times10}{62{\cdot}4\times60}=7{\cdot}48 \text{ cfs}$$

(or $Q=0{\cdot}212$ m^3/sec)

Assume **4 blades**, each ½ in. thick.

$$v_{m_2}=\frac{Q}{\text{area of flow}}=\frac{Q}{\frac{\pi}{4}\left[D_2^2-n^2\,.\,D_2^2\right]}$$

where $n=\dfrac{\text{diameter of boss}}{\text{outlet diameter of impeller}}=\dfrac{d}{D_2}$

$$\therefore\quad 10{\cdot}83=\frac{7{\cdot}48}{\frac{\pi}{4}D_2^2\,(1-n^2)}=\frac{7{\cdot}48\times4}{\pi\times1\times(1-n^2)}$$

$$\left[\text{or } 3{\cdot}305=\frac{0{\cdot}212}{\frac{\pi}{4}\times0{\cdot}305^2\times(1-n^2)}\right]$$

or $$n^2=1-\frac{7{\cdot}48\times4}{10{\cdot}83\times\pi}=1-0{\cdot}878=0{\cdot}122$$

$$\left[\text{or}\quad n^2=1-\frac{0{\cdot}212\times4}{3{\cdot}505\times\pi\times0{\cdot}305^2}=1-0{\cdot}878=0{\cdot}122\right.$$

or $n=0{\cdot}35$

∴ Diameter of boss

$=0{\cdot}35\times12=4{\cdot}2$ in. $\approx$ **4¼ in.** (or $0{\cdot}35\times305=$ **107 mm**)

Horsepower required to drive the pump

$$=\frac{w\,.\,Q\,.\,H}{550\times\eta_t}=\frac{62{\cdot}4\times7{\cdot}48\times9}{550\times0{\cdot}695}=11$$

(assuming pump overall efficiency as 69·5%)

∴ HP=**11** *Answer*

$$\left[\text{or Power}=\frac{1{,}000\times0{\cdot}212\times2{\cdot}74}{75\times0{\cdot}695}=\textbf{11·14 metric HP}\quad \textit{Answer}\right]$$

13.2 Cavitation in Propeller Pumps—Propeller or axial flow pumps have a limited suction lift, about three feet (or one metre) of liquid. The impeller is therefore located within the suction limit, very close to the pump inlet. Unlike radial pumps the axial flow pump cannot be operated throughout the range of performance curve. Operation must be confined to the range of maximum efficiency otherwise it becomes noisy, and there is danger of cavitation (See Art 7.16). Cavitation results from the same causes in turbines as well as in pumps and Thoma's co-efficient $\sigma=\dfrac{H_b-H_s}{H}$ derived for turbines is also applicable to pumps. Modern trend, however, is to introduce another criterion besides, Thoma's co-efficient σ, for pumps.

Net positive suction head = excess of absolute suction head over vapour pressure

Symbolically, $H_{sv} = H_b - H_s$

Then $$\sigma_{crit} = \frac{H_{sv}}{H} \qquad ..\ (13.1)$$

Specific speed of a centrifugal pump is given by

$$N_s = \frac{N \cdot \sqrt{Q}}{H^{\frac{3}{4}}}$$

or $$H = \frac{N^{\frac{4}{3}} \cdot Q^{\frac{2}{3}}}{N_s^{\frac{4}{3}}} = \frac{\text{const}}{N_s^{\frac{4}{3}}} \qquad ...(13.2)$$

Substituting this value of H in (13.1)

$$\sigma_{crit} = \frac{H_{sv}}{\text{const}} \cdot N_s^{\frac{4}{3}} \qquad ...(13.3)$$

$\therefore$ For two pumps

$$\frac{(\sigma_{crit})_2}{(\sigma_{crit})_1} = \left(\frac{N_{s_2}}{N_{s_1}}\right)^{\frac{4}{3}} \qquad ...(13.4)$$

Relationship of σ and N_s can be known empirically.

Thus $$\sigma = \frac{6{\cdot}3 \cdot N_s^{\frac{4}{3}}}{10^6} \text{ for single suction pumps} \qquad ...(13.5)$$

(FPS units)

and $$\sigma = \frac{4 \cdot N_s^{\frac{4}{3}}}{10^6} \text{ for double suction pumps} \qquad ...(13\ 6)$$

(FPS units)

Wislicenus* introduced another cavitation criterion, known as suction specific speed,

$$s = \frac{N \cdot \sqrt{Q_{gpm}}}{H_{sv}^{\frac{3}{4}}} \text{ (FPS units)} \qquad ..\ (13.7)$$

The relation between the factors s, N_s and σ_{crit} can be derived from equations (13.3) and (13.7)

$$\frac{N_s}{s} = \sigma_{crit}^{\frac{3}{4}} \qquad ...(13.8)$$

Stepanoff† has given the following values for s:

7,900 (FPS units) for single suction pumps,

and 11,200 (FPS units) for double suction pumps.

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*Trans. ASME Vol 61, 1939, p. 17.

†Centrifugal and Axial Pumps by A. J. Stepanoff, (John Wiley & Sons), p. 268.

13.3 Characteristic Curves of Axial Flow Pumps—Typical characteristics of an axial flow pump are shown in Fig 13.2*d*. It will be noticed that the head-discharge curve has a steeper slope than for a centrifugal pump of radial type. Unlike the latter, the BHP of an axial flow pump increases with head. Rising up of the BHP-discharge curve towards the left suggests that the motor may become overloaded if the lift were to be raised above the head corresponding to the maximum efficiency. The pumping set is, therefore, provided with a device to stop the motor before overloading. The efficiency-discharge curve indicates a maximum efficiency at less than half the shut off value (*i e.* when $Q=0$) and fall in efficiency for higher and lower heads.

Fig 13.2 *a*, *b* and *c* show the typical characteristic curves of a single-stage volute pump, multiple-stage turbine pump and mixed flow (screw type) pump respectively, which could be compared with the characteristic curves of a propeller pump (Fig 13.2*d*).

The speed of an axial flow pump may be varied widely without affecting efficiency or head. Since the discharge varies directly with speed (See Eqn 12.51), variation of speed may be employed for regulating discharge.

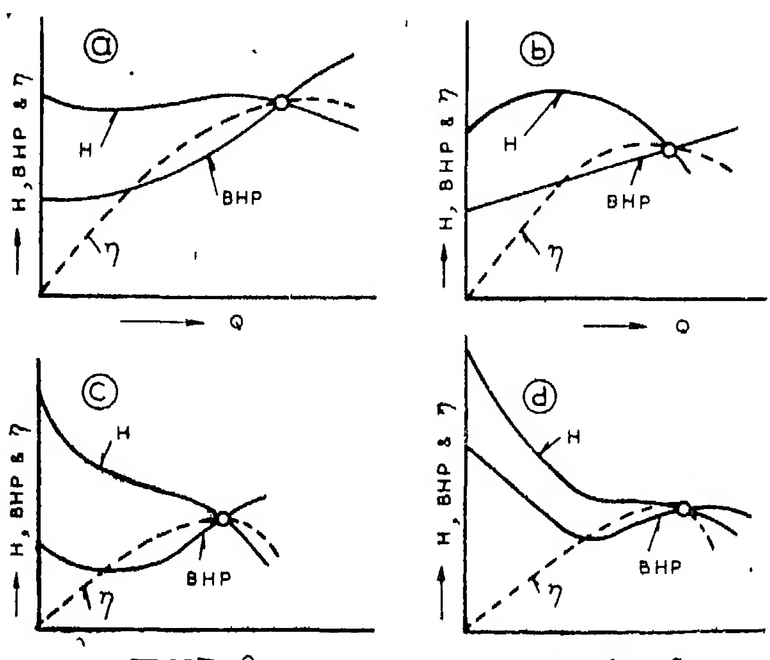


Fig 13.2 Typical Characteristic Curves *a*) Single Stage Volute Pump *b*) Multiple Stage Turbine Pump *c*) Mixed Flow (Screw) Pump *d*) Axial Flow (Propeller) Pump

13.4 Uses of Propeller Pumps—Propeller or axial flow pumps are widely employed for all types of low head pumping jobs such as land drainage, flood control, storm water disposal, irrigation, condenser circulation in power plants, etc. This pump, however, is not satisfactory for developing pressures at throttled discharge.

13.5 Screw Pumps or Mixed Flow Pumps (Fig 12.7*b*, 12.10 and 13.3 *a*) and *b*)—Mixed flow pumps are often called angular flow or semi-axial pumps because the water flows through the impeller axially as well as radially. Like the axial flow type, these are also used for low head and high discharge, and have a limited suction lift. The specific speed of mixed flow impeller varies from 3,900 to 7,750 FPS units (or 82 to 164 metric units) (See Table 12.1). The head of liquid is developed partly by centrifugal action and partly by lifting action of vanes. Basically, these pumps have a construction similar to that of Francis or mixed flow turbines, but the operation is just the reverse. Typical characteristics of a mixed flow (screw type) pump are shown in Fig 13.2*c*.

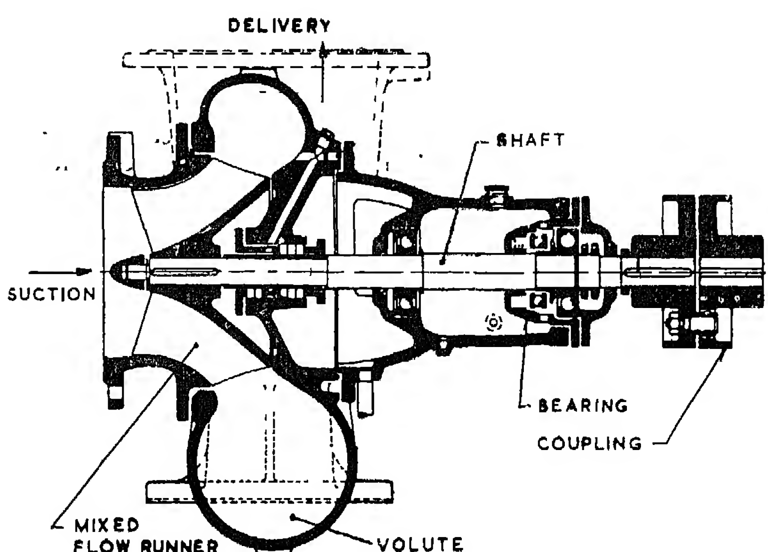


Fig 13.3 *a*) Section through a Screw (Mixed Flow) Horizontal Type Pump Manufactured by Sulzer Bros Winterthur (Switzerland)

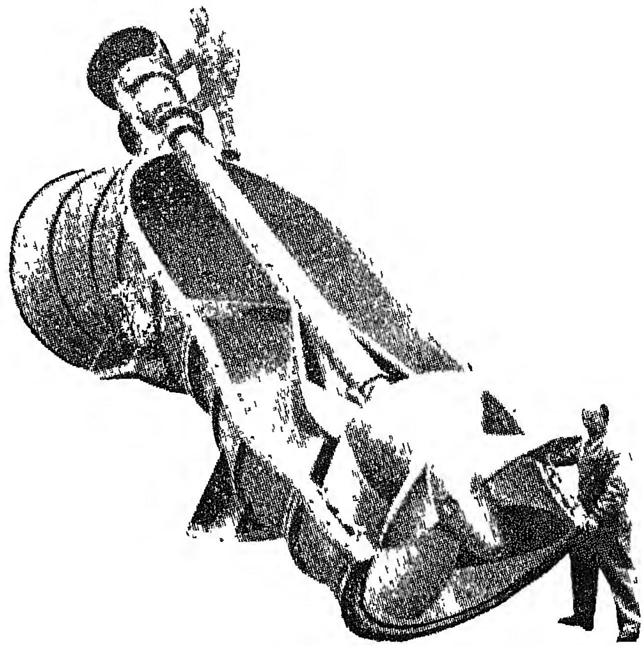


Fig 13.3 b) Sulzer Screw Pump Delivering 185,000 gpm (or 14,000 lit/sec) against a Head of 10·75 ft (or 3·28 m)

Problem 13.2 Two centrifugal pumps A and B, each single stage, single suction, mixed flow type, when tested separately gave the following performance results—

Pump A

Q gpm	0	1,000	2,000	3,000	4,000	5,000	6,000
H ft	38	37·5	35·5	33	30	27	23

Pump B

Q gpm	0	1,000	2,000	3,000	4,000	5,000	6,000
H ft	59	56	51	46	39	29·5	16

Draw H vs Q curve of each pump. If the two pumps work in parallel, draw the combined H vs Q curve. Find the BHP of the combined pumping set when the system is delivering 9,500 gpm. Take the efficiency of the combined set as 65%. *(UPSC—Jan 1953)*

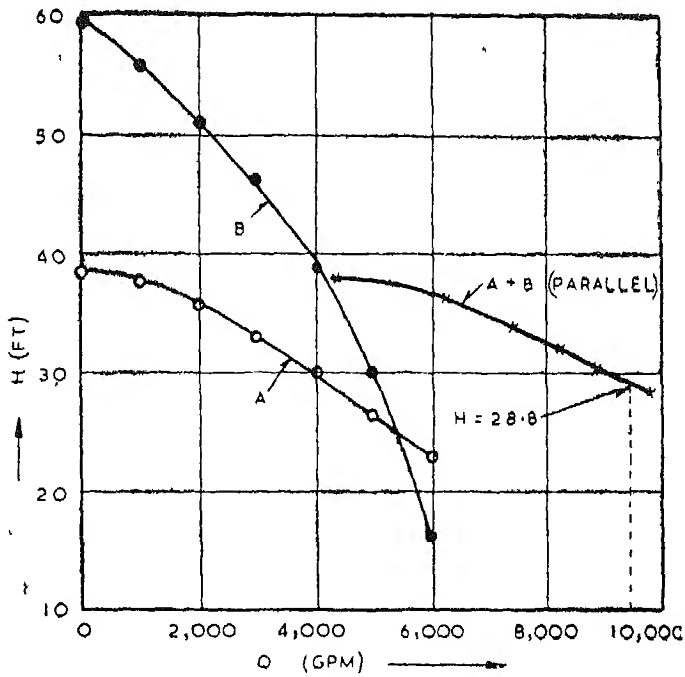


Fig 13.4 *H* vs Q Curves of Two Mixed Flow Pumps Working in Parallel

Solution

H vs *Q* curve of each pump is drawn in Fig 13.4. For pumps working in parallel, add the *Q* 's of the two pumps at the same heads, thus—

H ft	*Q* gpm		Total *Q* (*A*+*B*) gpm
	Pump A	Pump B	
38	0	4,150	4,150
34	2,700	4,600	7,300
30	4,000	4,950	8,950
26	5,250	5,325	10,575
24	5,700	5,450	11,150

For the combined set, the head *H*, when *Q* is 9,500 gpm is 28·8 ft. (from the new curve)

$$\therefore \quad \text{BHP of combined set} = \frac{(w \cdot Q) \cdot H}{550 \cdot \eta}$$

$$= \frac{\frac{9{,}500}{6 \cdot 0} \times 28 \cdot 8}{550 \times 0 \cdot 65} = \mathbf{127\ HP} \quad \textit{Answer}$$

OTHER WATER LIFTING DEVICES

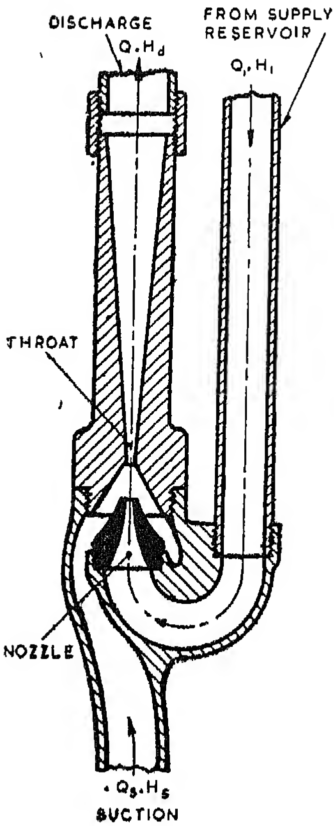


Fig 13.5 Jet Pump

13.6 Jet Pump (Fig 13.5)—Steam or water at a high pressure is forced through a fine aperture nozzle, thereby converting most of the pressure energy into kinetic form. It results in a lowering of pressure which causes suction to take place. Probably a part of the suction is due to the lowering of pressure resulting from condensation of steam.

Steam is generally used in jet pumps meant to feed boilers. The nozzle can lower the pressure to three fourths of atmospheric pressure *i.e.* about 8 ft suction head can be obtained. Steam also serves to preheat the water fed to the boiler.

With water at high pressure, a more perfect vacuum can be produced so that a suction lift of 18 to 20 ft (or 5·5 to 6 m) can be obtained. The jet should be near the surface of water. Several jets may be employed if a large quantity is to be pumped. This arrangement is often used in mines and also for pumping oil or petroleum. The capacity of a jet pump ranges upto about 700 gpm (or 53 lit/sec).

If Q_1 be the rate of flow from supply reservoir and Q_s the quantity sucked per second, the delivery $Q=Q_1+Q_s$.

If H_1 be the height of supply reservoir above the jet, H_s the suction head and H_d, the delivery head, then efficiency of the jet pump is given by—

$$\eta=\frac{Q_s\,(H_d+H_s)}{Q_1\,(H_1-H_d)} \qquad \ldots(13.9)$$

Problem 13.3 A jet pump fitted 8 ft (or 2·44 m) above the suction reservoir and 60 ft (or 18·3 m) below the supply reservoir lifts water through a total height of 8 ft 9 in. (or 2·67 m). Determine the efficiency of the jet pump when it delivers 100 gpm (or 7·57 lit/sec) of water while using 36·5 gpm (or 2·76 lit/sec) from the supply reservoir.

Solution

$H_1=60$ ft (or 18·3 m) $\qquad H_s=8$ ft (or 2·44m)

$H_s+H_d=8{\cdot}75$ ft (or 2·67 m)

$Q=100$ gpm (or 7·57 lit/sec) $\qquad Q_1=36{\cdot}5$ gpm (or 2·76 lit/sec)

$Q_s=Q-Q_1=100-36{\cdot}5=63{\cdot}5$ gpm

(or $Q_s=7{\cdot}57-2{\cdot}76=4{\cdot}81$ lit/sec)

Jet pump efficiency $\eta=\dfrac{Q_s\,(H_d+H_s)}{Q_1\,(H_1-H_d)}=\dfrac{63{\cdot}5\times8{\cdot}75}{36{\cdot}5\times(60-0{\cdot}75)}=0{\cdot}257$

or **25·7%** *Answer*

$$\left[\text{or} \quad \eta = \frac{4{\cdot}81 \times 2{\cdot}67}{2{\cdot}76 \times (18{\cdot}3 - 0{\cdot}229)} \times 100 = \mathbf{25{\cdot}7\%} \quad \textit{Answer}\right]$$

13.7 Air Lift Pump—It utilises compressed air for raising water. Because the density of a mixture of air in water is much lower than that of pure water, if such a mixture is balanced against a water column, the former will rise much higher than the latter.

Fig 13.6 shows diagrammatically the simplest method of raising water from a deep well by means of compressed air supplied by a compressor unit installed on the surface. The rapid stream of air at A produces a jet and entraps the water in the immediate vicinity and carries it upwards to B. The air-water mixture can rise above the water level because its density is less. The air is introduced at a considerable depth below the water surface in the well, so that the actual difference ($L-H$) may be sufficient to initiate flow to B, the desired delivery level.

For best results, the lift of the pump ($L-H$) should be less than H (See Fig 13.6).

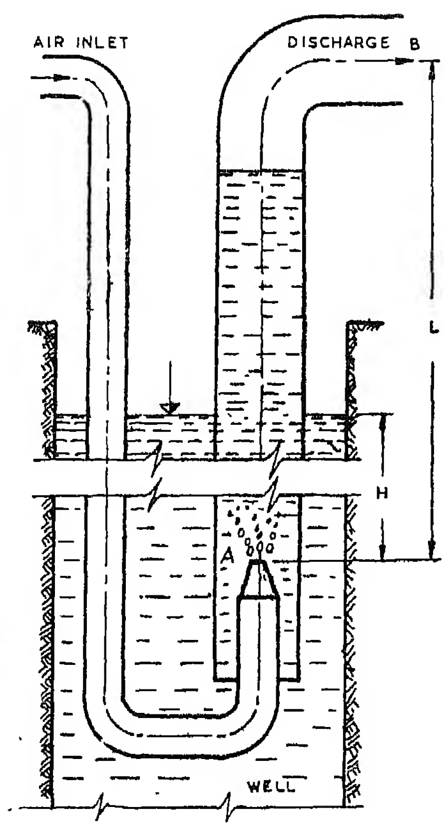


Fig 13.6 Air Lift Pump

TABLE 13.1

Lift vs Submergence

Lift ($L-H$)	ft	50	150	250	350	450
	m	15	45	75	105	135
Submergence H	ft	100	200	250	275	320
	m	30	60	75	85	100

Advantages of air-lift pump :

i) No moving parts, no valves. No damage due to solids in suspension in water.

ii) Raises more water for a given diameter than any other pump.

iii) Can be used to drain out water in mines where the compressor units are already available.

Disadvantages :

i) Efficiency is very low. The volume V of air in cu ft, at atmospheric pressure, required to lift one cu ft of water through a height $(L-H)$ depends upon the efficiency.

If $\quad \eta=30\%, \quad V\approx\frac{L-H}{20}$ cu ft

and if $\quad \eta=40\%, \quad V\approx\frac{L-H}{25}$ cu ft

ii) Possibility of air leakage.

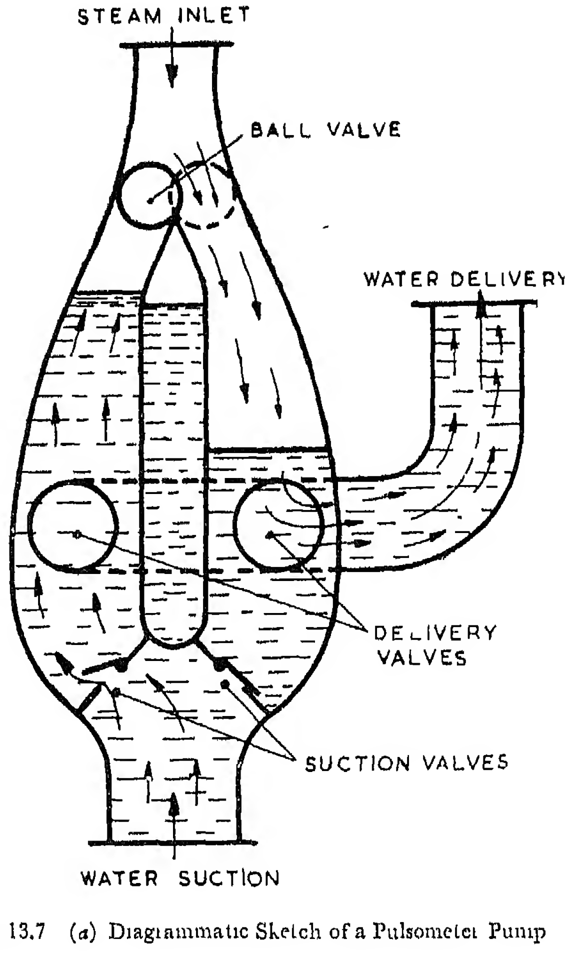


Fig 13.7 (*a*) Diagrammatic Sketch of a Pulsometer Pump

13.8 Pulsometer Pump (Fig 13.7*a* and *b*)—Condensation of steam is utilised in a pulsometer pump for raising water. The pulsometer pump comprises two chambers arranged side by side. While water is rising in one chamber, discharge takes place from the other. There is a steam inlet at the top with connections to either chamber. A ball valve is used to shut off one chamber while the steam passes to the other. There is a water inlet at the bottom connected to both chambers by simple

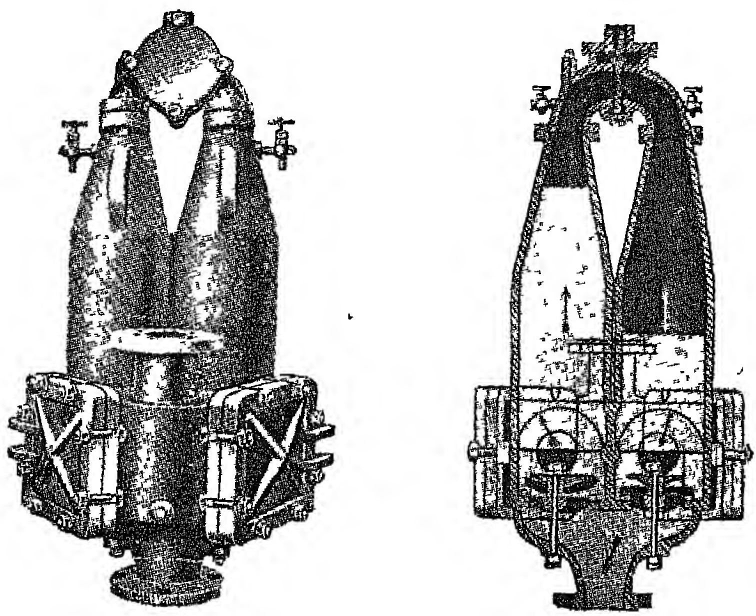


Fig 13 7 (*b*) Section through a Pulsometer Pump

non-return valves. As steam enters a chamber full of water, the surface of contact being small no appreciable condensation takes place. The steam pressure forces the water out through the delivery valve and continues to do so until the water level falls below that of the delivery valve. At this stage the exit is thrown open to steam and the violent escape of steam and the increased area of surface of contact result in sudden condensation. Immediately, the ball valve changes its seat cutting off steam supply, the delivery valve is closed and suction commences through a valve at the bottom. While the chamber is gradually filled with water, discharge takes place from the other chamber in a similar way. The two chambers work alternately and maintain the cycle. An air vessel is provided for cushioning.

Practical Data

Suction range	=6·5 to 13 ft (or 2 to 4 *m*)
Maximum suction lift	=26 ft.(or 8 *m*)
Temperature of water at which Suction stops and delivery begins	=120°*F* (or 50°*C*)
Delivery head per pulsometer	=150 ft (or about 50 *m*)

Steam pressure	=Delivery head in lb/sq in.+22 to 45 lb/sq in. depending on head (or in $kg/cm^2+1 \cdot 5$ to $3\ kg/cm^2$), i.e. about 50% higher than water head required.
Volume of steam required	=2 to 3 times the water delivered.

13.9 Hydraulic Ram—Hydraulic ram is a contrivance to raise a part of large amount of water available at some height, to a greater height. This is employed when some natural source of water like a spring or a stream is available at some altitude e.g. in a hilly region. Work done by a large quantity of water in falling through a small height is used to raise a small part of it to a greater height. Action of water hammer makes it feasible. No external power is, therefore, required to work this machine. The first hydraulic ram was invented in 1775 by John Whitehurst of Derby (England).

Fig 13.8 illustrates a typical ram installation. A is the source of supply connected to a supply pipe having a gate G. Water flows through the supply pipe to a waste valve or impulse valve W which, when open, allows the water to escape. As this valve closes, water is suddenly brought to rest and the resulting water hammer forces open the delivery valve fitted in an air chamber. Water is pushed through the delivery

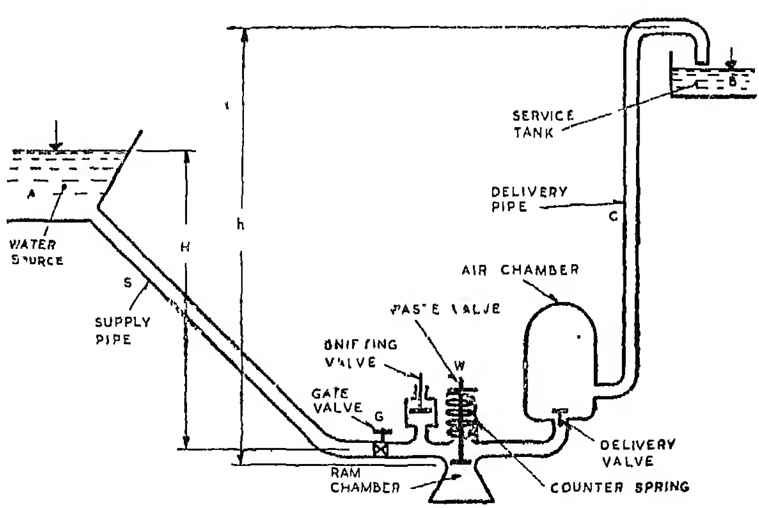


Fig 13.8 Hydraulic Ram

pipe, connected to the air chamber, into the service tank B. A snifting or air valve is provided, which admits air for the air chamber to make up for the loss due to absorption in water. All three valves mentioned above are non-return type.

13.10 Ram Principle and Operation—In principle, the hydraulic ram is an impulse pump. The impulse is developed at the expense of dynamic inertia possessed by a moving column of water. In other words, the momentum of a long column of water flowing through the supply pipe is made to force a part of the water to a level higher

than that of the supply source itself. In order to develop maximum impulse, supply pipe should be as long as possible. Installation of ram too close to the source of supply will reduce the impulse and consequently the delivery head.

The cycle of operations is explained below. Initially, the water is at rest and the delivery pipe *C* and waste valve *W* are closed. Ram is started by opening the gate valve *G*, of the supply pipe, setting the water in motion. The column of water in the supply pipe rebounds a short distance creating partial vacuum in the ram chamber. The vacuum causes the snifting valve to open, admitting a small amount of air. This removes the pressure difference across the waste valve which then opens of its own weight. Water begins to escape through the waste valve. As flow accelerates, pressure on the waste valve rises and it begins to close. As it closes, pressure increases accelerating the rate of closing. Consequently, the valve closes suddenly, producing water hammer in the supply pipe. A very high pressure is momentarily produced in the ram chamber, by which the delivery valve is forced open. Open delivery valve admits water to the air chamber. As the pressure gradually rises in the air chamber, inflowing water is brought to rest and the delivery valve is closed. Incoming water, however, compresses the air and as soon as the delivery valve is closed, the air pushes a part of the water into the delivery pipe. Water passing through the air chamber carries along with it the air admitted to the air chamber through the snifting valve at the beginning of the cycle. This ensures that there would always be some air in the air chamber. The cycle is complete and starts again at the beginning. The ram operation can be stopped by closing the gate valve *G* in the supply pipe.

This cycle of operation may be summed up as follows :

1) Delivery valve closed.
2) Vacuum is created by rebound and air is admitted through the snifting valve.
3) Waste valve opens and water starts flowing to waste through the valve.
4) Waste valve closes suddenly causing water hammer.
5) Momentary high pressure resulting from water hammer forces open the delivery valve.
6) Water enters the air vessel through the delivery valve.
7) Pressure rises in the air chamber, slowly bringing the incoming water to rest and closing the delivery valve, thus completing the cycle.

The operation of the hydraulic ram depends upon the successful creation and destruction of velocity in the supply pipe. The waste valve must close suddenly to enable kinetic energy to be utilised to a maximum. Theoretically, the pumping head remains constant during the operation.

Advantages—

a) No coal, oil or electricity is required as the motive power is the driving water.

b) It requires no lubrication and no packings as there are no piston glands.

c) Maintenance expenses are low and almost no labour is required for supervision.

d) Hydraulic rams can work continuously for 24 hours and thus give regular water supply.

e) These can be adjusted to work with any quantity from their maximum down to less than one-half and automatic adjustment is possible.

f) It has high efficiency and quiet working.

13.11 Some Hydraulic Ram Installations—

i) Blake's Hydram (Hydraulic Ram) manufactured by John Blake Ltd. England was installed at Taj Mahal, Agra in 1900 for raising water at the rate of 600,000 gallons per day (or 31·5 lit/sec) and working satisfactorily till now with very low maintenance cost.

ii) Blake's Hydram installed at Risalpur (Pakistan) in 1925 raises 1,080,000 gallons of water per day (or 56·7 lit/sec) to a vertical height of 60 ft (or 18·3 m) and to a horizontal distance of 5,000 ft (or 1,525 m).

13.12 Hydraulic Ram Calculations—

Let Q = quantity of water wasted by ram, in cfs (or m^3/sec)

q = quantity of water pumped by ram, in cfs (or m^3/sec)

h = elevation of service tank above waste valve, in ft (or m)

H = height of source above waste valve, in ft (or m)

L = length of supply pipe from the source of supply to the waste valve, in ft (or m)

D = diameter of supply pipe, in ft (or m)

H_{L_s} = head loss in supply pipe, in ft (or m)

H_{L_d} = head loss in delivery pipe, in ft (or m)

Size of the supply pipe should be so designed that the amount of liquid passing through per second is equal to three times the average quantity of liquid consumed by the ram.

$$D=\sqrt{1\cdot63\,(Q+q)}\text{ ft}\quad[\text{or } D=\sqrt{0\cdot5\,(Q+q)}\text{ m}]\qquad ...(13.10)$$

$$L=5\text{ to }10\,(h-H)\text{ ft}\quad[\text{or } L=1\cdot5\text{ to }3\,(h-H)\text{ m}]\qquad ...(13.11)$$

Length of the supply pipe, can also be determined as follows ;

$$L=c\,.\,h$$

where the co-efficient c which is a function of H may be obtained from the following table.

TABLE 13 2

Values of Pipe Length Co-efficient c

i) $h <$ 100 ft (or 30 m)

H	ft	2	4	6	8	10
	m	0·61	1·22	1·83	2·44	3·05
c		3	2·65	2·25	1·85	1·5

ii) $h >$ 100 ft (or 30 m)

H	ft	2	4	6	8	10
	m	0·61	1·22	1·83	2·44	3·05
c		3·5	3·0	2·6	2·25	2·0

Columbiana suggests the following values of lift and fall for maximum efficiency.

TABLE 13.3

Lift and Fall for Maximum Efficiency of Ram

H ft	h ft	L ft
3	20	30
4	30	30
5	40	40
·7	50	50
8	60	60
10	80	80
. 14	100	100

H m	h m	L m
1	6	10
1·5	12	12
2	15	15
3	23	23
4	30	30

Volume of air chamber = Volume of delivery pipe

Dynamic pressure on waste valve

$$=1 \cdot 35 \times \frac{w \, . \, v^2}{2g} \text{ lb/sq ft} \quad \left[\text{or } 6 \cdot 6 \times \frac{w \, . \, v^2}{2g} \text{ kg/m}^2\right] \quad ...(12.13)$$

where w = Specific weight of liquid in lb/cu ft (or kg/m³)

v = Velocity of water past waste valve in ft/sec (or m/sec)

Velocity of water t seconds after the opening of waste valve is given by*—

$$v = c \, . \, \frac{a}{a_v} \left\{ \frac{1 - e^{\frac{-2c}{k} t}}{1 + e^{\frac{-2c}{k} t}} \right\} \text{ ft/sec (or m/sec)} \quad ...(13.13)$$

where a = cross-sectional area of supply pipe,

and a_v = effective area of delivery valve

= actual area × co-efficient of discharge

$$\text{The co-efficient } = \sqrt{\frac{2g \, . \, H}{H \, . \, \frac{f \, . \, l}{m} \, . \, \frac{a_v^2}{a^2}}} \quad ...(13.14)$$

and $$k = 2l \, . \, \frac{a_v}{a} \, . \, \frac{1}{H \, . \, \frac{f \, . \, l}{m} \, . \, \frac{a_v^2}{a^2}} \quad ...(13.15)$$

where f = frictional factor,

l = length of pipe,

m = hydraulic mean radius

$= \frac{d}{4}$ for a pipe running full.

The area of delivery valve *i.e.*, area of a port opening should be such that the maximum velocity of flow is about 3·3 ft/sec (or 1 m/sec).

Efficiency of ram is given by—

$$\eta = \frac{q(h + H_{L_d})}{(Q+q)(H - H_{L_s})} \quad ...(13.16)$$

Clark gives the following figures for efficiency :

TABLE 13.4

$\frac{\mathbf{h}}{\mathbf{H}}$ vs η

Ratio of lift to fall $\left(\frac{h}{H}\right)$	4	8	12	16	20	24	26
Efficiency η in %	72	52	37	25	14	4	0

*Hazra, L.F., Bulletin of the University of Wisconsin (1908) No. 205, p. 211.

Efficiency of ram is also expressed as—

a) $\eta=\frac{q\,.\,h}{(Q+q)H}$ ...(by D' Aubuisson) ...(13.17)

(This is same as Eqn 13.16 neglecting frictional losses)

b) $\eta=\frac{q\,.\,(h-H)}{Q\,.\,H}$ ...(by Rankine) ...(13.18)

Practical Data for Ram—The lowest head under which a ram can work is two feet (or 0·6 m). The delivery head h should be six to twelve times the fall.

Quantity of water delivered by the ram is about $\frac{1}{12}$ to $\frac{1}{24}$ of the amount supplied depending upon the ratio of lift to fall.

Roughly $q=\frac{H\,.\,Q}{2h}$ ...(13.19)

Problem 13.4 A hydraulic ram installation gave the following particulars :

Fall or supply head	$H=9$ ft 10 in. (or 3 m)
Length of supply pipe	$L=20$ ft (or 6·1 m)
Diameter of supply pipe	$D=1$ in. (or 25·4 mm)
Lift of delivery head	$h=24$ ft 7 in. (or 7·5 m)
Length of delivery pipe	$L_d=24$ ft 7 in. (or 7·5 m)
Diameter of delivery pipe	$d_d=\frac{1}{2}$ in. (or 12·7 mm)

Time taken to pump 3 lb (or 1·36 kg) of water=33·8 sec

Water wasted during 33·8 sec =13·5 lb (or 6.13 kg)

Assuming a frictional factor 0·01, determine the efficiency of the ram.

Solution

Total quantity of water supplied by the reservoir

$$=Q+q=13{\cdot}5+3=16{\cdot}5 \text{ lb in } 33{\cdot}8 \text{ sec}$$

$$[\text{or } =6{\cdot}13+1{\cdot}36=7{\cdot}49 \text{ kg in } 33{\cdot}8 \text{ sec}]$$

$$=\frac{16{\cdot}5}{38{\cdot}8}\times\frac{1}{62{\cdot}4}=0{\cdot}00782 \text{ cusec}$$

$$\left[\text{or } \frac{7{\cdot}49}{33{\cdot}8}\times\frac{1}{1{,}000}=0{\cdot}221\times10^{-3}\ m^3/\text{sec}\right]$$

Velocity of water flowing in supply pipe

$$v_s=\frac{Q+q}{a_s}=\frac{0{\cdot}00782}{\frac{\pi}{4}\times\left(\frac{1}{12}\right)^2}=1{\cdot}439 \text{ ft/sec}$$

$$\left[\text{or } v_s=\frac{0{\cdot}221\times10^{-3}}{0{\cdot}785\times0{\cdot}0254^2}=0{\cdot}437\ m/\text{sec}\right]$$

Water pumped $q=\frac{3}{33{\cdot}8}\times\frac{1}{62{\cdot}4}=0{\cdot}00142$ cusec

$$\left[\text{ or } q=\frac{1 \cdot 36}{33 \cdot 8}\times\frac{1}{1{,}000}=0\ 402\times10^{-4}\ m^3/\text{sec}\right]$$

Velocity of water flowing in delivery pipe

$$v_d=\frac{q}{a_d}=\frac{0 \cdot 00142}{\frac{\pi}{4}\times\left(\frac{1}{24}\right)^2}=1 \cdot 043 \text{ ft/sec}$$

$$\left[\text{ or } v_d=\frac{0 \cdot 402\times10^{-4}}{0 \cdot 785\times0 \cdot 0127^2}=0\ 318\ m/\text{sec}\right]$$

Head lost in friction in supply pipe

$$H_{L_s}=\frac{4fL}{D}\cdot\frac{v_s^2}{2g}=\frac{4\times0 \cdot 01\times20}{\frac{1}{12}}\times\frac{(1 \cdot 439)^2}{64 \cdot 4}=0 \cdot 309 \text{ ft}$$

$$\left[\text{or } H_{L_s}=\frac{4\times0 \cdot 01\times6 \cdot 1}{0\ 0254}\times\frac{0 \cdot 437^2}{19 \cdot 62}=0 \cdot 094\ m\right]$$

Head lost in friction in delivery pipe

$$H_{L_d}=\frac{4\times0 \cdot 01\times24 \cdot 58}{\frac{1}{24}}\times\frac{(1 \cdot 043)^2}{64 \cdot 4}=0 \cdot 398 \text{ ft}$$

$$\left[\text{or } H_{L_d}=\frac{4\times0 \cdot 01\times7 \cdot 5}{0\ 0127}\times\frac{0\ 318^2}{19 \cdot 62}=0 \cdot 1216\ m\right]$$

$\therefore$ Efficiency of hydraulic ram $\eta=\dfrac{q(h+H_{L_d})}{(Q+q)(H-H_{L_s})}$

$$=\frac{0\ 00142\times(24 \cdot 58+0 \cdot 398)}{0\ 00782\times(9 \cdot 834-0 \cdot 309)}=0 \cdot 475 \text{ or } \mathbf{47 \cdot 5\%} \quad \textit{Answer}$$

$$\left[\text{or } \eta=\frac{0 \cdot 402\times10^{-4}\times(7 \cdot 5+0 \cdot 1216)}{0 \cdot 221\times10^{-3}\times(3-0\ 094)}=0 \cdot 475 \text{ or } \mathbf{47\ 5\%} \quad \textit{Answer}\right]$$

Problem 13.5 A hydraulic ram has a waste valve 4 in. in diameter which begins to close when the velocity of flow past the valve itself is 6 ft/sec. If the dynamic pressure on the valve per unit area is $1 \cdot 35\ \dfrac{w\ .\ v^2}{2g}$ lb, find the necessary weight of valve.

Solution

Diameter of waste valve base $d_w=4$ in.

Area of waste valve base $\qquad a_w=\frac{\pi}{4}\times4^2=12 \cdot 56$ sq in.

$$v=6 \text{ ft/sec}$$

$$w=62 \cdot 4 \text{ lb/cu ft}$$

Dynamic pressure on the valve $=\dfrac{1 \cdot 35\ w\ .\ v^2}{2g}$

$$=\frac{1{\cdot}35\times 62{\cdot}4\times 36}{144\times 64{\cdot}4}=0\ 328\ \text{lb/sq in.}$$

$\therefore$ Weight of valve $=0\ 328\ a_w$

$=0{\cdot}328\times 12{\cdot}56$

$=\mathbf{4{\cdot}12\ lb}$ *Answer*

Problem 13.6 A hydraulic ram with a supply pipe of 3 in. diameter has a waste valve of 4 in. diameter and $3\frac{1}{2}$ lb weight. The length of supply pipe is 30 ft. The travel of the valve is $\frac{1}{4}$ in. The number of beats per minute is 110. Determine the discharge through the delivery pipe against a head of 20 ft ? Assume that there is no slip.

(*Jadavpur—1954*)

Solution

$D=3$ in.	$h=20$ ft
$d_w=4$ in.	$L=30$ ft
Valve travel $s=\frac{1}{4}$ in.	$w=3\frac{1}{2}$ lb
Beats $=110$ per min	

Pressure required to close the waste valve

$$=\frac{\text{total valve weight}}{\text{area of base of waste valve}}=\frac{3\ 5}{\frac{\pi}{4}\times\left(\frac{4}{12}\right)^2}$$

$=40{\cdot}1$ lb/sq ft

Dynamic pressure on waste valve for closing $=\dfrac{w\ .\ v^2}{2g}$ lb/sq ft

(neglecting factor 1·35 in Eqn 13.12)

where $v=$ max velocity of water past waste valve just before closure

$$\therefore\quad \frac{w\ .\ v^2}{2g}=40\ 1$$

or $$v=\sqrt{\frac{40{\cdot}1\times 64{\cdot}4}{62{\cdot}4}}=6{\cdot}43\ \text{ft/sec}$$

Maximum velocity of water in the supply pipe v

$$=\text{max velocity past waste valve}\times\frac{\text{area past waste valve}}{\text{area of supply pipe}}$$

Now, area past waste valve $=\pi\ .\ d\ .\ s$

$$=\pi\times\frac{4}{12}\times\frac{1}{4\times 12}=0{\cdot}0218\ \text{sq ft}$$

Area of supply pipe $\quad a_s=\dfrac{\pi}{4}\times\left(\dfrac{3}{12}\right)^2=0{\cdot}0491$ sq ft

$$\therefore\quad v=6{\cdot}43\times\frac{0{\cdot}0218}{0{\cdot}0491}=2{\cdot}85\ \text{ft/sec}$$

Kinetic energy of water column in the supply pipe at the time of closing the waste valve $=\frac{1}{2}\ m\ .\ v^2$

$$=\frac{(w\ .\ a\ .\ L)\ .\ v^2}{2g}=\frac{62{\cdot}4\times 0\ 0491\times 30\times 2{\cdot}85^2}{64{\cdot}4}=11{\cdot}6\ \text{lb ft}$$

∴ Neglecting slip, weight of water that could be lifted against a delivery head of 20 ft

$$=\frac{11.6}{20}=0.58 \text{ lb}$$

This amount of water could be lifted for one beat of waste valve.

∴ Total amount of water lifted, or discharge

$$=110\times 0.58 \text{ lb/min}$$

or $\frac{110\times 0.58}{10}=$ **6·38 gpm** *Answer*

UNSOLVED PROBLEMS

13.1 What is a propeller pump ? Describe briefly such a pump with the help of a sketch.

13.2 Describe with the help of a sketch a mixed flow pump. What is the range of specific speed for such a pump ?

13.3 Can an axial flow pump be classified as a centrifugal pump ?

13.4 Where would you employ a Kaplan pump ? What is the approximate specific speed range for such a pump ?

13.5 Explain the causes of cavitation in axial flow pumps. With the aid of a sketch and suitable symbols indicate the factors which influence cavitation and methods of reducing its effects.

(AMI Mech E—April 1954)

13.6 If a propeller pump delivers a discharge Q against a head H when running at a speed N, deduce an expression for the speed of a geometrically-similar pump of such a size that, when working against unit head, it will transmit unit power to the water flowing through it. Show that this value is proportional to the specific speed of the pump.

Explain why it is that, in general, the specific speed of a propeller pump is greater than that of a centrifugal pump. Then show that for stipulated conditions of head and discharge, a propeller pump is likely to be smaller than an equivalent centrifugal pump.

(AMI Mech E—Oct 1959)

13.7 What kind of pump will be preferred for irrigation purposes ?

13.8 What do you know of the following ?—

a) Hydraulic ram, *b*) Air lift pump,
c) Jet pump, *d*) Pulsometer pump.

(AMIE—Nov 1953)

13.9 Enumerate the advantages and disadvantages of an air lift pump as compared with the usual reciprocating or centrifugal pump. Give a neat sketch of a good type of foot piece for an air-lift pump.

Derive an expression for the theoretical volume of air required to pump one cu ft of water against a total head of h ft by means of air lift pump when ratio of isothermal expansion of air during this operation is r.

13.10 Describe a hydraulic ram with the help of a line diagram.
(AMIE—May 1955)

What are the sources of loss in such a machine? Sketch curves to show how the efficiency depends upon the beat of the waste valve and the lift.

In what circumstances would you make use of such a machine and why?

13.11 Four single stage axial flow pumps, manufactured by SULZER are working at RAINI Station (Ramganga River UP) each of them raises 18,700 gallons of water per min. against a head of $15\frac{1}{2}$ ft and is to be driven by an AC motor running at 730 rpm. Assuming tip speed $2{\cdot}1\sqrt{2gH}$; boss dia half the tip dia. Overall efficiency 80%, mechanical losses 3%, uniform axial flow velocity and no whirl or shock at inlet, find :—

a) The diameter of the propeller,
b) The mean whirl velocity after leaving the propeller.

Draw the theoretical velocity diagrams at inlet and exit from the propeller blades at maximum radius.

Find also the h.p. required to drive the pump and the rating of A.C. driving motor. [(*a*) $20\frac{1}{5}$ in. (*b*) 7·75 ft/sec; 109·2 hp; 120 hp]
(Jadavpur University 1957—Final)

13.12 In an axial-flow or propeller pump, the rotor has an outer diameter of 2·42 ft and an inner diameter of 1·30 ft; it revolves at 500 rpm. At the mean blade radius, the inlet blade angle is 12 deg and the outlet blade angle is 15 deg. Sketch the corresponding velocity diagrams at inlet and outlet, and estimate from them (*i*) the head the pump will generate, (*ii*) the discharge or rate of flow in gal/min, (*iii*) the shaft h.p. input required to drive the pump, (*iv*) the specific speed of the pump.

Assume a manometric or hydraulic efficiency of 88% and a gross or overall efficiency of 81%.

(63 ft; 8,860 gpm; 209 HP; 2,100. *Note*: Specific speed of 2,100 for a propeller pump is very low.) [*AMI Mech E (Lond)—Oct 1958*]

13.13 An air lift pumping plant is required to discharge 1,500 gpm against a static head of 220 ft. The rising main is 346 ft long. The overall efficiency of the plant $\left(i.e.\ \dfrac{\text{WHP}}{\text{Compressor BHP}}\right)$ is 0·32. Estimate the volume of air per minute (at atmospheric pressure) that must be handled by the air compressor of 0·288 BHP is required to compress 1 cu ft of free air per minute to 150 lb per sq in. pressure. *(Rajasthan University—1956)*

13.14 A ram is used to lift 50 gpm through 500 ft of $2\frac{1}{2}$ in. pipe into a reservoir at a height of 80 ft above the ram, while drawing in 900 gallons of water per minute from a supply tank 10 ft above. Determine the efficiency. $f=0{\cdot}014$. (62·28%) *(Madras University—1954)*

13.15 Describe the principle of operation of the hydralic ram, and sketch graphs to show cyclical changes in pressure and velocity in the system. Discuss the limitations of this device, in relation to head and discharge.

A particular installation has a supply or drive pipe 23 ft long the supply head (above waste valve) is 8·7 ft, the delivery head (above waste valve) is 29 ft, and the waste valve closes when the velocity in the supply pipe is 8 ft/sec. If the discharge of useful water lifted is to be 18 gal/min, estimate : (*i*) the diameter of the supply pipe, (*ii*) the quantity of waste water per minute, (*iii*) the number of beats per minute. A simplified solution will serve, neglecting pipe friction, etc.

[*AMI Mech E* (*Lond*)—*Oct 1958*]

13.16 A ram working under a head of 15 ft (or 4·575 *m*) pumps 30 gallons of water per minute (or 2·27 lit/sec) into a steel tank placed 50 ft (or 15 25 *m*) above the ram. The amount of water used is 180 gpm (or 13·6 lit/sec) Taking a loss of head in the delivery pipe as 4·5 ft (or 1·37 *m*) of water, calculate the Rankine's efficiency of the ram.
(52·6%) (*Osmania University—1952* ; converted also to metric units)

13.17 *a*) Explain with the help of a diagrammatic sketch how a hydraulic ram works and also show how the pressure and velocity undergo changes.

b) The following particulars relate to a hydraulic ram installations—

Supply head	...	3 *m*
Drive pipe length	...	10 *m*
Drive pipe diameter	...	6 *cm*
Area past waste valve	...	36 cm^2
Head to close waste valve	...	0·5 *m*
Delivery head above waste valve	...	9 *m*

Compute the quantities of useful and waste water in litres per minute and find the Rankine efficiency of the hydram.

(*Bombay University—1957*)

SECTION IV

Testing of Hydraulic Machines

CHAPTER 14

HYDRAULIC MEASUREMENTS

14.1 Hydraulic Measurements—The term Hydraulic Measurements refers to the measurement of the following quantities required for the testing of water turbines and pumps :

i) Quantity of water flowing per sec or Discharge—Q (cusecs or m^3/sec)

ii) Head—H (ft or m).

In order to measure the discharge and head, the level of free surface of water has to be determined. This Chapter is devoted to the study of various methods of measuring discharge, head, water level and speed of the machine, the latter being used to find the brake horsepower. From the discharge and head obtained by the methods explained, available hydraulic-power can be calculated. The brake horsepower is determined by any of the methods described in Chapter 15. The efficiency of a hydraulic machine is finally calculated from the available power and brake horsepower.

A. Water Measurements

14.2 Introduction—The term Water Measurement refers to the measurement of quantity of flowing water, generally known as discharge. All the methods except the first two, employed for water measurement, give the velocity of flowing water and the discharge is determined by multiplying the cross-sectional area of the stream by the velocity obtained. The various methods of quantitative water measurements are enumerated hereunder :

i) Direct Weight Measurement,
ii) Calibrated Tank,
iii) Travelling Screen,
iv) *a*) Venturimeter *b*) Venturi Flume,
v) Pitot Tube,
vi) Orifice, Nozzle or Diaphragm,
vii) Pipe Bend,
viii) Rotameter,
ix) Quantity Meter,
x) Turbine Nozzle,
xi) Floats,
xii) Weirs,
xiii) Current Meters,
xiv) Allen Method or Salt Velocity Method,
xv) Titration or Chemical Method,
xvi) Gibson Method,
xvii) Thermometric and Thermodynamic Method,
xviii) Ultrasonic Flowmeter.

The methods (*i*) and (*ii*) are 'Direct Methods' *i.e.* discharge can be directly measured by them. All other methods are known as 'Indirect Methods' which give the velocity of water.

The methods (*i*) to (*viii*) of which only the first three are accurate, are employed in the laboratory. Method (*ix*) is only used for household purposes. The methods (*x*) to (*xv*) find wide application in practice, and (*xvi*) to (*xviii*) have been recently tried.

Water measurement in *closed conduits*—Methods (*i*), (*ii*), (*iva*), (*v*) to (*x*) and (*xvi*) to (*xviii*). Water measurement in *open channels or streams*—Methods (*iii*), (*ivb*), (*xi*) to (*xv*).

It may be noted that the methods are generally named after the principle, process or a part or whole of the apparatus employed. Sometimes they are known after the names of the inventors.

14.3 Direct Weight Measurement—Water discharging from the turbine or pump is received into a tank. For a specified period of time, not less than 100 seconds, the water is diverted to another special measuring tank by turning a swivelling elbow attached to the outlet of the discharge-pipe (*See* Fig 14.1). The quantity of water flowing in this time is determined by weighing the tank with and without the contents. Thus the rate of flow is known.

This is the most accurate method, chief advantage being that temperature has no influence on the accuracy.

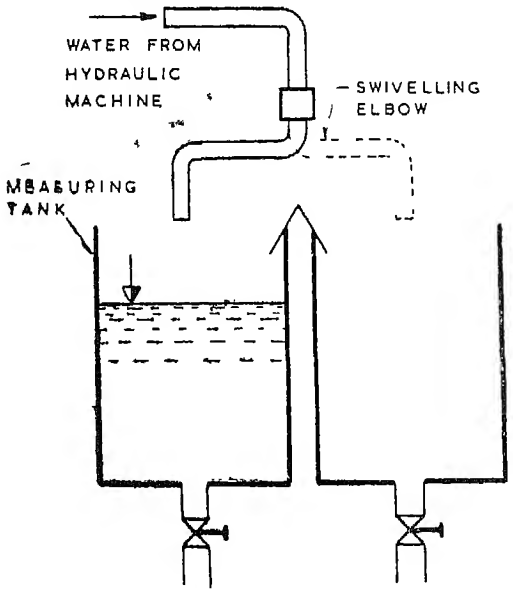


Fig 14 1 Measurement of Continuous Flow of Water by Weighing

14.4 Calibrated Tank—Discharge from the Hydraulic Machine is led into a tank which has been previously calibrated *i.e.*, the weight or volume of the contents is known as a function of the height of liquid inside the tank. The quantity of water flow in a specified time is obtained by measuring the difference in the height of water level produced in the same period.

Time is measured with a stop-watch and height with a piezometer tube, pointer gauge or a hook gauge (*See* Fig 14 2) Floats may be employed for large tanks.

If the constant cross-sectional areas of tank and attached piezometer tube are A and a respectively and the level changes from h_1 to h_2 in time t then,

$$Q=\frac{v}{t}=\frac{(A+a)(h_2-h_1)}{t}$$

$$=\frac{(A+a)\cdot h}{t} \qquad \ldots(14.1)$$

This is the next most accurate method of water measurement provided the tanks are carefully calibrated by actual weighing. It is important that there should be

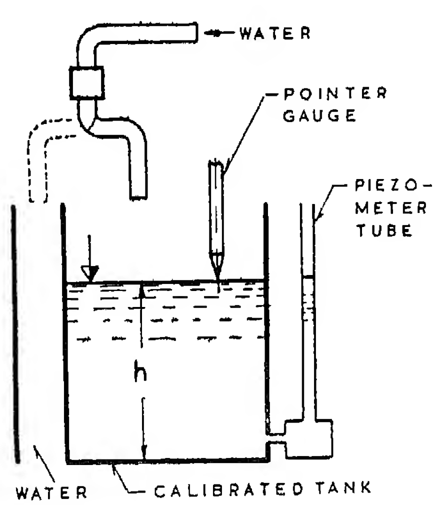


Fig 14.2 Water Measurement by Calibrated Tank fitted with Piezometer Tube

no volumetric losses or additions during the process of measurements. Cross-sectional area should be exactly maintained by preventing deformation of tank sides. Water level should not fluctuate.

14.5 Travelling Screen—In principle, this method consists of the measurement of the mean velocity of water flowing across a given cross-sectional area. Practically, the apparatus consists of a metal screen mounted on rails on either side of a long straight uniform canal through which the discharged water is allowed to run (See Fig 14.3).

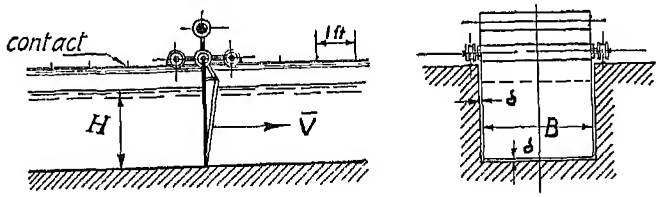


Fig 14.3 Travelling Screen

When the screen is lowered into water, it is carried along with the moving water at a constant speed equal to the mean velocity of flow if the flow is steady. This velocity can be measured either with an electric chronograph or by measuring the actual distance travelled in a certain time period.

The clearance between screen and sides of canal is, generally, very small : $\delta = \frac{5}{64}''$ to $\frac{3}{16}''$ (or 2 to 5 *mm*). The rate of discharge is given by

$$Q = Q_s + \Delta Q$$

where $Q_s = Q$ screen $= B \,.\, H \,.\, \bar{v}$ (See Fig 14.3)

and $\Delta\ Q$ = Water escaping through clearance.

The obvious disadvantage is the high cost of installation of screen and construction of a suitable canal not less than 50 ft (or about 20 *m*) long. But the method is accurate, quick and easy.

14.6 *a*) **Venturimeter** is employed to determine the rate of flow in pipes of practically all diameters. It was invented by Clemens

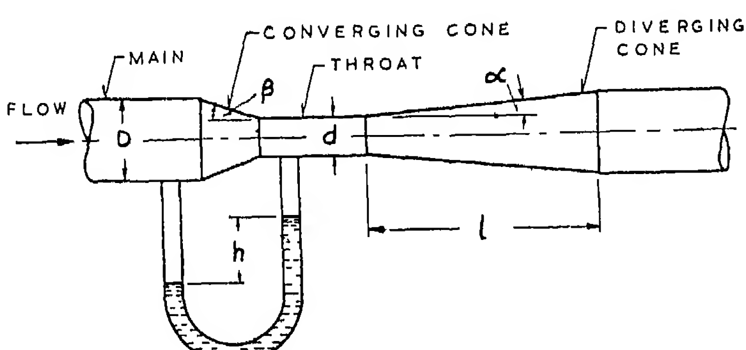


Fig 14.4 Horizontal Venturimeter

Herschel in 1887 and was named in the honour of G. B. Venturi, an Italian philosopher on account of his achievements in hydraulic research in connection with this meter. In its usual form the Venturimeter consists

of two truncated conical pipes connected together by a throat as shown in Fig 14 4 The meter is generally inserted in the pipe line in its horizontal position. The diameter of the larger ends of the frustra being equal to that of the pipe. A U-shaped mercury manometer is used to measure the difference of pressure between the main and the throat

Applying Bernoulli's Theorem between main and throat (See Fig 14.4),

$$\frac{p_1-p_2}{w}+(z_1-z_2)=\frac{v_2^2-v_1^2}{2g}$$

∴ Measured difference of pressure between main and throat,

$$\frac{p_1-p_2}{w}=h=\frac{v_1^2}{2g}\left\{\left(\frac{A}{a}\right)^2 \quad 1\right\}=\frac{v_1^2}{2g}\cdot\left(\frac{A^2-a^2}{a^2}\right)$$

where A and a are the cross-sectional areas of main and throat respectively.

or $$v_1=\sqrt{\frac{2gh}{\frac{A^2-a^2}{a^2}}}$$

Theoretically,

$$Q_{th}=A\,.\,v_1=\frac{A\,.\,a}{\sqrt{A^2-a^2}}\cdot\sqrt{2gh}$$

and accounting for losses, the actual discharge

$$Q=C\,.\,\frac{A\,.\,a}{\sqrt{A^2-a^2}}\cdot\sqrt{2gh} \qquad \text{...(14.2)}$$

where C=co-efficient of discharge or generally known as meter's co-efficient.

The co-efficient C varies with the size of Venturitube and the Reynolds' number. The following table gives the approximate values of co-efficient C, when the pressure difference between main and throat or the Reynolds' number at the throat is known.

TABLE 14.1

Values of Co-efficient C

h	ft of water	—	18	6	2
	m of water	—	6	2	0 66
R_e		10^4	10^5	10^6	10^7
C		0 95	0·975	0·985	0 99

In Fig 14.4, $\alpha=2·5°$ to $6°$, $\beta\approx10°$, $l\approx6\,D$

In order to avoid the loss of head resulting from vortices and eddy currents due to boundary layer formation, the angle of divergence α is

kept small. The ratio of contraction $\frac{D}{d}$ is also limited to about 2. The increase in throat velocity with the reduction in d causes a decrease in throat pressure. If it falls below 8 ft (or 2·44m) of water absolute, dissolved gases separate and vapour is formed, thereby, causing a discontinuity of flow.

The Venturimeter can be used for a vertical or inclined pipe. Eqn 14.2 will hold good in such cases too as the energy equation will then be—

$$\frac{v_2^2-v_1^2}{2g}=\frac{p_1-p_2}{w}+(z_1-z_2)=h$$

where h=differential manometer reading.

b) **Venturiflume**—The venturi principle can be used to determine the flow in an open channel. The width of the channel is narrowed down from B to b (Fig 14.5). H and h indicate the heights of the flowing water at the normal and restricted sections respectively. If V and v be the velocities of water at the above two sections, then the discharge through the channel is given by

$$Q=(B\,.\,H)\,.\,V=(b\,.\,h)\,.\,v$$

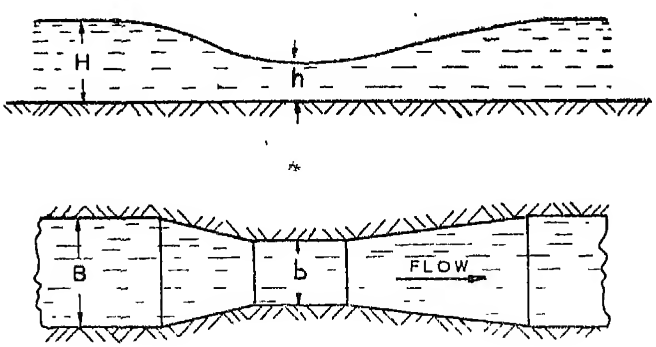


Fig 14.5 Venturiflume

Applying Bernoulli's Theorem between the main and the restricted section.

$$H+\frac{V^2}{2g}=h+\frac{v^2}{2g}$$

$\therefore$ the pressure difference,

$$(H-h)=\frac{v^2-V^2}{2g}=\frac{v^2}{2g}\left\{\left(\frac{A}{a}\right)^2-1\right\}$$

Hence $$Q=A\,.\,V=\frac{A\,.\,a}{\sqrt{A^2-a^2}}\,.\,\sqrt{2g\,(H-h)} \qquad \ldots(14.3)$$

A particular precaution is necessary. Width of the throat should be such that depth of water is always more than critical so that a standing wave may not be formed.

14.7 Pitot Tube—The tube invented by a French Scientist Henry Pitot in 1732, is L-shaped glass tube as shown in Fig 14.6 (*a*). When

the tube is immersed in flowing water, the rise of water in the tube will be proportional to the square of velocity of stream. This can be proved by applying Bernoulli's Theorem between point 1 and point 2. Point 1 is to be taken on the centre line of the nose of the tube and at some distance from it. The water at this place has got a velocity of v. Point 2 lies immediately at the nose of the tube where the water is assumed to be at rest. This point is known as stagnation point.

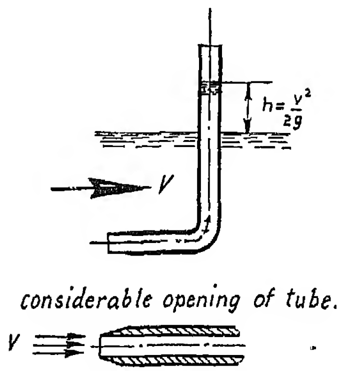


Fig 14.6 (a) Simple Pitot Tube

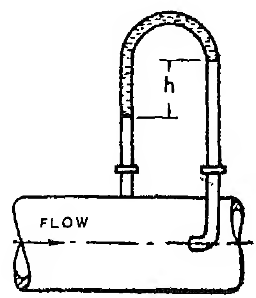


Fig 14.6 (b) Pitot Tube with one end normal and the other end parallel to the flow

$$\therefore \qquad \frac{v^2}{2g}+\frac{p_1}{w}+z_1=0+\frac{p_2}{w}+z_1$$

or $$\frac{v^2}{2g}=\frac{p_2-p_1}{w}=h$$

or $$v=\sqrt{2gh} \qquad \ldots(14.4)$$

The difference of pressure between points 1 and 2 is therefore,

$$\frac{p_2-p_1}{w}=\frac{v^2}{2g}$$

or $$p_2-p_1=\frac{w\,.\,v^2}{2g}=\frac{1}{2}\,\rho\,.\,v^2 \qquad \ldots(14.5)$$

If the Pitot tube is used to measure the velocity of stream, there must be some device to separate the two pressures p_1 and p_2. This is difficult to find out accurately by a single tube shown in Fig 14.6(*a*).

The method employed to have accurate result is to connect another tube, the opening of which is normal to the direction of the velocity, shown in Fig 14.6 (*b*). The water will be drawn in the second tube by means of suction. If, now, the two tubes are connected by an inverted U-tube manometer, the difference of heights h, used in Equation 14.6 will yield accurate results to determine v.

In practice $\quad v=\phi\sqrt{2\,gh} \qquad \ldots(14.6)$

where $\quad \phi=$Pitot tube co-efficient

The value of ϕ is about 0·95 to 0·98. However by using the method shown in Fig 14.6 (*b*) the value of ϕ becomes nearer to unity.

The value of ϕ will differ according to the shape of the nose of L-tube and the position of the second tube opening with reference to the nose and the stem. **Prandtl tube** which is a modification of Pitot tube has unity co-efficient ϕ. This is accomplished by balancing the reduced pressure caused by the flow past the nose, to the increased pressure due to the presence of stem. Fig 14.7 (*a*) shows such a design. It consists of an inner tube (diameter about $\frac{1}{8}$ inch or about 3 mm) which leads to

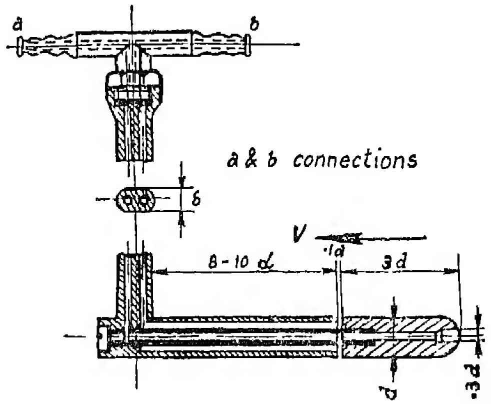


Fig 14 7 (a) Pitot Tube of Prandtl Design

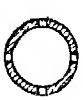


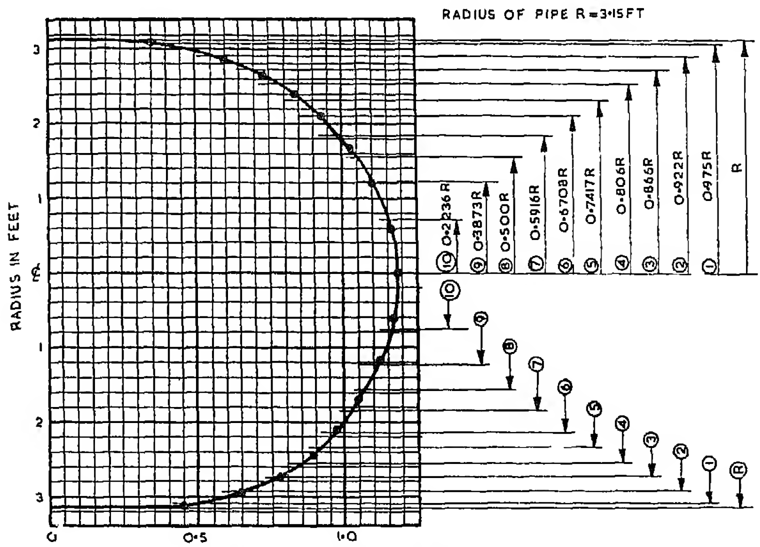
Fig 14.7 (b) Section of Prandtl Tube at a Distance of 3d from the Nose.

connection *a* and an outer casing having openings normal to the direction of flow. These openings are provided at a distance of 3*d* from the nose and lead to connection *b*. The section taken at a place of opening is shown in Fig 14.7 (*b*). The connections *a* and *b* are meant for fixing a U-tube manometer by a rubber tubing.

The Pitot tube can be used in an open channel as well as in a closed pipe. As the velocity in the closed pipe is not the same at all points of cross-section, an average of a few readings at different points is determined. For rough and ready flow approximations, a single Pitot tube reading can be taken in the centre of the pipe which would be the maximum velocity of flow. The mean velocity of flow will then be equal to approximately 0 84 of the maximum velocity. But for very accurate work and where flow is more turbulent, traversing of pipe is done which is explained below.

Pitot Tube Traversing in Pipes—Pitot tube traversing is used to ascertain the pipe factor $\left(\frac{v_m}{v_c}\right)$ of flow in pipes, where v_m is the mean velocity along a cross-section of the pipe and v_c is the velocity at the centre. The pitot tube is inserted into the pipe and readings of the impact heads taken at various points to be ascertained as explained below. A graph is drawn between the impact bend and the distance of the pitot-tube mouth from the wall of the pipe, a smooth curve being drawn through the test points as shown in Fig 14.8. A circle is then drawn, the diameter of which corresponds to that of the pipe and this circle is divided into a convenient number of rings, each of equal area, and from

the test curve the impact pressure is obtained at the centre of area of each annular ring. From these values the corresponding square roots of the impact pressures are found and their average taken which is proportional to v_m. Denoting the average value of the square roots of the



H (in ft), Deflection of Indicating Liquid (S G.=1·595)

Fig 14.8 Pitot Tube (or Pitot Meter) Traversing, from the British Pitotmeter Co. Ltd.

impact heads by $\sqrt{H_m}$ and the square root of the impact head at the centre by $\sqrt{H_c}$,

$$\frac{v_m}{v_c}=\sqrt{\frac{H_m}{H_c}}$$

or

$$v_m=\sqrt{\frac{H_m}{H_c}}\cdot v_c$$

$$=\sqrt{\frac{H_m}{H_c}}\cdot\phi\cdot\sqrt{2g\cdot H(x-1)} \qquad \ldots(14.7)$$

where ϕ=Instrument co-efficient

H=Difference between the levels of liquid in the limbs of the manometer

and x=Specific gravity of the indicating liquid.

The discharge can be measured by multiplying v_m by the area of cross-section.

In practice it is not necessary to draw a circle with concentric rings and then find the centres of areas of these rings, since the required points can be obtained mathematically. The radii of the circles giving these points are given by

$$r_1 = r\sqrt{\frac{2n-1}{2n}}, \qquad r_2 = r\sqrt{\frac{2n-3}{2n}}, \text{ and so on} \quad ...(14.8)$$

where r is the mean pipe radius and n is the number of rings.

The graph in Fig 14.8 has been plotted for a pipe 3·15 ft (or 0 96 m) in radius. The pipe factor is computed as below :

Point Number	1	2	3	4	5	6	7	8	9	10
H	0·405	0·605	0·700	0·780	0·850	0·915	0·975	1·040	1·090	1·155
$\sqrt{H}$	0 636	0·778	0·837	0·883	0·922	0·957	0·987	1·020	1·041	1·074
	10	9	8	7	6	5	4	3	2	1
	1·155	1·110	1·065	1·010	0·960	0 905	0·845	0·770	0·660	0·460
	1 074	1·054	1·032	1·005	0·980	0·951	0·919	0·877	0·812	0·678

$$\text{Mean} \quad \sqrt{H} = \frac{18·520}{20} = 0·926$$

$$\text{Centre} \quad \sqrt{H} = 1·0816$$

$$\therefore \quad \frac{\sqrt{H_m}}{\sqrt{H_c}} = \frac{v_m}{v_c} = \frac{0·926}{1·0816} = 0·856$$

Now $\qquad v = \phi\sqrt{2g \,.\, H(x-1)}$

where $\qquad \phi$ = Instrument co-efficient (0 98)

H = Deflection of indicating liquid in ft (1·17 at centre)

x = Specific gravity of indicating liquid (1·595)

$$\therefore \quad v_c = 0\ 98\sqrt{2 \times 32·2 \times 1·17\ (1·595-1)}$$
$$= 6·567 \text{ ft/sec}$$

But $\qquad \dfrac{v_m}{v_c} = 0·856$

$$\therefore \quad v_m = 6·567 \times 0·856$$
$$= 5·62 \text{ ft/sec}$$

14.8 Orifice, Nozzle or Diaphragm—An orifice is a sharp-edged hole in a thin plate. The hole is of smaller diameter (max 80%) than that of pipe line where it would be inserted between two flanges. Care should be taken that the orifice is fitted concentrically to the pipe.

A number of orifices having different hole diameters can be made and be used to measure the discharge of different rates. The approach to the orifice must be straight for a distance of atleast five times the pipe diameter. Piezometer holes of about $\frac{3}{16}$ inch. (or 5 *mm*) diameter, as shown in Fig 14.9 are located, one at a distance equal to the pipe diameter upstream from orifice and the other approximately half the pipe diameter down-stream.

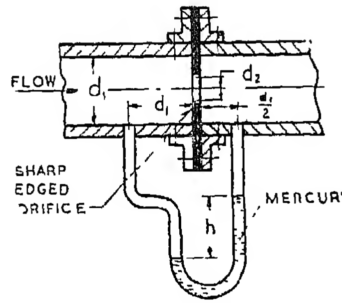


Fig 14.9 Orifice

Applying Bernoulli's Theorem between points 1 and 2,

$$\frac{p_1}{w}+\frac{v_1^2}{2g}=\frac{p_2}{w}+\frac{v_2^2}{2g}$$

$$\therefore\ h=\frac{p_1}{w}-\frac{p_2}{w}=\frac{v_2^2-v_1^2}{2g}$$

But from equation of continuity,

$$Q=a_1\,.\,v_1=a_2\,.\,v_2,\text{ etc}\qquad\therefore\quad v_1=\frac{a_2\,.\,v_2}{a_1}$$

$$\therefore\ h=\left(v_2^2-v_2^2\,.\,\frac{a_2^2}{a_1^2}\right).\frac{1}{2g}=\frac{v_2^2}{2g}\left(1-\frac{a_2^2}{a_1^2}\right)=\frac{v_2^2}{2g}\left\{1-\left(\frac{d_2}{d_1}\right)^4\right\}$$

$$\therefore\ v_2=\sqrt{\frac{2gh}{\left\{1-\left(\frac{d_2}{d_1}\right)^4\right\}}}$$

and $$Q=a_2\,.\,v_2=a_2\,.\sqrt{\frac{2gh}{\left\{1-\left(\frac{d_2}{d_1}\right)^4\right\}}}$$

A co-efficient C may be introduced to account for the loss of head due to the orifice then,

$$Q=C\,.\,a_2\,.\sqrt{\frac{2gh}{\left\{1-\left(\frac{d_2}{d_1}\right)^4\right\}}}\qquad\ldots(14.9)$$

C is a function of Reynolds' number and ratio $\frac{d_2}{d_1}$. The following table gives the values of co-efficient C and the expression $\sqrt{1-\left(\frac{d_2}{d_1}\right)^4}$ for different values of ratio $\frac{d_2}{d_1}$.

TABLE 14.2

$\frac{d_2}{d_1}$	C	$\sqrt{1-\left(\frac{d_2}{d_1}\right)^4}$
0·25	0 604	0·998
0·30	0·605	0·995
0·35	0 606	0 993
0·40	0·606	0·987
0·50	0·607	0·967
0 60	0·608	0·933
0 70	0 611	0 872
0·80	0 643	0 767
0 90	0·710	0 586

Normally, $C \approx 0\ 6$ and ratio $\frac{d_2}{d_1} = 0{\cdot}5$

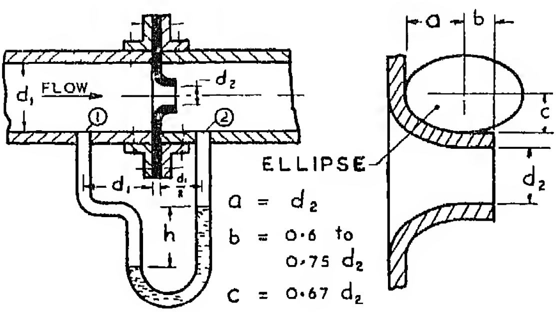


Fig 14.10 Nozzle

A nozzle is a special type of orifice with rounded edges on the upstream side as shown in Fig 14.10. The curvature of the nozzle is a quadrant of an ellipse whose construction is shown in Fig 14.10.

Applying Bernoulli's Theorem between points 1 and 2 as in case of an orifice, the velocity at point 2 is given by

$$v_2 = C_v \cdot \sqrt{\frac{2gh}{1-\left(\frac{d_2}{d_1}\right)^4}} \qquad \ldots(14.10)$$

where C_v is the co-efficient of velocity.

and $\qquad Q = \frac{\pi}{4} \cdot d_2^2 \cdot v_2$

The following table gives some approximate values of C_v obtained by John *R.* Freeman with cold water in the year 1890.

TABLE 14.3

d_2 inches	d_2 *m m*	$\frac{d_2}{d_1}$	C_v
1·75	44·4	0·467	0·995
2 00	50 8	0·533	0·997
2·50	63 5	0·667	0·994

Problem 14.1 In a flowmeter water is flowing at a temperature of 68°*F*. The manometer fluid is bromoform, having a specific weight of 168 0 lb/cu ft. The diameter of the pipe is 6·085 inches and the diameter of the orifice is 3·625 inches. If the deflection of the manometer fluid is 30 inches, what is the rate of flow ?

Solution

Ratio $\frac{d_2}{d_1}=\frac{3\cdot625}{6\ 085}=0\ 598$

Difference of pressure, $\frac{\Delta p}{w}=h=\frac{30}{12}\times\left(\frac{168-62\ 4}{62\cdot4}\right)=4\cdot24$ ft of water

At 68°*F*, density of water $=1\ 94$ slugs/ft^3

and co-efficient of viscosity$=2\cdot1\times10^{-5}$ lb sec/sq ft.

$$\therefore\ R_e=\frac{\rho\cdot d}{\mu}\cdot\sqrt{2gh}=\frac{1\cdot94\times3\ 625\times10^5}{2\cdot1\times12}\times\sqrt{64\cdot4}\times\sqrt{4\ 24}$$
$$=640{,}000$$

$C=0\ 653$ for above R_e

$$\therefore\ Q=C\cdot\frac{\pi}{4}\times d_2^2\cdot\sqrt{2gh}=0\cdot653\times1\cdot182$$
$$=\mathbf{0\ 772\ cu\ ft/sec}\quad \textit{Answer}$$

14.9 Pipe Bend—An ordinary pipe bend can be used to measure the flow of liquid. It can be shown that the net effect of the variation of velocity and centrifugal action is to create a pressure difference between the outside and the inside of the bend. If this pressure differenee h is experimentally measured (Fig 14 11), it can be found that the rate of flow through the pipe,

$$Q=f\,(h)$$

Thus the value of Q for any h may be obtained from a calibration curve.

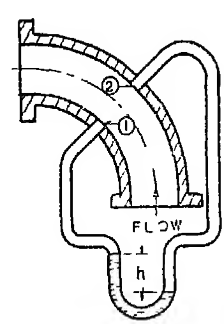


Fig 14.11 Pipe Bend

Further, it can be deduced from theoretical considerations that,

$$Q=C_d\cdot\sqrt{2gh}\cdot\frac{R^2-r^2}{\sqrt{R\cdot r}}\ \pi\cdot\left[\ R-\sqrt{R^2-r^2}\ \right]\quad ...(14.11)$$

where, C_d=co-efficient of discharge to account for loss of head in bend.

h=measured difference of pressure (Fig 14.11),

R=mean radius of bend,

r=inside radius of pipe.

This equation is derived, for horizontal bend, as follows—

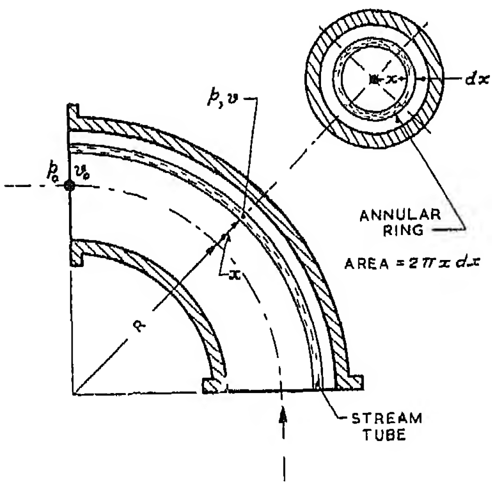


Fig 14.12 Determination of Discharge Formula for Pipe Horizontal Bend

Consider a small stream tube as shown in Fig 14.12.

$$\frac{p_0}{w}+\frac{v_0^2}{2g}=\frac{p}{w}+\frac{v^2}{2g}$$

$$\therefore \quad \frac{dp}{w}=-\frac{2v\,.\,dv}{2g}=-\frac{v\,.\,dv}{g} \quad \ldots(1)$$

Consider the equilibrium of a small element of stream tube

Area × Pressure = Centrifugal force

$$\therefore \quad dp=\frac{w}{g}\left(\frac{v^2}{R+x}\right)dx \quad \ldots(2)$$

From (1) and (2)

$$\frac{dp}{w}=-\frac{v\,.\,dv}{g}=\frac{v^2\,.\,dx}{g\,(R+x)}$$

or $$-\frac{dv}{v}=\frac{dx}{R+x}$$

$\therefore$ $\log v=\log\,(R+x)+\text{const}$

$$\therefore \quad v(R+x)=\text{const} \qquad \text{or } v=\frac{C}{R+x} \quad \ldots(3)$$

$\therefore$ From (2) and (3)

$$g\,\frac{dp}{w}=\frac{C^2}{(R+x)^3}\,dx$$

or $$g\,.\,h=-\frac{C^2}{2}\left[\frac{1}{(R+x)^2}\right]_{-r}^{+r}=\frac{C^2}{2}\left[\frac{1}{(R-r)^2}-\frac{1}{(R+r)^2}\right]$$

$$=\frac{C^2}{2}\cdot\frac{4R\,r}{(R^2-r^2)^2}$$

$$\therefore \quad C=\sqrt{2gh}\;.\;\frac{R^2-r^2}{2\sqrt{R\,r}}$$

Now consider an annular ring (Fig 14.12)

$$\text{area}=2\pi\,x\,.\,dx$$

$$\text{Mean velocity}=\sqrt{\frac{C}{R+x}\times\frac{C}{R-x}}=\frac{C}{\sqrt{R^2-x^2}}$$

$$\therefore \quad dQ=\text{Vel}\times\text{Area}=\frac{C}{\sqrt{R^2-x^2}}\,2\pi x\,.\,dx$$

$$Q=C\,.\,2\pi\int_0^r\frac{x}{\sqrt{R^2-x^2}}\,.\,dx=C\,.\,2\pi\,.\left[-\sqrt{R^2-x^2}\right]_0^r$$

$$\therefore \qquad Q = C \cdot 2\pi \cdot \left\{ \sqrt{R^2} - \sqrt{R^2 - r^2} \right\}$$

$$= \sqrt{2gh} \cdot \frac{R^2 - r^2}{2\sqrt{R \cdot r}} \cdot 2\pi \cdot \left\{ R - \sqrt{R^2 - r^2} \right\}$$

Practically, $$Q = C_d \cdot \sqrt{2gh} \cdot \frac{R^2 - r^2}{\sqrt{R \cdot r}} \cdot \pi \cdot \left\{ R - \sqrt{R^2 - r^2} \right\}$$

14.10 Rotameter—A rotameter is a device to find the velocity flow in a pipe with the aid of a rotating free float. It is essentially an orifice meter with fixed pressure drop and variable orifice area.

Fluid is allowed to flow vertically upward through a transparent tapered tube placed vertically with the large end at the top (Fig 14.13). The float is freely suspended inside the tube. The maximum diameter of float is slightly less than the minimum bore of the tube. When there is no flow, float rests at the bottom. But when fluid has some velocity float rises upwards to make way for the fluid motion. Rise of float is a function of the velocity of flow and the tube can be graduated in gpm (or lit/sec).

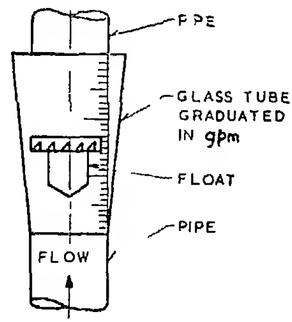


Fig 14.13 Rotameter

The float is provided with slantwise slots to enable it to occupy a stable position at the centre of the tube, by virtue of gyroscopic equilibrium. This prevents rubbing and sticking of the float to the tube.

From the velocity of flow, rate of flow can be easily determined.

14 11 Quantity Meters which directly measure the volume of liquid passing through them are usually employed for closed pipes.

Actual designs of the meters which are commercial gadgets, are patents of manufacturers. In general, these are of two types :

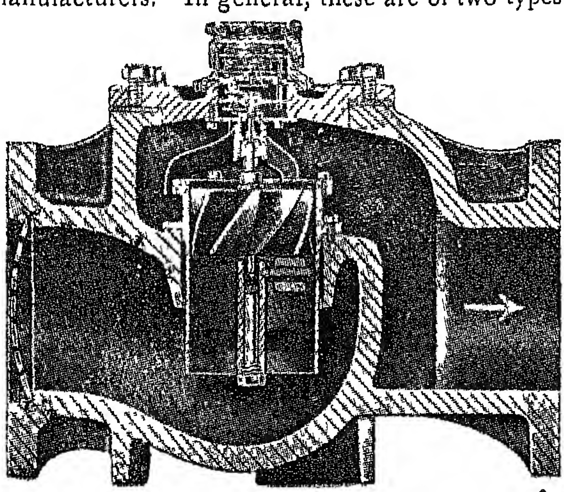


Fig 14.14 Kent's Watermeter 'Torent' Helical Rotor-Type

a) **Rotary Type**—Its action is like that of a propeller. When the water flows across the meter, propeller rotates operating a series of gears. Volume of liquid in gallons (or litres) can be directly read from it. The meters of this type are mostly employed in practice (See Fig 14.14).

b) **Positive Type**—It consists of a piston which fits closely against the walls of a measuring chamber fitted in the meter. The piston is pushed forward by the flowing water and its movement is connected to a series of gears which further connect to pointers on the dial graduated in gallons (or litres) and its fraction. At the end of each stroke the piston actuates the valve through the medium of rocker arm and so reverses the flow.

14.12 Turbine Nozzle (See Art 6.11)—Nozzles are employed in impulse turbines such as the Pelton turbine, to direct the jet to the buckets of the runner. If the nozzle is standard, the rate of flow Q can be directly obtained provided percentage opening of nozzle is known.

The curve showing the variation of Q with s, the spear travel is supplied with the nozzle.

14 13 Float—This is the oldest and the simplest method to find the surface velocity of water. Time required by a float to traverse a known distance is noted by the help of a stop watch. Velocity of float will then be taken to be the surface velocity of water. This method is best suited when the river is free from weeds.

Float

Surface Float | Sub-Surface Float | Rod Float

a) **Surface Float**—Any object which can float having its centre of gravity near the surface of water is employed for the purpose *e.g.* empty bottle, empty tin, cork washers, wooden circular disc etc. The float must travel in a straight line as far as possible. The surface float gives the surface velocity of water. Divide the width of river into a number of segments and determine the surface velocity of each segment. Calculate the mean surface velocity and then the mean velocity of flow by the following emperical relation

$$\bar{v} = \left(\frac{C}{C+25}\right) \cdot v_s \qquad \ldots(14.12)$$

where $\bar{v}$ = mean velocity

v_s = mean surface velocity

C = Chezy's constant

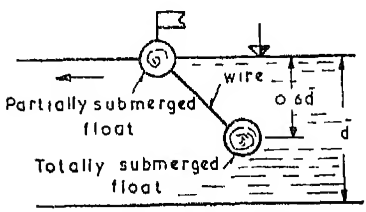


Fig 14.15 Sub-Surface Float

b) **Sub-Surface Floats** (Fig 14.15)—In case of heavy wind the surface float will not give accurate results. Sub-surface floats are, then, recommended. It consists of a double float—a totally submerged and a partially submerged, both of spherical form, connected with each other by a wire. The bottom float which is totally submerged, should be heavier than water and comparatively

large in size. Its purpose is to keep the wire *taut* without drawing down the upper float which is partially submerged. The bottom float should float at about 0 6 of the mean depth of river.

c) **Rod Float** (Fig 14 16)—It consists of a wooden pole or hollow cylinder made from tin sheet, weighted at bottom end, so that it floats in an upright position with its unweighted end emerging out of free water surface. The length of rod float should be such so that it reaches the bottom of stream without touching the sand, mud or weeds at the river bed.

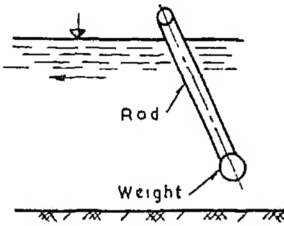


Fig 14 16 Rod Float

Rod float gives the mean vertical velocity of river section. The following empirical relation was deduced by Francis* for the determination of mean velocity of flow.

$$v=v_r\left(1{\cdot}012-0{\cdot}116\sqrt{\frac{d'}{d}}\right) \qquad \ldots(14.13)$$

where v=mean velocity of flow on the vertical plane in which the float moves

v_r=mean velocity of rod float

d=depth of water

d'=distance from the bottom of float to the bed of channel

and $d' \leq \frac{1}{4}d$

This method is suited for a discharge upto about 2,000 cfs (or about 60 m^3/sec). It is not suitable for large rivers or for a turbulent stream. For a stream having excessively turbulent flow, titration method (Art 14.17) is employed.

14.14 Weir—Water in open channels is measured through weirs. In design, the weir is a dam over which the water flows. The weirs may be classified in the following principal groups—

i) Rectangular Weir—

a) without end-contractions (*See* Fig 14.17),

and *b*) with end-contractions (*See* Fig 14.18),

ii) Triangular weir or V—Notch (*See* Fig 14.19),

iii) Trapezoidal Weir (*See* Fig 14.20) or any other special type such as compound weir.

Measurement of head to determine the velocity and the capacity of flow over the weir must be accurately made at some distance shown in Fig 14.17. The common method of head measurement on a weir is by a pointer or hook gauge. In practice pointer or hook gauge is lowered until it is in level with the top of the weir, generally, known as *weir crest*. This furnishes the point of zero reading. The pointer gauge is, then, raised until its pointer touches the surface of flow. This differential reading establishes the actual head for the rectangular weir.

As far as possible, a rectangular weir without end-contractions should be used in preference to any other type, because this has the

*Francis J.B. "Lowell Hydraulic Experiments" 4th Ed. D. Van Nostrand Co Inc, New York, 1883.

soundest experimental basis and gives most dependable results. Weir with end contractions is used where the walls of the channel are not built perpendicular to the flow. Triangular weir or V-notch should be employed for a small rate of flow. It is preferred over rectangular weir, because the discharge in this case is independent of wetted surface, as $Q \propto h$ and not $L \cdot h$ as in case of rectangular weir. The combination of weir with end contractions and V-notch results in a trapezoidal weir. The *Cippoletti* weir which is of trapezoidal form is mostly used in practice, because it has an advantage to compensate for the reduction due to end contractions in a rectangular weir of the same crest length. It has a slope of 1 : 4, horizontal to vertical.

The following points should be considered while installing a rectangular weir—

i) The measuring canal upstream of the weir should run straight for a distance at least twenty times the maximum head above the crest ;

ii) The measuring canal should have a constant cross-section with parallel vertical smooth walls and a smooth horizontal bottom.

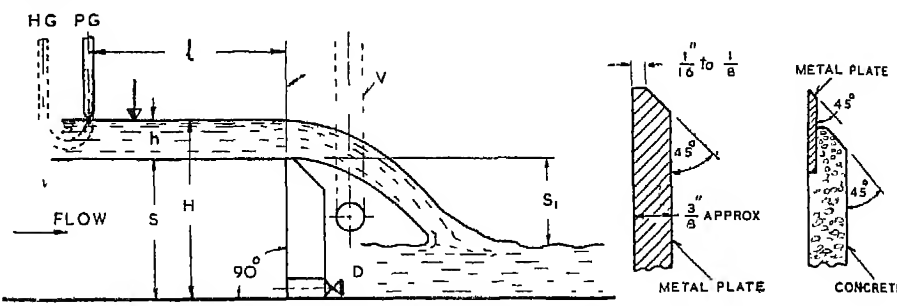


Fig 14 17 Rectangular Weir Without End Contractions

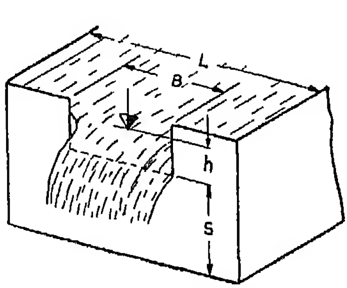


Fig 14 18 Rectangular Weir With End Contractions

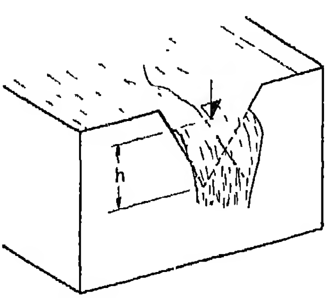


Fig 14 19 Triangular or V-Notch

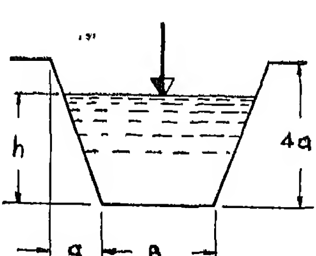


Fig 14.20 Trapezoidal Weir of Cippoletti Design

iii) The water flow upstream of the weir should be as calm and regular as possible. If necessary, stilling devices such as racks, baffles etc. should be provided.

iv) The actual crest of the weir should be made of rust proof metal, brass or stainless steel and should have an accurately finished sharp edge (See Fig 14.17). Edge should be bevelled to 45° and must be $\frac{1}{16}$ inch to $\frac{1}{8}$ inch (or 1 to 3 mm) thick.

v) The weir should be placed perpendicular to the canal walls and bottom. Its upstream face should be absolutely smooth and even. A drain opening at the foot of the weir is recommended.

vi) To have perfect discharge, the weir should be so arranged that even under the highest head above the crest, a perfect overflow is obtained *i.e.*, the height S_1 of the crest above the downstream level should be sufficient to ensure that the nape is completely free on the underside, with atmospheric pressure prevailing there.

vii) In order that the pressure under the nape is equal to that above it *i.e.*, atmospheric pressure, ventilation holes are provided on the underside of nape.

Cross-sectional area of ventilation holes is equal to about 0·5 percent of $(B \cdot S_1)$. Generally two 1 in. (or about 25 mm) dia pipes are sufficient for discharges upto 3 cusecs (or 85 lit/sec).

viii) Canal on the downstream side of weir, like the upstream canal, should have a uniform width and cross-section.

ix) For a rectangular weir with end-contraction (See Fig 14.18)

$$0{\cdot}3 \leq \frac{b}{B} \leq 0{\cdot}8$$

x) It should be calibrated with other accurate methods.

Practical Data :

With reference to Fig 14.17,

h=height above crest, generally, h=1 to 32 in. (or 25 to 800mm)

s=height of weir or weir board $\geqq$ 1 ft (or 0·3 m)

$H=h+s$=depth of water on upstream side of weir,

S_1=height of drop on downstream side of weir,

$$1 \text{ ft (or } 0{\cdot}3 \text{ m)} \leq S_1 \leq \frac{h_{max}}{2}$$

l=distance of pointer gauge (*PG*) or hook gauge (*HG*) from the weir edge,

$$6\, h_{max} \geqq l \geqq 4\, h_{max}$$

PG stands for pointer gauge

HG stands for hook gauge

V stands for Ventilation hole or duct

D stands for drain opening.

Calculation of discharge—

If Q =Discharge in cusecs or m³/sec

B = Width of weir crest in ft or m

h = height of water level above weir crest in ft or m

v_0 = velocity of approach in ft/sec or m/sec

g = gravitational acceleration in ft/sec^2 or m/sec^2

C_d = co-efficient of discharge = 0·6

then,

The following formulae are employed in practice to find the discharge over different types of weirs. The velocity of approach is neglected in these formulae. In case the velocity of approach is taken into account, the value of $h^{\frac{3}{2}}$ is replaced by

$$\left\{\left(h+\frac{v_0^2}{2g}\right)^{\frac{3}{2}}-\left(\frac{v_0^2}{2g}\right)^{\frac{3}{2}}\right\}$$

a) Rectangular weir without end-contractions—

$$Q=\tfrac{2}{3}\,.\,C_d\,.\,\sqrt{2g}\,.\,B\,.\,h^{\frac{3}{2}} \qquad \ldots(14.14)$$

b) Rectangular weir with end-contractions (Francis Formula)—

i) *Feet Units*—

$$Q=3{\cdot}33\,.\,(B-0\,1\,n\,.\,h)\;h^{\frac{3}{2}} \qquad \ldots(14.15)$$

where n = number of ends contracted

and $3{\cdot}33=\tfrac{2}{3}\,.\,C_d\,.\,\sqrt{2g}$ taking $C_d=0{\cdot}623$

ii) *Metric Units*—

$$Q=1{\cdot}84\;(B-0\,1\,n\,.\,h)\;h^{\frac{3}{2}} \qquad \ldots(14{\cdot}15a)$$

where $1{\cdot}84=\tfrac{2}{3}\,.\,C_d\,.\,\sqrt{2g}$ taking $C_d=0\;623$

c) 90° V—Notch

i) *Feet Units*—

$$Q=2{\cdot}53\,.\,h^{\frac{5}{2}} \qquad \ldots(14.16)$$

taking $C_d=0{\cdot}593$

ii) *Metric Units*—

$$Q=1{\cdot}4\;h^{\frac{5}{2}} \qquad \ldots(14.16a)$$

taking $C_d=0{\cdot}593$

d) *Cippoletti* weir or trapezoidal type—

i) *Feet Units*—

$$Q=3{\cdot}33\;B\,.\,h^{\frac{3}{2}} \qquad \ldots(14.17)$$

derived from Francis formula (Eqn 14.15) by neglecting the term $0{\cdot}1\,n\,.\,h$

ii) *Metric Units*—

$$Q=1{\cdot}84\;B\,.\,h^{\frac{3}{2}} \qquad \ldots(14.17a)$$

Emperical Formulae for rectangular weirs—

a) Bazin Formula—

i) Feet Units—

$$Q=\left(0{\cdot}405+\frac{0\ 00984}{h}\right)\sqrt{2g}\ .\ B\ .\ h^{\frac{3}{2}} \qquad ..\ (14.18)$$

ii) Metric Units—

$$Q=\left(0{\cdot}405+\frac{0{\cdot}03}{h}\right)\sqrt{2g}\ .\ B\ .\ h^{\frac{3}{2}} \qquad ..\ (14.18a)$$

b) Rehbock Formula—

i) Feet Units—

$$Q=\left(3{\cdot}227+0{\cdot}435\ \frac{h_e}{s}\right)\ .\ B\ .\ h_e^{\frac{3}{2}} \qquad ..\ (14.19)$$

where $h_e=h+0{\cdot}0034$ ft

ii) Metric Units—

$$Q=\left(3{\cdot}227+0\ 435\frac{h_e}{s}\right)\ .\ B\ .\ h_e^{\frac{3}{2}} \qquad ...(14.19a)$$

where $h_e=h+0{\cdot}0011\ m$

c) Society of Swiss Engineers and Architects Equation

i) Feet Units—

$$Q=\frac{2}{3}\sqrt{2g}\ .\ B\ .\ h^{\frac{3}{2}}\times0{\cdot}615\left(1+\frac{1}{3\ 05\ h+1{\cdot}6}\right)\times\left\{1+0{\cdot}5\left(\frac{h}{h+s}\right)^2\right\} \qquad ...(14.20)$$

(for rectangular weir *without* end-contractions)

$$Q=\frac{2}{3}\sqrt{2g}\ .\ B\ .\ h^{\frac{3}{2}}\left\{0{\cdot}578+0{\cdot}037\left(\frac{B}{L}\right)^2+\frac{3\ 615-3\left(\frac{B}{L}\right)^2}{3\ 05\ h+1{\cdot}6}\right\}\times\left\{1+0{\cdot}5\left(\frac{B}{L}\right)^4\left(\frac{h}{h+s}\right)^2\right\} \qquad ...(14.20a)$$

(for rectangular weir *with* end-contractions)

ii) Metric Units—

Replace the value of 3 05 h by h and use the above formulae (Eqn 14.20 and 14.20*a*)

Problem 14.2 In a given waterfall the quantity of water was measured by a rectangular notch above the fall. The notch had a length of 12 ft (or 3·66 *m*) and the mean head over the notch was measured as 20 in. (or 0·508 *m*). The velocity of approach at the point where the head was measured, was 2 miles per hour (or 3·22 *km* per hour). Compute the discharge in gpm (or m^3/sec). Take $C_d=0{\cdot}6$.

Solution

$L=12$ ft (or 3·66 *m*)

$H=\frac{20}{12}=1{\cdot}67$ ft (or 0·508 *m*)

$v_0=2$ miles per hour $=\dfrac{2\times1760\times3}{60\times60}$ ft/sec

$=2{\cdot}93$ ft/sec

$\left(\text{or} \quad v_o = \dfrac{3\ 22 \times 1{,}000}{60 \times 60} = 0\ 894\ m/\text{sec}\right)$

Head due to velocity of approach, $\dfrac{v_o^2}{2g} = \dfrac{2\ 93^2}{64\ 4} = 0\ 13$ ft

$\left(\text{or} \quad \dfrac{v_o^2}{2g} = \dfrac{0\ 894^2}{19{\cdot}62} = 0{\cdot}0408\ m\right)$

Discharge $Q = \dfrac{2}{3} \cdot C_d \sqrt{2g}\ L \cdot \left\{\left(H + \dfrac{v_o^2}{2g}\right)^{\frac{3}{2}} - \left(\dfrac{v_o^2}{2g}\right)^{\frac{3}{2}}\right\}$

$= \frac{2}{3} \times 0{\cdot}6 \times 8{\cdot}02 \times 12 \times \{(1\ 67 + 0{\cdot}13)^{\frac{3}{2}} - (0\ 13)^{\frac{3}{2}}\}$
$= \frac{2}{3} \times 0{\cdot}6 \times 8\ 02 \times 12 \times 2\ 363$
$= 91\ 2$ cu ft/sec

or $Q = \dfrac{91{\cdot}2 \times 62{\cdot}4 \times 60}{10} = $ **34,100 gpm** *Answer*

$\Big[$or $Q = \frac{2}{3} \times 0{\cdot}6 \times 4\ 43 \times 3\ 66 \times \{(0\ 508 + 0{\cdot}0408)^{\frac{3}{2}} - (0\ 0408)^{\frac{3}{2}}\}$

$=$ **2 32 m³/sec** *Answer* $\Big]$

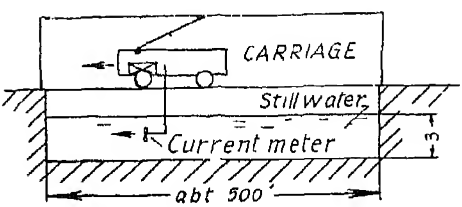


Fig 14 21 Towing of Current Meter in Still Water

14 15 Current Meters—

Principle—Current meter is a miniature reaction turbine. When placed in a stream of moving water it rotates with a speed which is a function of the velocity of flow at that point (centre of meter shaft to be more precise). It can be calibrated by observing its rpm, when moving

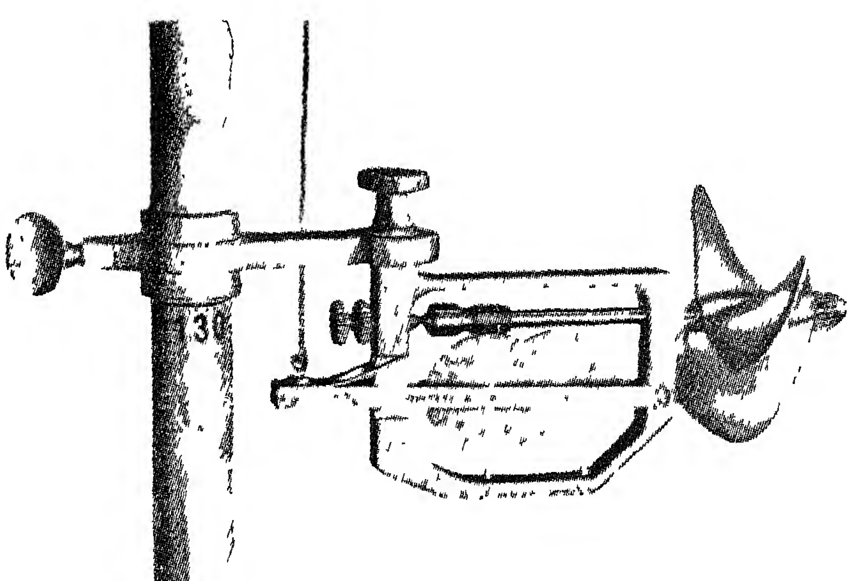


Fig 14 22 Propeller Type Current Meter

with a carriage mounted on rails, across still water at known velocities (See Fig 14 21). Then it can be used to measure velocities of flow at different points in a stream. Once the velocities at several points in a cross-section are known, calculation of discharge is a simple mathematical or graphical problem.

Types—Current meters are, generally, of two types, differing only in construction

a) **Propeller Type or Screw Type** (Fig 14 22)—A shaft parallel to the direction of flow carries a number of vanes or propeller blades. The shaft is capable of rotating in bearings fixed to a stationary frame.

b) **Cup Type** (Fig 14.23)—A *shaft normal to the direction of flow* and carrying a number of cups around its periphery, is allowed to rotate on bearings fixed to a frame.

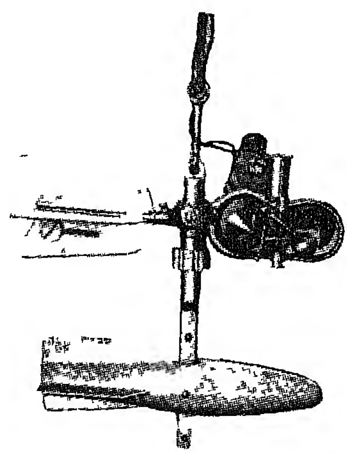


Fig 14.23 Cup Type Current Meter

Propeller type is more sensitive than cup type because it gives a higher rpm for the same velocity of flow.

The meters are usually named after the firms manufacturing them. Propeller type current-meter is manufactured by Ott (Kempten, West Germany), Amsler (Schaffhausen, Switzerland), Stoppany (Berne, Switzerland) in Europe ; by Maskell and Moff in U K. and by the Fteley-Stearns in USA. Cup type current meters are mostly manufactured in USA by Gurley or Price.

Measurement of Speed—Meter is submerged under water and motion of water in the stream actuates it, driving the rotary elements at a speed proportional to the velocity of flow. Bearing friction should be eliminated and the bearings in which the spindle carrying cups revolves, should be watertight Upper end of the bearing spindle extends above the bearing into an airtight chamber containing an arrangement of electrical contacts closed by a cam or eccentric device. In the propeller type, a commutator or eccentric arrangement fixed to the shaft carrying the blades makes and breaks an electric circuit.

The number of such makes and breaks depends on the rpm of the shaft. A dry cell of 4-volts and a bell or a buzzer constitute the rest of the circuit. By counting the number of beats in a known time, noted by a stopwatch, the rpm can be calculated. Then the velocity can be determined from the calibration curve.

When a number of current meters are working simultaneously, individual counting is not possible and an automatic recorder is necessary.

Arrangement generally employed consists of a tape chronograph and a chronometer. (A **chronograph** is a device for recording events and correlating them with time base. It comprises a strip chart and drive, a timing pen for marking equal increments of time on the chart and a number of recording style which record isolated events or the duration of operations on the same chart). A paper strip is continually fed to a chronograph at a constant speed by means of clockwork and chronometer. After a certain number of revolutions a signal is received energising a coil wound on an electromagnet which deflects a pen marking a stroke on the paper strip. Time is printed on the paper and by counting the number of strokes in a known interval, rpm can be reckoned.

Experimental Details—Current meters are used to measure the quantity of flow in channels, rivers or large pipes *e.g.* water turbine penstocks. The velocity across a large section of a stream is by no means uniform. To get a mean velocity a number of gauging points have to be chosen. The following empirical relation may be used for this purpose.

$$z=K \cdot \sqrt{A} \quad \text{.. (14.21)}$$

where, z=no. of gauging points required,

A=cross-sectional area in sq ft or sq meters,

K=a constant whose value is 4·25 to 7·6 for feet units and 14 to 25 for metric units.

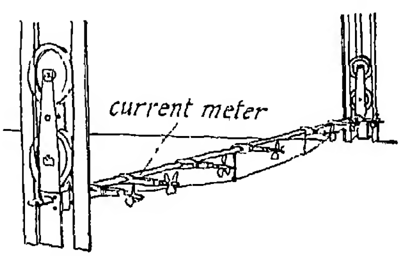


Fig 14.24 Horizontal Placing of Current Meters

Meters must be mounted on supporting rods to ensure that the axes of meters are parallel to the axis of channel or pipe in which flow takes place. Care is taken to see that no twisting or vibration of shaft or supporting rod occurs. Only the effect of flowing water should be impressed on the meters.

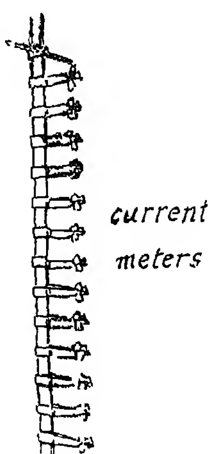


Fig 14.25 Vertical Placing of Current Meters

A relay of meters is fixed to a single supporting rod which can be stretched along the breadth of the stream with its ends fixed to the sides or along the depth with one end firmly fixed to the bottom. The former setting is said to be horizontal (Fig 14.24) and the latter vertical (Fig 14.25). The former gives the velocity variation along the breadth of the channel and this can be determined at many different depths by successive experiments. The latter distribution gives the velocity

distiibution along the depth and this can be carried out at several positions along the width. The second method is more commonly adopted, because the width of river is generally not uniform.

Calculation of Discharge—

a) **Open Channel** – With reference to Fig 14.26, the discharge is given by,

$$dQ = V \cdot dA$$
$$= V \cdot dB \cdot dH$$

or $$Q = \int_0^H \int_0^B V \cdot dB \cdot dH \qquad \ldots(14.22)$$

This integral is generally evaluated by graphical methods after the velocities at a number of points over the cross-section of the stream, have been gauged with the help of current meters.

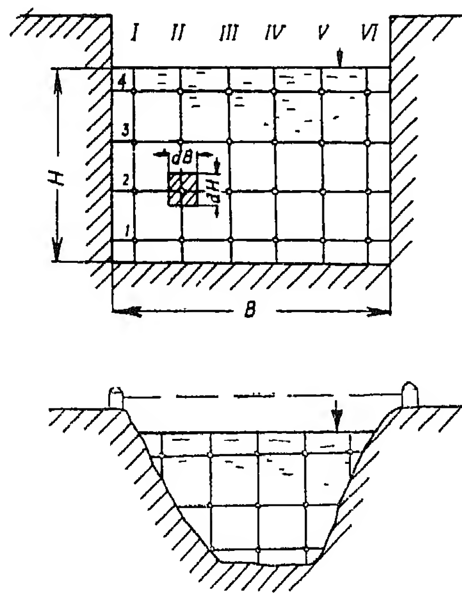


Fig 14.26 Stream Cross-sections Showing Gauging Points

If a vertical supporting rod is used, curves showing $V = f(H)$ are drawn for several positions along the breadth (See Fig 14.27).

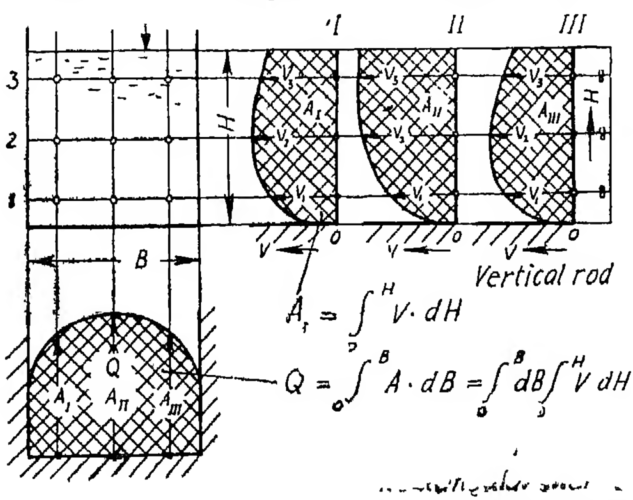


Fig 14.27 Graphical Determination of Q in a Rectangular Channel

Then the integral $\int_0^H V \cdot dH$ is evaluated for each position by measuring with a planimeter the area enclosed by the corresponding curve.

These areas are, then, plotted against breadth and the area under this curve gives

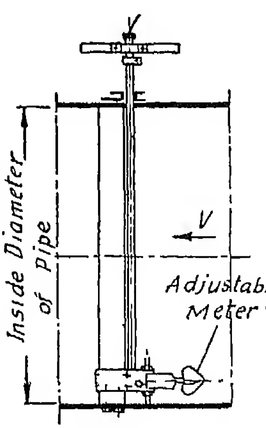
$$\int_0^B \int_0^H V \,.\, dH \,.\, dB$$

This, therefore, is the required discharge. If a horizontal supporting rod is used, curves showing $V=f(B)$ are drawn and $\int_0^B V \,.\, dB$ is evaluated first. Then another curve showing variation of this integral with H is drawn and area under this curve gives

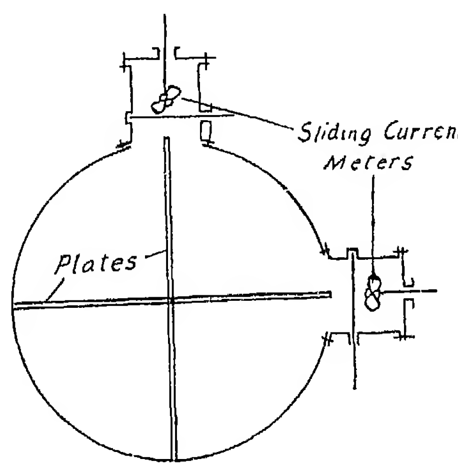
$$\int_0^H \int_0^B V \,.\, dB \,.\, dH = Q$$

b) **Closed Pipe**—(See Fig 14.28).

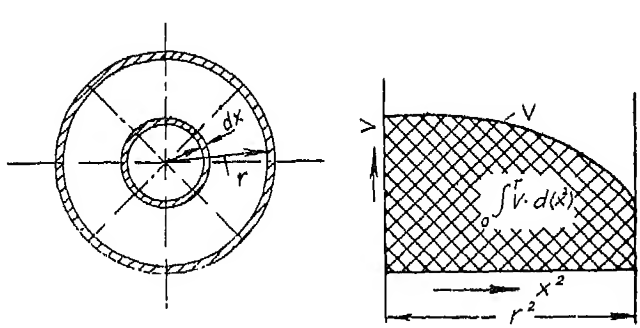
$dQ = V \,.\, dA = V \,.\, 2\pi\, x \,.\, dx$



(*a*) Measurement of Discharge in Pipe by one Current Meter



(*b*) Measurement of Discharge in Pipe by Two Current Meters



(*c*) Graphical Determination of Q in a Pipe

Fig 14.28 Measurement of Discharge in a Pipe

$$\therefore \quad Q=\int_0^r 2\pi x \,.\, V \,.\, dx=\pi\int_0^{r^2} V \,.\, dx^2$$

Velocities at a number of points at different distances from the centre are gauged and the function—

$$V=f\,(x^2)$$

is plotted. The area under this curve multiplied by π gives the discharge.

$$\text{Thus} \quad Q=\pi \,.\int_0^{r^2} V \,.\, dx^2 \qquad \text{...(14.23)}$$

$$=\pi\times[\text{ area under curve } V=f\,(x^2)]$$

Precautions—

i) Width of canal or river should be measured by accurate theodolites.

ii) Depth of water at various points should be measured by a pointer gauge. In order to ensure that the pointer tip exactly touches the water surface, the pointer is made a part of a dry cell circuit which is completed at the touch of water. To facilitate contact, local addition of an electrolyte like Sodium Chloride *i e.* NaCl is recommended. Changes in depth due to evaporation should be considered especially in the tropics.

iii) A sufficient number of gauging points should be chosen to ensure an accurate velocity picture.

iv) The distance of the axis of the meter nearest to a wall or bottom should be at least $\frac{3}{4}$ propeller diameter and at the most 8 inches (or 0·2 *m*). All meters must be completely submerged under water.

v) Current meter revolutions should be observed for not less than 60 seconds. Inaccurate stopwatch reading is a possible source of error.

vi) Flow should be kept steady during the period of observation. Any irregularity should disqualify the observation and it should be repeated.

vii) The axis of the meter should be parallel to the direction of flow, otherwise

$$V=\frac{V_e}{\cos\alpha} \quad \text{(See Fig 14.29)}$$

where,

V=velocity of water

V_e=velocity indicated by meter

α=angle of deviation

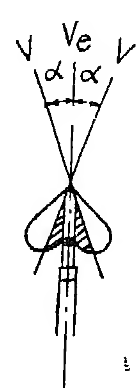


Fig 14.29 Determination of Velocity When the Axis of Current Meter is Inclined to the Direction of Flow

This correction may be applied but experience shows that the corrected value thus obtained is slightly greater than the actual value.

viii) In case of acceptance tests for water turbines, gauging should be carried on the head race side because the tail race is usually turbulent and full of eddies.

ix) Whenever possible, especially in laboratories, walls of the channel should be vertical and smooth. Bottom should be square with the walls, smooth and horizontal. A minimum straight stretch of 5 ft (or 1·5 *m*) must precede the current meter.

x) Current meter must be robust and bearing should be frictionless allowing no lateral displacement of shaft.

xi) The cam and electrical "make and break" device should be shielded from water.

xii) Vanes or cups should be scrupulously cleaned so that meter may not be blocked with dirt.

xiii) Frequent and careful calibration is necessary.

xiv) Ends of velocity curves should be drawn according to construction shown in Fig 14.30.

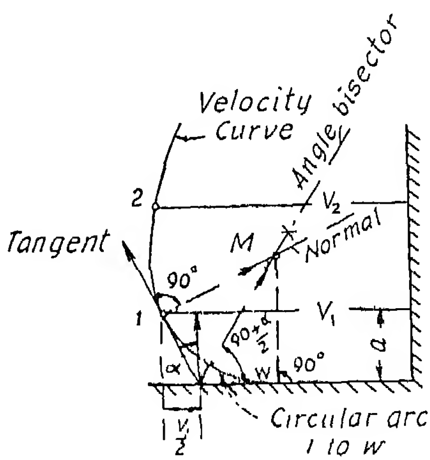


Fig 14.30 Construction of Velocity Curve End of Fig 14.27

Uses—Current meter measurements are not expensive and yield quite passable accuracy. The only disadvantage is that calculation takes time. They are suited to a wide range of discharge. The method is widely prevalent in Europe and turbine manufacturers recommend it for acceptance tests.

14.16 Salt Velocity Method or Allen Method—This method first suggested and used by Prof. C. M. Allen of Worcester (USA) is applicable to pipes and open channels.

Working Principle—Salt solution has greater electrical conductivity than pure water. If some concentrated salt solution is suddenly introduced into a pipe or a channel, it will be carried downstream with the water at a speed equal to the mean velocity of flow. At some distance further downstream, electrodes connected to the terminals of a battery are lowered into the water and current is allowed to pass through the water. When the injected salt flows past this point, conductivity becomes maximum and this is indicated by a maximum reading in an ammeter placed in the circuit (See Fig 14.31). The conductivity is observed at two points and if l be the distance between them and t the time interval shown by a stopwatch between the moments of maximum ammeter readings at the two points, then velocity of salt

$$=\frac{l}{t}$$

$\therefore$ The mean velocity of flow, $v=k\ .\ \frac{l}{t}$ (Generally, $k=1$)

If A be cross-sectional area of stream of pipe,

$$Q=k\ .\ \frac{l}{t}\ .\ A \qquad \ldots(14.24)$$

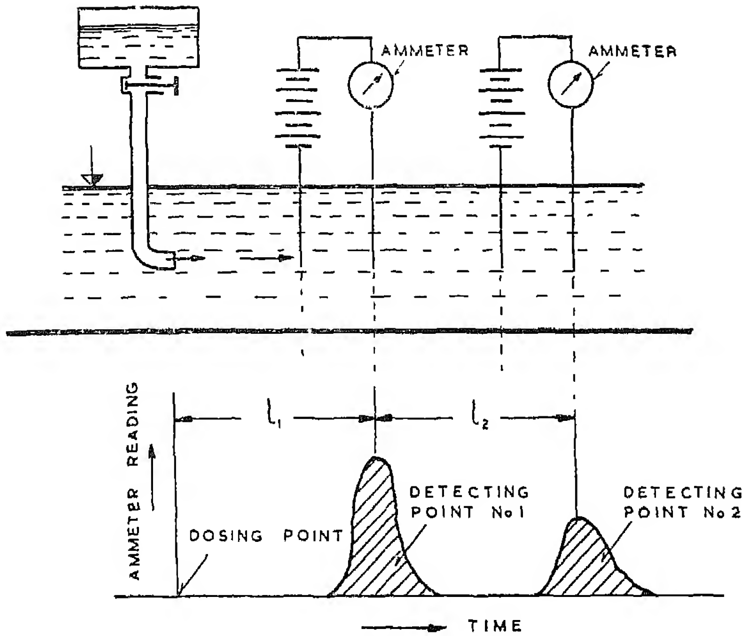


Fig 14 31 Salt-Velocity Method

The injection of salt should be carried out in an instant for ideal results. A near approximation is achieved in practice by a quick acting valve. The method is expensive but is widely used in USA.

Practical Example—

At Walchen Sea Research Station USA, the discharge was measured by salt-velocity method and gave following data—

Length of head race canal	=450 ft (or 137 *m*)
Salt introduced	=10 tons (or 10·16 metric tonnes)
No. of quick acting valves used	=38
No. of vertical steel electrodes used	=30 pairs
Time taken	=2 days
Discharge measured	=1600 to 2500 cfs (or 45 to 70 m^3/sec)

14.17 Titation or Chemical Method—When the water flow is excessively turbulent, normal methods cannot be employed and chemical methods may be used.

A chemical, generally, a common salt (sodium chloride) solution of known strength, is steadily introduced into the stream at some definite point. The solution should be sufficiently strong containing at least one part of the salt in four parts of water. After some time, not less than ten minutes, a sample of down stream water is taken at some point sufficiently distant from the point where salt is being introduced. This is to allow the salt the necessary time and distance to diffuse uniformly over the whole section.

Now, if k_o=weight of salt per unit volume of natural water
k_1=weight of salt per unit volume of salt solution
k_2=weight of salt per unit volume of sample water
q=rate of discharge of salt solution
and Q=rate of flow of water in the stream
then, $Q \cdot k_o + q \cdot k_1 = (Q+q) \cdot k_2$
or $Q(k_2-k_o)=q(k_1-k_2)$
or
$$Q=q \cdot \left(\frac{k_1-k_2}{k_2-k_o}\right) \qquad \ldots(14.25)$$
Generally, $k_o=0$ and, therefore, $Q=q\left(\frac{k_1}{k_2}-1\right)$

If $k_1 >>> k_2$
$$Q \approx q \cdot \frac{k_1}{k_2} \qquad \ldots(14.25a)$$

Degree of concentration of salt or the strength of solution can be determined chemically by titration, or by electrical or optical methods.

Lately, experiments are performed by using sodium bichromate. This salt is selected because it is neither present as such nor as impurity in natural water. However, it is more expensive than common salt.

The titration or chemical method is used for rivers generally having a discharge of not more than 3,500 cusecs (or 100 m³/sec). The quantity of salt required is roughly 100 to 200 lb (or 50 to 100 kg) per 3,500 cusecs (or 100 m³/sec).

14.18 Gibson Inertia-Pressure Method—This method was first suggested and used by Dr. N. R. Gibson of Niagra Falls Power Co. Pressure generated due to inertia of moving water when it is suddenly brought to rest by closing a valve at the end of the pipe, is measured. Velocity of water is calculated from it as follows :

From the equation of water hammer,
$$-\frac{dv}{dt} \cdot \frac{L}{g} = H - H_o$$
where, H_o=normal head
H=head after closing the valve
L=Length of pipe

The inertia head $h_i = H - H_o$
$$\therefore \quad -\frac{dv}{dt} \cdot \frac{L}{g} = h_i$$
or
$$dv = -\frac{g}{L} \cdot h_i \cdot dt$$

If v be the original velocity and t the time taken by water to come to rest,
$$0 - v = -\int_0^t \frac{g}{L} \cdot h_i \cdot dt$$

or $$v=\frac{q}{L}\int_0^t h_t \, . \, dt \qquad ...(14.26)$$

The integral $\int_0^t h_t \, . \, dt$ can be evaluated by measuring the area under the curve $h_t=f(t)$.

Then the discharge,

$$Q=\frac{\pi}{4} \, . \, d^2 \, . \, v \qquad ...(14.26a)$$

where d is the diameter of pipe carrying the water.

This method is suitable for hydraulic turbines with closed conduits. Basically the Gibson method is used to establish the initial rate of flow from a diagram on which are recorded the variations of pressure occuring in a penstock during shutting down of a turbine in a known time. In other words the discharge in the turbine supply conduit is deduced from pressure measurements taken during the period in which the regulating gear is closed.

If the conduit or penstock is of varying cross-section the accurate discharge is obtained by multiplying Eqn 14.26a by a co-efficient, known as *pipe factor* which is the summation of lengths divided by area.

The apparatus* for the measurement of pressure variations, consists of a calibrated U-tube mercury manometer connected to the conduit as shown in Fig 14.32. After the turbine inlet valve is closed the variations of pressure difference from the U-tube are recorded on a sensitive film wound onto a clockwork or electrically driven drum. A pendulum swinging before the camera lens marks time signals on the film, which appear as vertical lines on the final print.

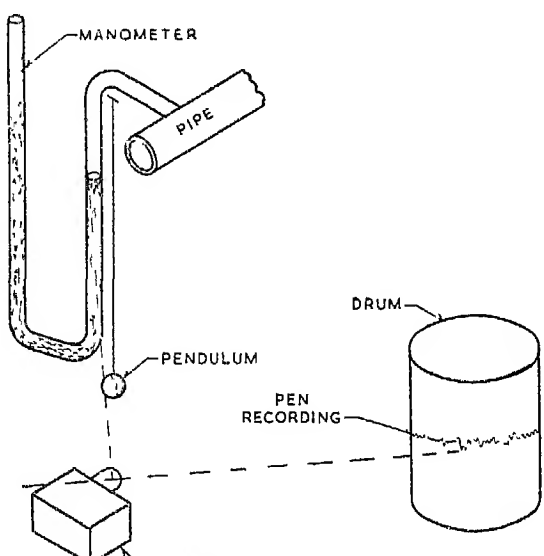


Fig 14.32 Gibson Apparatus

Advantages

1) Gibson method does not require extensive preliminary work excepting two tappings into the penstock.

2) The conduit does not require to be opened.

* "Turbine Efficiency Measurement" Part two from Water Power, February 1959, page 55.

Disadvantages

1) Since the method involves load rejection, normal power station operation during the tests is impossible.
2) The turbine and the regulating gear are subjected to heavy stresses.
3) Evaluation of results is somewhat long and arduous process.

This method has now been accepted as reliable and accurate way of measuring the discharge.

14 19 Thermometric and Thermodynamic Methods—These methods are based on a very simple principle that the energy lost in the machine heats up the water passing through. The measurement of rise of temperature of water ($\frac{1}{1400}$ deg C per foot water-gauge head loss) gives a measure of the losses. From this efficiency of water turbine can be worked out. For low head machines very accurate measurements are required while sensitive mercury-in-glass thermometers can perhaps do for very high head installations.

Assuming electrical output and efficiency, the turbine discharge could be deduced.

a) **Thermometric Method**—Thermometric method was visualised in principle about 1914 by Poirson in France and later experimental work was carried out by Barbillon, Fontaine, Gaillard, Poirson and Volle in France about 1920, by Umfenbach in Germany in 1937 and Katzman in Canada after the Second World War. This method was tried at National Research Laboratories, Ottawa in 1948 and in Switzerland in 1955.

The efficiency was determined by direct measurement of temperature at inlet and outlet from the turbine. The expression* for the efficiency is as follows—

i) Where the thermometers are inserted in the penstock and tail race giving tempteratures of water θ_e and θ_s respectively—

$$\eta = 1 - \frac{J\,C\,(\theta_s - \theta_e)}{H_n} \qquad \ldots(14.27)$$

where J=mechanical equivalent of heat

C=specific heat of the working fluid

H_n=net or effective total head across the turbine.

ii) Where the temperature of water is measured as in (*i*) but the sample from the penstock is fully expanded to atmospheric pressure before its temperature is measured—

$$\eta = \frac{J\,C_{p_2}\,(\theta_1 - \theta_2) + (z_1 - z_2)}{H_e\,(1-\beta) + (z_1 - z_2)} \qquad \ldots(14.28)$$

where C_{p_2}=specific height of the working fluid at the constant pressure at outlet test point,

H_e=total head at inlet test section

* See "Thermodynamic Method of Measuring Turbine Efficiency" by Dr. D. N. Singh, Mechanical Engineering Research Laboratory (MERL) Fluid Report No. 70 Glassgow, August 1958.

z_1 and z_2=datum levels of test sections
β=thermodynamic co-efficient.

There must, of course, be no heat exchange between the water samples and the surroundings. The absolute calibration of thermometers must be made.

b) **Thermodynamic Method**—In 1950 Campmas, Fontaine and Volle investigated the principle and Willm and Campmas applied the results of these investigations to modify the thermometric method. Dr. D. N. Singh* fully developed this method (See Fig 14.33) recently. The thermodynamic expansion of water *under* pressure is made to measure the temperature of water. The apparatus (See Fig 14.33) consists of a water calorimeter in which a steady and continuous running sample of water, withdrawn from the penstock through a pitot-tube probe at a section where the total energy of water is fairly constant, is expanded adiabatically. This expansion in the calorimeter and the rate of flow through it, can be controlled by the operation of two valves so that the temperature of water was raised to a temperature identical with that in the tail race. Carefully matched platinum resistance thermometers are used in the calorimeter and the tail race, requiring no absolute calibration, when connected as adjacent arms on a simple Wheatstone bridge, a null reading on a galvanometer is obtained, showing equality in the temperatures at the two stations.

The total head of water in the calorimeter between the two valves ; H_1 is then measured. By closing the outer valve, the flow through the calorimeter is stopped and the total static head H_e noted. The efficiency is then given by

$$\eta=\frac{H_1\ (1-\alpha)+(z_1-z_2)}{H_e\ (1-\beta)+(z_1-z_2)} \qquad \text{...(14.29)}$$

where z_1 and z_2=datum levels at the two test sections
α and β=thermodynamic co-efficients which correct the variation of specific volume of the working fluid and change of its internal energy with temperature and pressure.

$$\alpha=\frac{\omega_1-\omega_o}{\omega_1}+\frac{\omega_o}{(P_1-P_2)}\int_{P_2}^{P_1}\left[\ T_1\ \frac{\partial}{\partial T}\left(\frac{1}{\omega}\right)+P\frac{\partial}{\partial P}\left(\frac{1}{\omega}\right)\right]dP \qquad \text{...(14.30)}$$

and $$\beta=\frac{\omega_e-\omega_o}{\omega_e}+\frac{\omega_o}{P_e-P_s}\int_{P_e}^{P_s}P\ \frac{\partial}{\partial P}_{(\theta=\theta_e)}\cdot\left(\frac{1}{\omega}\right)dP \qquad \text{...(14.31)}$$

where P=absolute pressure
T=absolute temperature

Suffices e=inlet test section
s=outlet test section
o=any arbitrary reference point (say 4°C for water)
1=inlet test point
2=outlet test point.

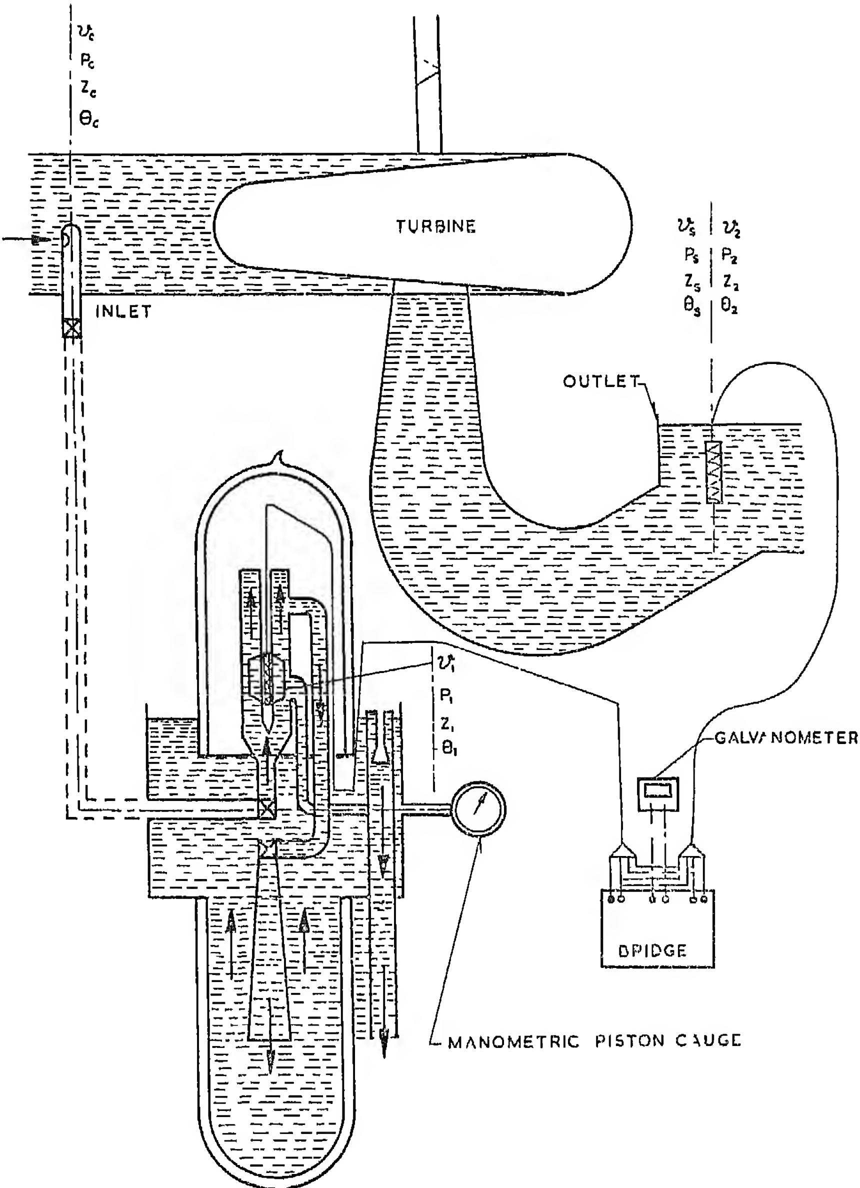


Fig 14.33 Thermodynamic Method Applied to Water Turbine Measurement

Advantages of Thermodynamic Method over Thermometric Method—

a) Absolute calibration of thermometers is not required.

b) Value of specific heat of water is not required.

c) Variations of specific weight of water are taken care of by the co-efficients α and β.

d) Principal terms are measured directly as pressures.

14.20 Ultrasonic Flowmeter*—The ultrasonic flowmeter contracts the transit times of ultrasonic waves travelling with and against the flow by comparing the phase of two sets of waves. The difference in phase angle is directly proportional to the velocity of flow of the fluid and it is also proportional to the frequency of the ultrasonic waves and the distance.

Principles

a) Differential Arrangement—If two ultrasonic transducers are placed in a conduit (Fig 14.34) in which the measurement of velocity of fluid is to be made, then the time taken for a pulse of sound emitted from one to reach the other is

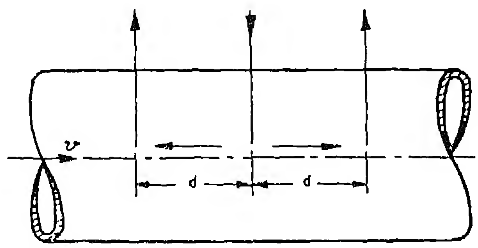


Fig 14.34 Ultrasonic Flowmeter Principle (Differential Arrangement)

$$t_o = \frac{d}{v_s}$$

where d = distance between two transducers

v_s = velocity of propagation of sound in the fluid.

If the fluid is flowing with a velocity v in the direction of propagation, the time of transit will become

$$t_1 = \frac{d}{v_s + v}$$

As v is much less than v_s, t_1 will be nearly equal to t_o. In practice v_s varies with temperature and pressure, therefore t_o is not accurate.

For accurate results a second transducer is placed upstream (See Fig 14.34) of the transmitting transducer, also spaced d from it. The time taken by the sound pulse in travelling to the upstream transducer—

$$t_2 = \frac{d}{v_s - v}$$

The difference between times of arrival of the upstream and downstream signals—

$$\Delta t = t_2 - t_1 = \frac{2\,dv}{v_s^2 - v^2} \approx \frac{2\,dv}{v_s^2} \quad ...(\because v_s >>> v) \quad ...(14.32)$$

In practice, use of continuous waves instead of pulses is made. The transmitting transducers are fed from a common source and the phases of the signals arriving at the two receiving transducers are compared. The phase difference

$$\Delta\phi = \frac{2\,\omega\,.\,dv}{v_s^2} \quad ...(14.33)$$

where $\omega = 2\pi f$ = angular frequency of transmitted signal.

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* (1) "Turbine Efficiency Measurement"—Water Power, Page 114, March 1959.

(2) "Measurement of Liquid Flow by Ultrasonics" by R. E. Fischbacher—Water Power, Page 212, June 1959.

b) Oscillating Loop System—This method has been developed to eliminate the effect of velocity of sound. A pair of transducers are used as in (*a*), each pair being formed into an oscillating loop. A pulse is emitted from transmitting transducer T_1 (See Fig 14.35) and is received by receiver T_2 after time t_1. This pulse is amplified and virtually instantaneously fed back to the transmitting transducer for retransmission. A pulse is therefore emitted every t_1 seconds *i.e.* a train of pulses is emitted at reception frequency—

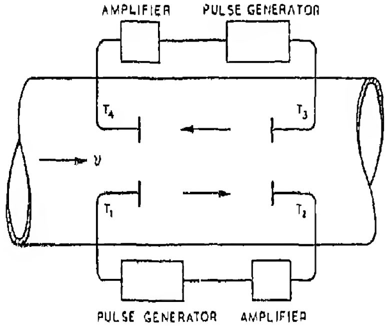


Fig 14.35 Ultrasonic Flowmeter Principle (Oscillating Loop System)

$$f_1=\frac{1}{t_1}=\frac{v_s+v}{d}$$

Similarly another train of pulses is generated by another transducer pair in an upstream direction at a frequency—

$$f_2=\frac{1}{t_2}=\frac{v_s-v}{d}$$

The frequency difference or beat frequency between two loops

$$\Delta f=f_1-f_2=\frac{2v}{d} \qquad \text{...(14.34)}$$

which is independent of velocity of sound v_s.

Fig 14.36 shows the principle of ultrasonic flowmeter. Two transducer rods are placed in the conduit in which measurement of water velocity is to be made. A transducer, when acting as transmitter, converts high-frequency alternating current into mechanical vibrations of the same frequency. Conversely, mechanical vibrations picked up by a transducer are converted into alternating current. It is found that the magnitude of the phase angle between the transmitted and received signal is a measure of the average velocity of water passing through the conduit, but in practice, to avoid the necessity of measuring the phase angle with the water stationary, two phase-angle measurements are made in quick succession : first with the downstream transducer receiving the signal of the upstream transducer, and second, with the functions of the two units interchanged. Part of the signal from the oscillator passes through one channel of the amplifier and mixer to the phase angle meter. The signal from the receiving transducer passes to the phase-angle meter through the second channel of the amplifier and mixer. The beat method of reducing frequency maintains the phase relationship of the two signals.

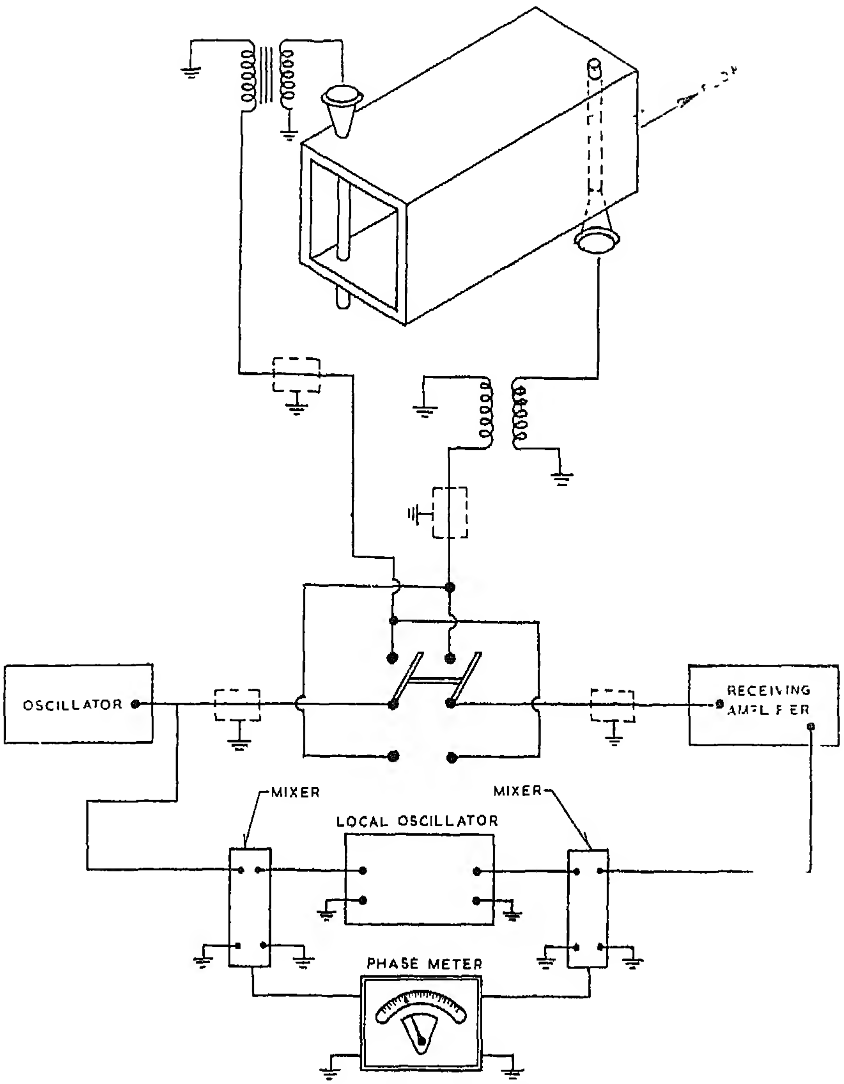


Fig 14.36 Ultrasonic Flowmeter Details

B. PRESSURE MEASUREMENTS

14.21 Gauge and Absolute Pressure—Local atmospheric pressure is taken as the datum for measurement of pressure in hydraulic engineering. Atmospheric pressure, therefore, is zero pressure on this scale. Pressure measured on this scale is referred to as *gauge* pressure. It is so called because the ordinary pressure gauges, the instruments for pressure measurement, yield readings on this scale. Sub-atmospheric pressures produce negative readings on the pressure gauges. This is often referred to as *vacuum* or suction pressure.

Gauge pressure should be distinguished from absolute pressure which is reckoned from the absolute zero of pressure.

∴ Absolute Pressure = Atmospheric Pressure + Gauge Pressure.

For sub-atmospheric pressures,

Absolute Pressure = Atmospheric Pressure − Vacuum Pressure.

14.22 Units of Pressure—The pressure of a column of liquid of height h and specific weight w is $w \cdot h$ per unit area. This affords two ways of expressing pressure.

The intensity of pressure is expressed in force per unit surface area. The FPS unit of intensity of pressure is lb per sq in. and the metric unit is kg per sq cm. The atmospheric pressure varies from place to place, and time to time but when not specified, an average value of 14·7 lb per sq in. (or 1·033 kg/cm^2) is taken for engineering purposes.

For engineering purposes, it has been found more convenient to express pressure in terms of the height of equivalent liquid column. This height is known as pressure head and is generally expressed in feet or meters of liquid (column). The advantages of using head as an expression for pressure become obvious when it is realised that the statement "pressure head of liquid h ft (or m)" implies that the pressure energy per pound (or kg) of liquid is h ft lb (or kgm).

Pressure head is also measured relative to the atmospheric pressure. Standard atmospheric pressure is equivalent to 34 feet (or 10·36 m) of water or 30 in. (or 760 mm) of mercury.

The relation between two expressions for pressure is obvious.

Intensity of pressure in lb per sq in.

$$= \frac{\text{Pressure head in ft} \times \text{Sp wt in lb per cu ft}}{144}$$

or $$p = \frac{H \cdot w}{144} \text{ lb in.}^{-2} \qquad \ldots(14.35)$$

Intensity of pressure in kg per cm^2

$$= \frac{\text{Pressure head in } m \times \text{Sp wt in kg per } m^3}{10,000}$$

i.e. $$p = \frac{H \cdot w}{10,000} \text{ kg } cm^{-2} \qquad \ldots(14.35a)]$$

14.23 Instruments for Measurement of Pressure—The instruments employed for pressure-measurement are usually of three types :

i) Manometers,

ii) Mechanical Gauges,

and *iii*) A combination of a Piston Gauge and a Manometer.

In manometers, the unknown pressure to be measured, is balanced against a column of liquid of known specific weight. The height of the balancing column is a measure of the unknown pressure. The liquid generally used is mercury. Choice of liquid depends, however, to some extent on the magnitude of pressure to be measured. For high pressures, mercury is preferred. For low pressures, liquids of low specific weight such as carbon tetrachloride (CCl_4)—sp gr = 1·59, or Acetylene-tetrabromide ($CHBr_2$ $CHBr_2$)—sp gr = 2 95 are used.

For very high pressures, the height of the balancing liquid column becomes inordinate, and use of manometers is inconvenient. Mechanical devices are then used. Mechanical gauges directly yield pressure readings, are portable, have a wider operating range and are fairly accurate if carefully calibrated.

14.24 Manometers—The more important types of manometers are the following :

i) Piezometer tube,

ii) U-tube or double column manometer,

iii) Single column manometer and inclined tube manometer,

and *iv*) Differential manometer and Inverted U-tube.

i) **Piezometer tube**—The balancing liquid column is contained in a glass tube, one end of which is open to the atmosphere and the other connected to the side of the vessel containing the fluid of which the pressure is to be measured (See Fig 14.37). Height of liquid in the tube is proportional to the pressure at the gauge point. Pressure may be directly read from the graduations on the tube or on a scale attached to it. Care should be taken to ensure that the end of the tube connected to the vessel under pressure is flush with the inside surface of the vessel. The diameter of the tube should not be less than $\frac{3}{4}$ in. (or 20 mm) to avoid error due to capillary action. Height of column should be read at the centre of the meniscus *i.e.*, lowest point of curve for water and the highest point of curve for mercury.

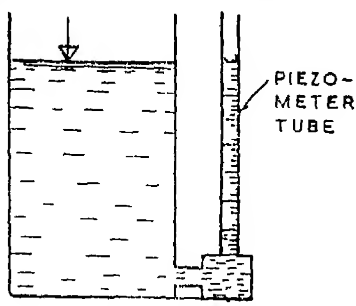


Fig 14 37 Piezometer Tube

ii) **U-tube or Double Column Manometer**—Generally speaking, if the pressure to be measured is more than 5 ft (or 1.5 m) gauge, a U-tube may be used (See Fig 14.38). One end of the U-tube is connected to the gauge point and the other left open to the atmosphere. The liquid contained in the U-tube is heavier than the fluid of which the pressure is being measured. Mercury is commonly used. Diameter of the tube should be about $\frac{3}{8}$ in. (or 10 mm).

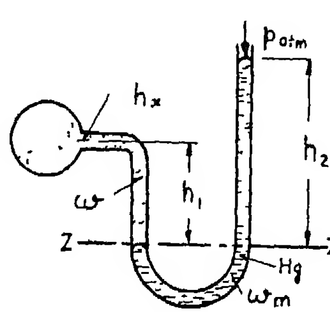


Fig 14 38 U-tube Manometer

Pressure at the gauge point x is given by :

$$h_x = \frac{h_2 \cdot w_m}{w} - h_1 \qquad ...(14.36)$$

where h_x is the required pressure at x in ft (or m) of liquid in the vessel,

w is the specific weight of liquid in the vessel,

w_m is the specific weight of the measuring liquid in U-tube,

h_1 and h_2 are the heights of liquid in the two limbs above the line $z-z$ as shown in Fig 14.38.

iii) **Single Column Manometer**—It may be regarded as a special type of double column manometer in which the cross sectional area of one limb is so large in comparison with that of the other, that any changes in the former may be neglected. The smaller limb is a Piezometer tube and the larger limb is a shallow container (See Fig 14.39). The ratio of diameter is 8 to 10. The required pressure

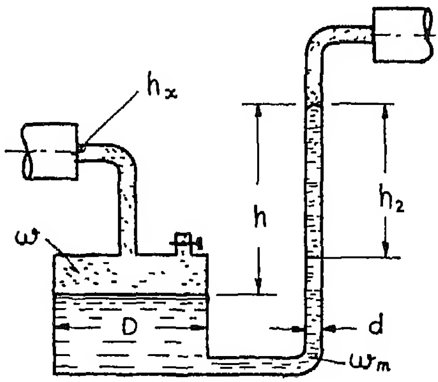


Fig 14 39 Single Column Manometer

$$h_x=h_2\left(\frac{w_m}{w}-1\right).\left\{1+\left(\frac{D}{d}\right)^2\right\} \qquad ...(14.37)$$

Where h_2 is difference of final and initial readings in the Piezometer tube.

This type of manometer is best suited for measuring the difference of pressure between two points.

iv) **Inclined Tube Manometer** is simply a single column manometer with the Piezometer tube inclined. In modern designs, it is possible for the inclined limb to swivel round its connection with the shallow container (See Fig 14 40).

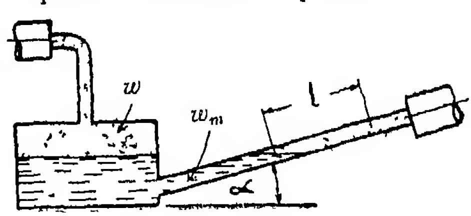


Fig 14 40 Inclined Tube Manometer

It may be used for measuring small differences of pressure accurately.

$$\triangle p=l\ (w_m-w)\sin\alpha \qquad ..(14.38)$$

If l, the travel of meniscus is in feet (or m) and w_m in lb/ft^3 (or kg/m^3),

then $\triangle p$ is in lb/ft^2 (or kg/m^2)

v) **Differential Manometer**—When the difference of pressure between two points is to be measured, they may be connected to the two ends of a U-tube as shown in Fig 14.41. This arrangement constitutes a differential manometer. The required difference of pressure is

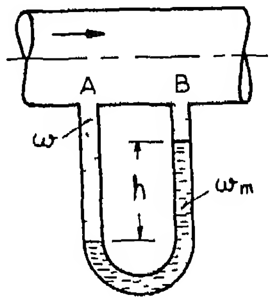


Fig 14 41 Differential Manometer

$$\triangle h=h_A-h_B=h\left(\frac{w_m}{w}-1\right) \qquad ..(14.39)$$

For precision pressure measurement, various micro-manometers are available in the market ; many of them named after their inventors *e.g.*, Rosenmueller, Prandtl, Chattock etc.

14 25 Pressure Connections—The following points should be kept in mind while making connections for the measurement of pressure :

a) The drilling should be perpendicular to the pipe wall and must flush with the inner surface (See Fig 14 42),

b) Diameter of hole should be about 1/8 in. (or 3 to 5 mm). Smaller holes are liable to be choked and larger holes may cause errors in measurement by change in pressure at the gauge point,

c) For connecting the gauge piping outside $\frac{1}{2}$ in. (or 10 mm) bore in pipe with gas threads will suffice,

d) Connecting pipes must be full of liquid. No air pockets should be present.

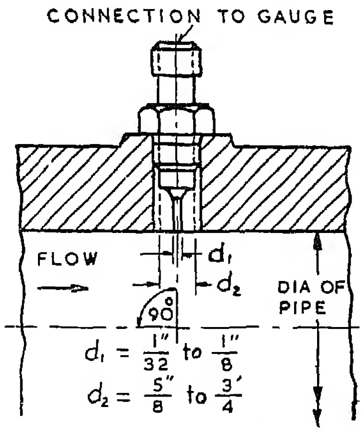


Fig 14.42 Gauge Connection to a Pipe Line

14 26 Mechanical Gauges—For reasons explained in Art 14.22 and 14 23, mechanical gauges are more suited than manometers for the measurement of high pressure, specially when it is more than two atmospheres. The most accurate and reliable region on the scales of mechanical gauges is between 40 to 70 percent of the maximum. The three important types are :

i) Bourdon tube pressure gauge,

ii) Diaphragm pressure gauge (Aneroid Barometer),

iii) Dead weight pressure gauge.

The first of these is the most widely used but the last is the most accurate and may be used for the purpose of calibration.

Mechanical gauges employed for measuring sub-atmospheric pressures are sometimes called Vacuum Gauges.

Graduations on the dials of mechanical gauges may be in feet (or m) of liquid column or in lb per sq in. (or kg/cm^2). For vacuum gauges, pressure scale in inches (or mm) of mercury may also be provided.

i) **Bourdon Tube Pressure Gauge**—A flexible metal (steel or bronze) tube of elliptical cross-section is the pressure responsive element in this gauge (See Fig 14.43). When the gauge is connected to the gauge point, liquid under pressure flows into the tube. The elliptical cross-section tends to become circular due to increase in internal pressure. This results in a tendency of the tube to straighten out. By a simple pinion and sector arrangement, this elastic deformation of the tube is made to cause a proportional displacement of the pointer-needle which indicates the pressure on a graduated dial. The instrument must be calibrated before use.

ii) **Diaphragm Pressure Gauge**—A corrugated diaphragm is used instead of the Bourdon Tube (See Fig 14.44). Elastic deformation of the diaphragm under pressure is transmitted by a similar arrangement to a pointer-needle. This type of gauge is used to measure relatively low

pressures. The instrument must be calibrated before use. The Aneroid barometer operates on a similar principle.

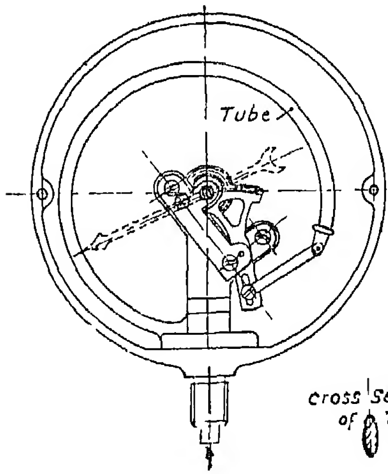


Fig 14.43 Bourdon Tube Pressure Gauge

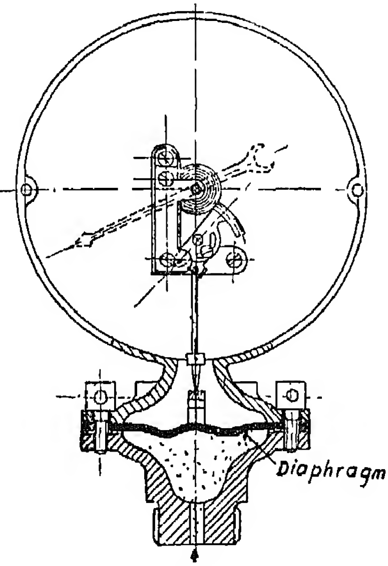
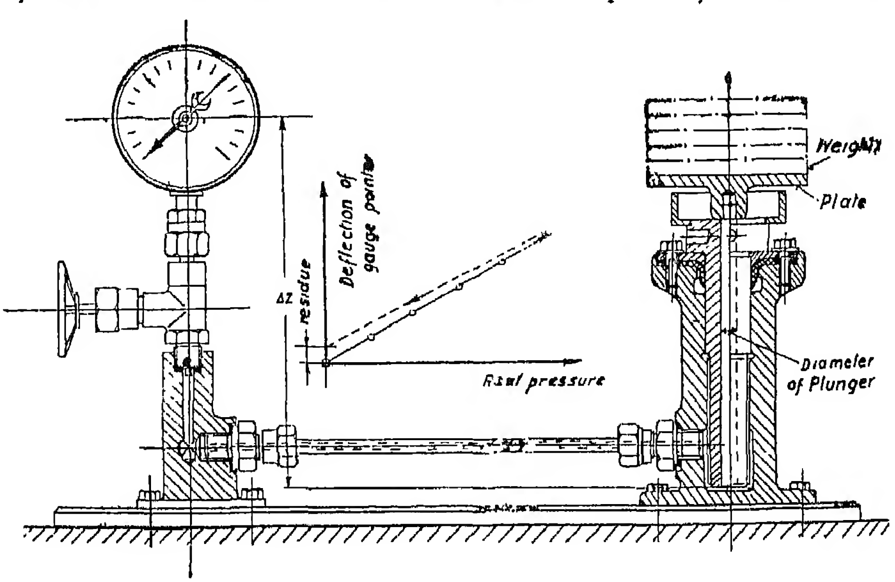


Fig 14.44 Diaphragm Pressure Gauge

iii) **Dead Weight Pressure Gauge**—This is the direct method of pressure measurement. The instrument consists of a piston and a cylinder of known area connected to the fluid space by a tube. The



14.45 Dead Weight Pressure Gauge and Calibration Instrument

fluid exerts a force on the piston equal to the pressure times the piston area. This force can be balanced by weights on the top of the vertical piston. This is most accurate device and is, therefore, used for precision work and for calibrating other pressure gauges. The pressure of liquid is balanced by known weights (See Fig 14.45), and

Pressure in lb per sq in. (or kg/cm^2)

$$=\frac{\text{Total load on plunger in lb (or kg)}}{\text{Cross sectional area of plunger in sq in. (or } cm^2)}$$

The only error involved is that due to frictional resistance to motion of plunger in the cylinder and of liquid in the connecting pipes. This can, however, be avoided by taking adequate precautions.

14.27 Manometric Piston Gauge—This is a new invention made after 1950 in France by Willm and the later developed by Singh* at the University of Glasgow for measurement of pressures in hydro power stations.

This instrument combines the principles of a dead-weight piston gauge and a water manometer. It has the advantages of both, giving a wide overall range together with great accuracy. The instrument gives rapid reading and is almost dead beat. It has been tested in Scotland and the prototype found to be accurate to 1 part in 20,000 while measuring a pressure of about 1,300 feet (or 400 *m*) of water.

This instrument can also be used for the accurate measurement of pressures elsewhere.

Let :

1) w_1=weight of piston, tray & slung weight suppoıt in lb (or kg)

2) w_2=added weight in lb (or kg)

3) w_3=weight of container & water in lb (or kg)

4) w_4=thrust on displacer in lb (or kg)

5) h=water manometer reading in in. (or cm)

6) H, p=gauge pressure to be measured in feet (or *m*) of water or lb/sq in. (or kg/cm²)

7) w=weight of 1 cu ft (or m^3) of water in lb (or kg)

8) A, D=cross-sectional area & dia of displacer in in. (or cm)

9) a, d=cross sectional area & dia of piston in in. (or cm)

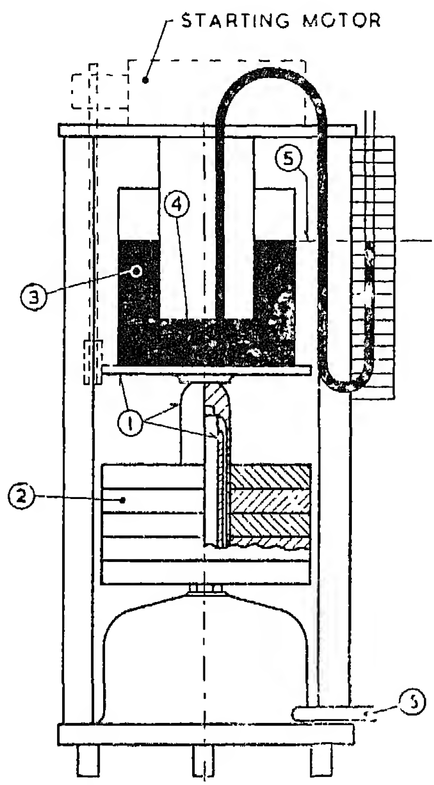


Fig 14.46 Manometric Piston Gauge Working Principle

*Singh, D. N. "Manometric Piston Gauge" Fluid Report No. 66, National Engineering Laboratory, Glasgow, Scotland (Department of Scientific and Industrial Research), Feb. 1958. Details and photographs by courtesy of the University of Glasgow and D. S. I. R. of U.K.

Then pressure measured $= K_1 (w_1 + w_2 + w_3) + K_2 \cdot h$ .. (14.40)

where, $K_1 = \dfrac{144}{w \cdot a}$ $\left(\text{or } = \dfrac{10{,}000}{w \cdot a}\right)$

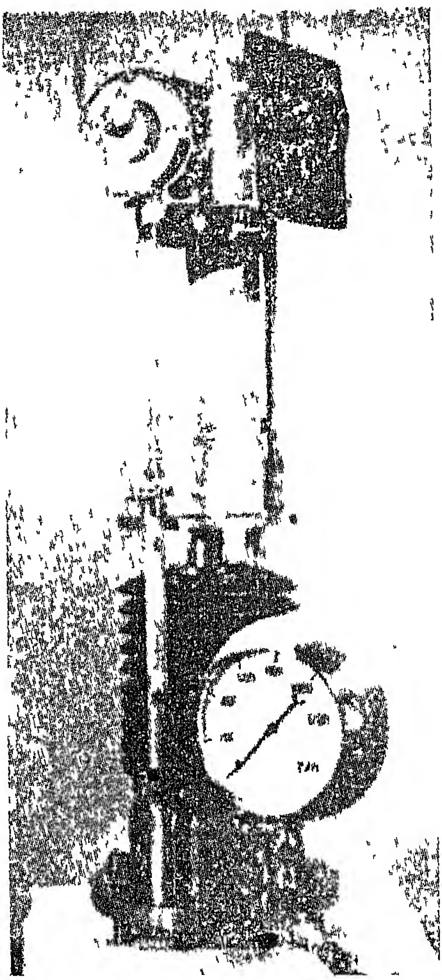


Fig 14.47 Manometric Piston Gauge, developed by Dr. D.N. Singh, arranged for Measurement of Pressures above 40 lb per sq in. (or 2·81 kg/cm²)

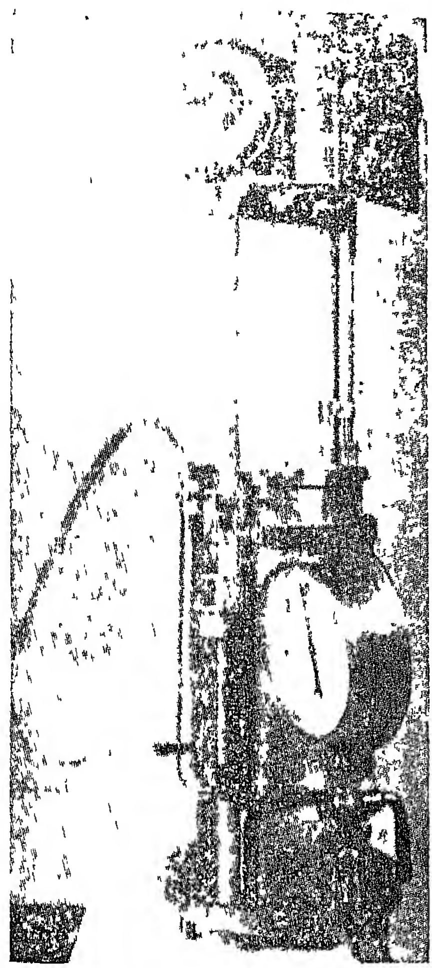


Fig 14.48 Manometric Piston Gauge, developed by Dr. D.N. Singh, for Measurements of Sub atmospheric Pressures and Pressures below 4 lb per sq in. (or 2·81 kg/cm²)

and $K_2 = \dfrac{D^2}{12\, d^2}$ $\left(\text{or } = \dfrac{D^2}{100\, d_2}\right)$

$= \dfrac{A}{12a}$ for measurement of head in feet of water

$\left(\text{or } = \dfrac{A}{100a} \text{ for measurement of head in } m \text{ of water}\right)$

and $\quad K_1=\frac{1}{a} \quad$ and $\quad K_2=\frac{w \cdot D^2}{12^3 \cdot d^2}$

$$=\frac{w \cdot A}{12^3 \cdot a} \text{ for measurement in lb/sq in.}$$

$$\left(\text{or } =\frac{w \cdot A}{10^6 \, a} \text{ for measurement in kg cm}^2\right)$$

But if there is any constant source of pressure supply available and the following observations made :

Weight required to balance constant pressure without container—w_{2x} lb (or kg)

Weight required to balance constant pressure with container in position and some reading in the manometer—w_{2y} lb (or kg) & h_y in. (or cm)

and now if, $w_1+(w_{2x}-w_{2y})-\frac{K_2}{K_1} \cdot h_y=W$, say, where $\frac{K_2}{K_1}=\frac{A \cdot w}{12^3}$

Then any pressure measured$=K_1\left(W+w_2+\frac{K_2}{K_1} \cdot h\right)$ .. (14 40a)

where K_1 and K_2 have the same significance as above.

N.B. K_1 and K_2 should be suitably amended if any liquid other than water is used in the container.

C. Level Measurements

14.28 Measurement of Level or Height of Free Surfaces—The height of a free surface of liquid can be accurately measured by one of the following instruments :

i) Pointer Gauge,
ii) Hook Gauge,
iii) Floats.

i) **Pointer Gauge**—A long rod with a fine point at one end moves vertically in a bearing (See Fig 14.49). A scale with a vernier is fitted to the rod. As soon as the point touches the liquid surface the rod is screwed in its position and the height is read on the scale. It is difficult for the eye to judge correctly when the point is just touching the liquid surface. For accurate measurement, two points may be used and an electric circuit so arranged that as soon as the points touch the liquid, the circuit is completed and an ammeter placed in the circuit immediately shows a reading. When still water surface is being measured, a pinch of common salt dropped in the region will considerably improve its conductivity.

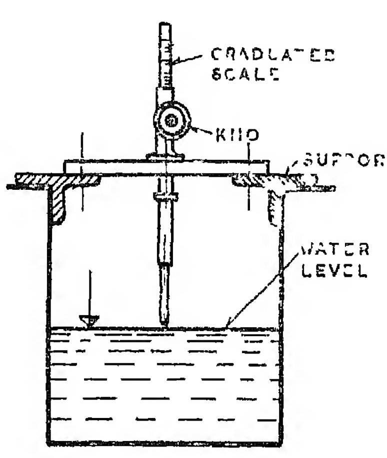


Fig 14.49 Pointer Gauge

ii) **Hook Gauge**—When the observer must rely on judgment of the eye, it is often advantageous to use a hook gauge (See Fig 14.50) instead

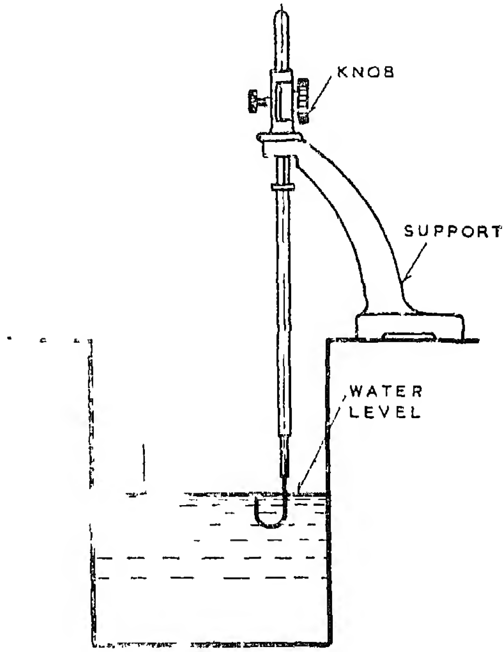


Fig 14.50 Hook Gauge

of a pointer gauge. Height is read when the point of the hook just emerges above the liquid surface. This can be judged fairly accurately.

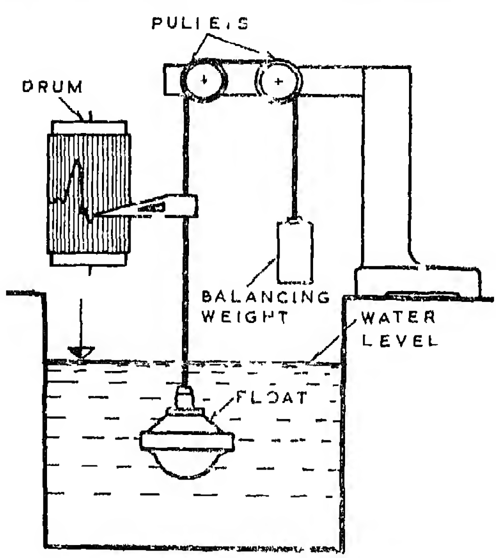


Fig 14.51 Float

iii) **Float**—Float in its simplest form is merely a hollow box or sphere of sheet metal, is allowed to float on the free surface of the liquid (See Fig 14.51). A wire connected to it at one end passes over a pulley and carries a suspended weight at the other end. Position of the weight

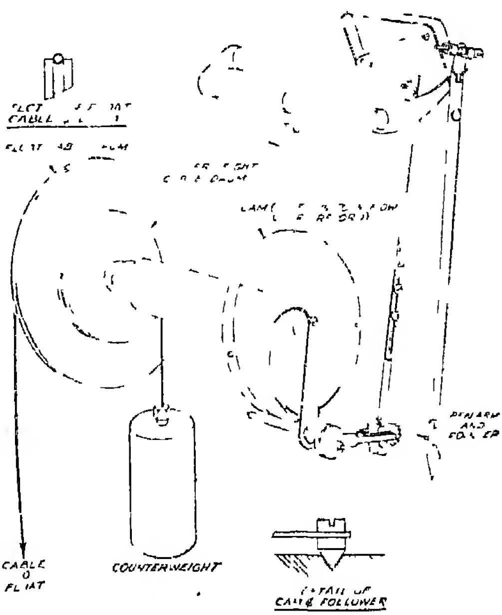


Fig 14.52 Diagramatic Sketch of Mechanical Level Recorder

or of any particular point of the wire can be used to find out the level of the liquid. If the friction at the pulley is small, this arrangement may yield accurate results.

Mechanical level recorders working on the float principle are now available. A typical example is shown in Fig 14 52.

D. Speed Measurement

14.29 Measurement of Speed—Revolutions per minute (N) of the shaft are measured by one of the following instruments :

i) Revolution Indicator,

ii) Tachometer,

iii) RPS or RPM Counter,

iv) Electrical or Optical Instruments.

In (*i*) revolutions are indicated on a dial but there is no indication of time for the measurement of which a stop watch is needed.

Tachometer gives speed in rpm while it is being driven, generally, by means of a belt from the shaft.

RPM Counter is a combination of (*i*) and (*ii*). Depending on the design, it takes time upto about three seconds to attain full speed after which the indicator needle stops, and the reading is preserved even if it is disconnected. Its working is similar to that of a stop watch.

Electric Tachometer comprises a generator with a constant permanent-magnet field and loaded only with high-resistance voltmeter provides an output voltage which is nearly proportional to speed. These machines are made in both A.C. and D.C. types.

Another type of electric tachometer consists of a contact maker driven by the shaft. The pulses generated by the contact maker build up charge in a condenser, and a voltmeter reads the EMF across the condenser plates. Power supply or batteries are essential to operate this type of electric tachometer.

A Swedish firm, Nydqvist & Holm (NOHAB), has developed an electronic revolution counter having a counting capacity of 30,000 pulses per second. The pulses are produced by means of a rotating disc.

UNSOLVED PROBLEMS

14.1 Define "Hydraulic Measurement."

A. Water Measurements

14.2 Name the different methods of water measurements. Which one of them is mostly used in practice ?

14.3 Name the different methods which could be employed for measuring the discharge of streams, rivers and channels. *(AMIE—Nov 1954)*

14.4 Describe "Direct Methods" of discharge measurement.

What is meant by "Indirect Method" of discharge measurement ?

14.5 Why Direct Weight Measurement method is the most accurate one ? Describe this method briefly.

14.6 Describe Calibrated Tank and Travelling Screen methods for discharge measurement.

14.7 Show by means of a sketch the difference between a Venturimeter and a Venturiflume. Can a Venturimeter be used in its vertical position ? *(AMIE—Nov 1954)*

14.8 Why divergence piece of a Venturimeter is longer than its convergence piece ? What precautions are taken to determine the throat diameter of a Venturimeter ?

14.9 What is a Venturiflume ? State the equation, giving discharge through such a flume and say what precautions are necessary for this to work properly. *(UPSC—Dec 1958)*

14.10 Why is it necessary to connect an additional pipe perpendicular to the flow in case of Pitot tube ? How does this construction effect the instrument co-efficient ?

14.11 What is a Prandtl-Pitot tube ? Sketch the same. *(Punjab University—1957) and (Delhi University—1957)*

14.12 What is meant by Pitot-tube Traversing ? Explain how is it carried out.

14.13 Deduce the formula for discharge through an orifice and indicate what factors are taken care of by the co-efficient employed in it. *(UPSC—Sept 1957)*

14.14 What are quantity meters and where are they used ?

14.15 Where would you employ the following in practice to measure the rate of flow of water ?

a) Turbine Nozzle
b) Travelling Screen
c) Pipe Bend
d) Calibrated Tank
e) Orifice
f) Direct Weight Measurement
g) Rotameter
h) Gibson Method.

Describe each of the above methods briefly.

14 16 Describe the following methods to measure the water discharge :

a) Rod method for river flow
b) Orifice or nozzle for pipe flow
c) Weir for open channel flow (*Delhi University—1958*)
d) Travelling screen
e) Double float method for river flow
f) Titration method for the stream having excessively turbulent flow
g) Current meter for pipe flow
h) Turbine nozzle for flow through Pelton turbine.

(*Punjab University—1958*)

14.17 What is the difference between surface float, sub-surface float and rod float used for measuring the discharge of a river ? How is the average velocity of water determined in such cases, if the water surface velocity is given ?

14.18 What is a suppressed weir with end contraction ? Where is it necessary to use such a weir in practice ? (*AMIE—May 1955*)

14.19 Where do you recommend the use of a triangular notch in practice ?

14 20 Why is the ventilation of rectangular weir necessary ?

(*AMIE—May 1955*)

14 21 What are the limitations of Rehbock's weir formula ?

14.22 Describe a "Cippoletti Weir." How does it differ from rectangular sharp crested weir ? (*UPSC—1958*)

14.23 How will you avoid the effect of end-contractions in case of rectangular weir ?

14.24 Draw typical calibration curve (Q vs H) of a rectangular weir.

(*AMIE—May 1955*)

14.25 Describe discharge measurement of canal by current meters.

(*AICTE—1958*) ; (*AMIE—Nov 1958*) *and* (*Punjab University—1959*)

14.26 What do you know of the following ?

a) Pitot tube *b*) Current meter. (*AMIE—Nov 1953*)

14.27 Describe how the measurement of discharge is carried out by means of current meters.

What is the approximate number of gauging points for a canal of rectangular section 20 ft wide by 10 ft deep ? Show how you would place the current meters at these gauging points.

How would you calculate the average discharge from the different readings obtained from the meters ? (*AMIE—Nov 1954*)

14.28 Describe the Titration and Salt Velocity (Allen) methods.

14.29 Describe the method of measuring quantity of water in a stream where the flow is excessively turbulent. (*AMIE—May 1955*)

14.30 Describe the salt-velocity and the Gibson pressure method for discharge measurement. Where are such methods employed and why ? (*AMIE—Nov 1958*) *and* (*Punjab University—1957*)

14.31 Describe a suitable method of gauging the flow of water in—

a) a river with a maximum discharge of 4 lakhs cusecs,

b) a distributory canal for 100 cusecs,

c) a laboratory channel carrying 0·5 cusecs.

(*UPSC—1958*)

14.32 Describe the difference between the thermometric and thermodynamic methods of discharge measurement.

14.33 Describe ultrasonic flowmeter.

14.34 The diameters of main and throat of a horizontal Venturimeter are 3 in. and 2 in. respectively. It is employed to measure the flow of kerosene having a specific gravity 0·8. If the gauge pressures at inlet and throat are 100 and 80 lb/sq in. respectively, determine the discharge through the meter. Assume the meter co-efficient as 0·97. (1·44 cfs)

14.35 A right-angled triangular notch, and a sharp-edged rectangular weir 30 cm broad, are to be used alternatively for gauging the flow estimated to be about 20 litres per second. Find in each case, the percentage error in computing the discharge that would be introduced by an error of 2 *mm*, in observing the head over the notch and the weir. Take C_d the co-efficient of discharge for the notch and the weir as 0·593 and 0·623 respectively.

(2·74% same in both cases) (*UPSC—Jan 1952*)

14.36 The rate of discharge over a V-notch is given by $Q = CH^n$, where C and n have constant values. Show that, for small changes, the fractional change in discharge is n times the fractional change in head.

A flow of 1·25 cusecs is to be measured with an accuracy of $\pm 2\%$, Take $Q = 2{\cdot}48\ H^{2{\cdot}18}$, find the permissible error in observing the head. $\left(\frac{\delta Q}{Q} = ;\ \frac{\delta H}{H},\ \pm 0{\cdot}073 \text{ in.}\right)$

14.37 A Cippoletti (trapezoidal) weir having a width of 16 in. is used to measure the discharge of water from a small water turbine. The weir was fixed in a rectangular channel 2 ft wide and 18 in. deep. Calculate the rate of flow in gpm if the water level in the channel is 9 in. above the weir crest. ($C_d = 0{\cdot}63$)

B. Pressure Measurements

14.38 What is the difference between "Gauge Pressure" and "Absolute Pressure" ?

14.39 What is the difference between a Piezometer tube and a differential manometer ?

14.40 Where do you employ a mechanical gauge to measure the pressure ?

14.41 With which instrument will you measure the pressure in the suction pipe of a pump ?

14.42 Describe with the help of a line sketch how a Bourden tube pressure gauge works.

14.43 What are the precautions you will take while making connections for connecting a pressure gauge to a pipe line ?

14.44 Can you use a dead-weight pressure gauge instead of a Bourden tube pressure gauge to measure the head of the water flowing under pressure in a pipe line ?

14.45 It is a standard practice when using a dead-weight tester to rotate the weights and piston and to tap the gauge lightly while obtaining readings. Explain why.

14.46 What is manometric piston gauge ?

C. Level Measurements

14.47 Explain the difference between a pointer gauge and a hook gauge.

14.48 Explain with the help of a sketch the working of a mechanical float.

D. Speed Measurements

14.49 What are the different arrangements to measure the speed of a machine ?

14.50 What is the difference between a tachometer and a rpm counter ?

14.51 A centrifugal tachometer having ranges of 25 to 300, 250 to 3,000 and 2,500 to 30,000 rpm is to be used to determine the speed of a shaft which is believed to be in the vicinity of 3,000 rpm. What range setting should be tried first and why ?

14.52 How does the electronic revolution counter work ?

14.53 Describe the principle of Electric Tachometer.

CHAPTER 15

TESTING & CHARACTERISTICS OF TURBINES & PUMPS

15.1 Introduction—Various tests which are required to be carried out on the turbines and pumps are given in this Chapter. The tests are made in accordance with the different specifications given in the test codes of various countries.

The measurement of necessary data for carrying out the various tests have been given in different chapters of the book. The measurements of discharge, pressure or head, level of water and speed of the machines, have been given in Chapter 14. From these data power and efficiency of the units are determined which will be explained in Chapter 15 B.

The data obtained above are generally represented in the form of curves, known as characteristic curves. The requirements of various curves and their plotting are explained in Chapter 15C and 15D.

A. Testing of Turbines & Pumps

15.2 Purposes of Tests—There are generally five purposes for which the tests are made. The type of test will depend upon its purposes.

(1) *Investigation of Losses*—In order to find out the efficiency of units, the different losses such as mechanical and hydraulic losses are determined first.

(2) *Design and Research*—These tests give valuable information to the designer and research worker regarding the performance of the units under different conditions. Such conditions include the varying of load, speed and head. The theory can be better understood by these tests, which are of two types *viz model tests* and *acceptance tests*.

(3) *Determination of Results Under Specified Conditions*—These tests are carried out at site and are called acceptance or take-over tests. They are undertaken to see that guarantees given by the manufacturers in the contract are fulfilled. Such tests comprise the determination of efficiency for given head and speed.

(4) *Determination of Best Operating Conditions*—These tests are employed to find out the best efficiency at different conditions of speed, load and head. They are therefore, to be performed in the neighbourhood of best efficiency.

(5) *Sale of Machinery*—The tests are performed before the prospective buyer, to give him an idea of the performance of the machine. The results of these tests are recorded in booklets which serve as a handy reference to the sales engineer.

15.3 Testing Codes—Tests, whether in the field or in the laboratory, are based on the same fundamental principles. These principles and the rules of procedure are formulated in the standard test codes of various countries.

These codes have much in common. They give definitions of terms used ; the principles of testing ; the requirements and conditions of testing ; method of taking necessary measurements ; description of measuring instruments and notes on their calibration ; tolerances on measurements and advice for the solution of the other points of dispute which may come up between the purchaser and the manufacturer. Codes of different countries differ mainly in the extent and presentation of the matter. Each is a guide and reference to the Engineer. When referred to in the specifications given by the purchaser, it becomes a contract document. These codes can be modified by the purchaser and the manufacturer between themselves.

Testing codes may go a little further in apportioning the cost of these tests between the buyer and the supplier. The buyer sends his representative to the manufacturing works to watch the model tests conducted in their laboratories and the manufacturer has to send his engineering personnel and staff for testing at site.

On the testing of water turbines, the best known codes in the English language are :

i) Swiss Rules for hydraulic turbines, published by the Swiss Electrotechnical Institution.

ii) The British Standard Test Code for hydraulic turbines, published by the British Standards Institution.

iii) The Testing Code of the Machinery Builders Society U. K.

iv) The Testing Code for hydraulic prime movers, published by the American Institute of Mechanical Engineers.

v) The Standard Test Code for Hydraulic Power Plants issued by the authority of the Councils of the Institution of Civil Engineers and the Institution of Mechanical Engineers, London.

15.4 Acceptance or Take-Over Tests are those which are performed at the site before the unit is commissioned for service. They are undertaken to see that the guarantees given by the manufacturers in the contract are fulfilled. During erection, pressure tests are undertaken to check the strength of the individual parts. Before commissioning a turbine, a number of tests, both mechanical and electrical are carried out on the turbine and its auxilliaries. The intake and penstock are cleared of timber and other trash capable of causing damage. The governor and lubricating oil systems are cleared of any dirt or foreign matter that may have found its way in. All equipment is checked for operation and when found satisfactory, water is allowed to come in the turbine and the set is gradually brought up to full speed. This is done first by hand and then by the governor.

The set is subjected to a load test. The load may be a commercial one or a water-resistance. The load is varied according to the specifications and the behaviour of the set studied.

The governor performance of sudden rejection of loads is studied. Loads at the fractions *viz.*, 25, 50, 75, 100 per cent of the full-rated output are built-up and when conditions are steady, suddenly thrown off, the pressures and speeds being recorded during and after the rejection. The pressure rise and speed rise must remain within the specified tolerances.

The overspeed device, for shutting down in the event of racing is tested by manipulating the governor so as to run the machine upto the maximum speed, which is generally one-third more than the rated speed.

15.5 Model Tests—Such tests are conducted in laboratories on small models of the big units. They are necessary to obtain complete data about the performance of turbines or pumps, before they are manufactured. Tests on actual-sized units are very much restricted in their scope. The limitations may be due to the cost of the tests and due to time, in case the plant is to be commissioned at an early date. In addition to these, there are hydraulic limitations. Head may not be variable, the speed may be constant and the load available may not be very steady which is desirable for accurate results.

Models are tested under a small head. This head is created by pumping units. It is desirable to use high head for impulse turbines and low head for reaction turbines. Generally the test turbines are of such a size that the BHP lies between 5 and 50.

The discharge is measured by any one of the methods explained in Chapter 14A. The output is absorbed by any of the brakes to be explained in Chapter 15B. The prony brake still gives the best results at variable speeds and torques.

The test consists in running the turbine for a given position of its opening, from standstill to maximum runaway speed and measuring

the data for each speed, when steady conditions prevail. The efficiency is calculated from the above data. The computation of different data and drawing of characteristic curves will be explained in Chapter 15C. The various values for the actual-sized turbine are calculated by the principles of geometrical similarity explained in Chapter 9.

For pumps, the model tests are carried out for large units in which case their performance data are required before they are actually manufactured. The computation of such data and drawing of characteristic curves are given in Chapter 15D.

15.6. Test Beds—The testing of model water turbines and pumps is carried out in a laboratory, specially constructed for the purpose in the manufacturer's works. Each type of turbine and pump require different type of test-beds. The type of test will depend on the purpose of test explained in Art 15.2. Some of the test-beds are given in the following articles. A few more test-beds such as aerodynamic test-beds, cavitation test-beds are also very important for the research engineer.

Aerodynamic test-bed is used for the Kaplan turbine employing a wind tunnel, where air is used instead of water. Such test-beds have become very common now-a-days, as it is very convenient to work with air because both pressure and velocity can be measured more easily at numerous points with which all-round observations are possible since the runner is not submerged in water as in the case of hydraulic experiments.

With cavitation test-bed the development of turbine runner blade profile is possible. Photographs are taken with the help of stroboscope, with an exposure upto $\frac{1}{200{,}000}$ sec, to see whether the runner is effected by cavitation.

15.7 Test Bed for Pelton Turbine—Fig 15.1 shows the test bed used for testing Pelton turbine runner by Escher Wyss & Co Ltd Zurich.

The pump 1 (See Fig 15.1) of centrifugal type supplies the required head and quantity of water from sump 6. The quantity of water is

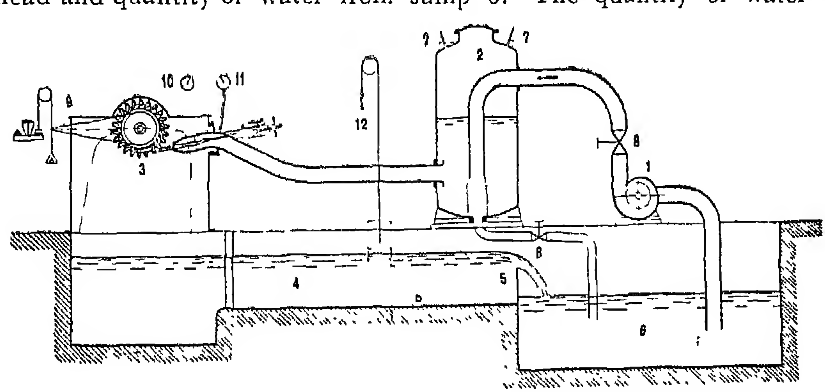


Fig 15.1 Test Bed for Pelton Turbine (Escher Wyss & Co Ltd.)

1. Centrifugal Pump
2. Air Vessel
3. Pelton Wheel
4. Measuring Channel
5. Measuring Weir
6. Sump
7. Compressed Air Vents
8. Delivery or Regulating Valve
9. Dynamometer or Brake
10. Tachometer
11. Pressure Gauge
12. Float

regulated by valve 8. The water is passed through a pressure vessel 2 which has a compressed air on the top surface of water. The compressed air is supplied by a compressor (not shown in Fig) through the pipes 7. With the help of air vessel, working head of the turbine is made constant. The head is varied with the help of compressor. If the pump is connected to the turbine without air vessel, the fluctuation of pressure takes place. In this case the working head of the turbine is made constant by adjusting the delivery valve 8 of the pump or by varying the speed of electric motor driving the pump. The head before the water emerges out of turbine nozzle is recorded by a pressure gauge 11. The Pelton runner 3 is fitted on the open tank. The speed of wheel is measured by a tachometer 10 and the torque is measured by a brake 9. The water after giving the work to the Pelton runner falls in the channel on the tail side 4. It flows back to the pump sump over a weir 5. The head of water over the weir is measured by a float 12.

15.8 Test Bed for Reaction Turbine—Fig 15.2 shows the test bed used for reaction turbines, both Francis and Kaplan type, by Escher Wyss.

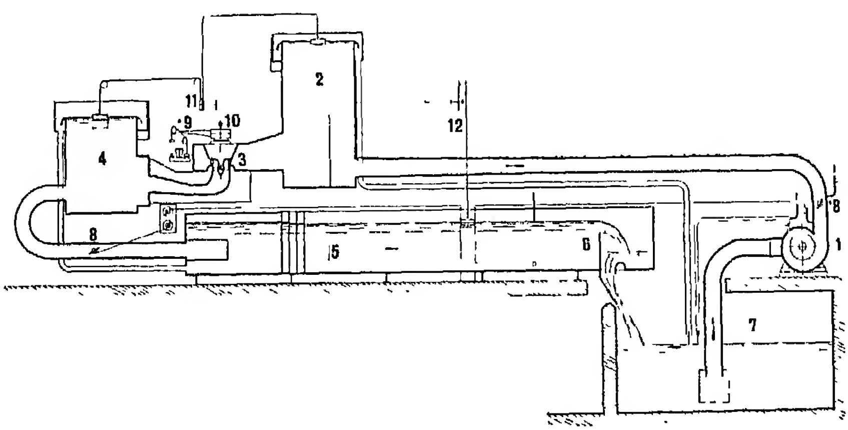


Fig 15.2 Test Bed for Reaction Turbines (Escher Wyss & Co Ltd.)

1. Centrifugal Pump	5. Measuring Channel with Stilling Devices	9. Dynamometer or Brake
2. Head Race Tank	6. Measuring Weir	10. Tachometer
3. Reaction Turbine	7. Sump	11. Head Measuring Scale
4. Tail Race Tank	8. Delivery or Throttle Valve	12. Float

The pump 1 of centrifugal type supplies the required head and quantity of water from the pump sump 7. The water is regulated by the throttle valve 8. The water enters the head race tank 2 which supplies water to the turbine 3. The speed of turbine is measured by tachometer 10 and the torque by brake 9. The water after doing the work passes on to the tail race tank 4 through the draft tube. The difference of water level in the two tanks 2 and 4, gives the head working on the turbine, which is measured by scale 11. The throttle valve regulates the supply for the tail race side. The water flows to the channel 15 which has stilling devices and at the end there is measuring weir 6. The head on the weir is measured by float 12.

15.9 Test Bed for Pumps—The pump sump is covered with a platform which is made generally of wooden sleepers and angle irons,

The pump to be tested is fixed over the sleepers. The discharge is measured by calibrated tank (cf Art 14.4) if the pump is small. In case of large-sized pump, the water is passed through a channel having a measuring weir (See Art 14.14) at its end. The head over the weir is measured by hook gauge (See Art 14.28 ii). The suction head is measured by piezometer tube dipped in a mercury trough. The delivery head is measured by dead weight pressure gauge (cf Art 14.26 iii) or piezometer tube (See Art 14.24 i). The type of fluid used in piezometer tube depends upon the delivery head. If the head is more, mercury is employed, otherwise fluid whose specific gravity is near about two is preferred to give a fairly long column in the tube, in order to have smaller percentage error.

Driving motor of D.C. type or variable speed A.C. type, is used for the tests, so that a wide range of speed variation can be obtained. The driving motor is provided, for the loading, with either mechanical dynamometer or fitted with ammeter and voltmeter. In the later case the efficiency of electric motor must be known in order to get the input to the pump.

No change in pipe diameters or directions of flow should occur at the places where the gauges are located, otherwise the readings will not be correct. All readings on suction and discharge piping before and after pump flanges must be referred to centre line of pumps. When the suction diameter is larger than the discharge diameter, a correction factor, $\frac{v_d^2 - v_s^2}{2g}$, for the velocity head must be made at all capacities and must be added to the total pressure readings.

The speed of the pump is measured by a tachometer or RPM-counter (See Art 14.29) from the shaft of driving motor.

15.10 Determination of Total Head—

a) **Turbines**—The net head for different types of water turbines is given in Fig 15.3 to 15.6, taken from the Swiss Rules for Hydraulic Turbines, SEI Publication No. 178e, 1947.

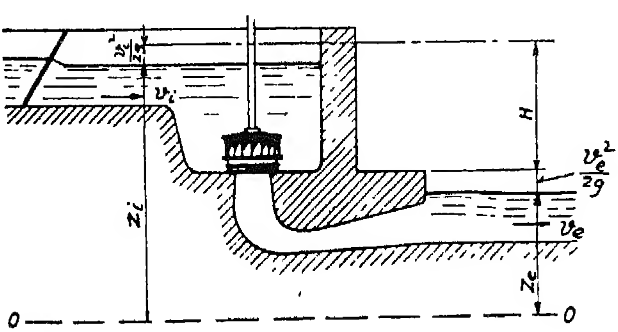


Fig 15.3 Reaction Turbine
Open Flume, Vertical Shaft, Draft Tube Bend

$$\text{Net Head } H = z_i - z_e + \frac{v_i^2 - v_e^2}{2g}$$

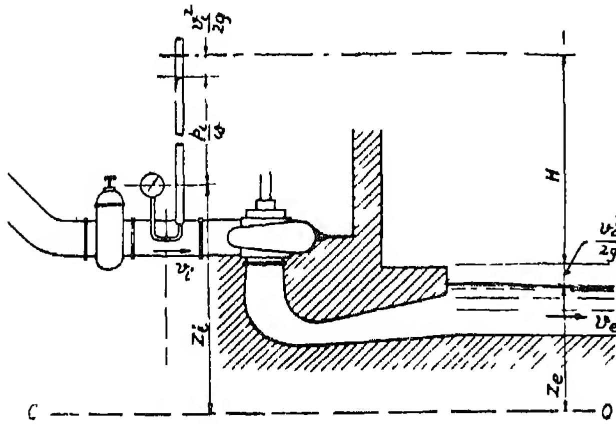


Fig 15 4 Reaction Turbine
Spiral Casing, Vertical Shaft, Draft Tube Bend

$$\text{Net Head } H = z_i - z_e + \frac{p_i}{w} + \frac{v_i^2 - v_e^2}{2g}$$

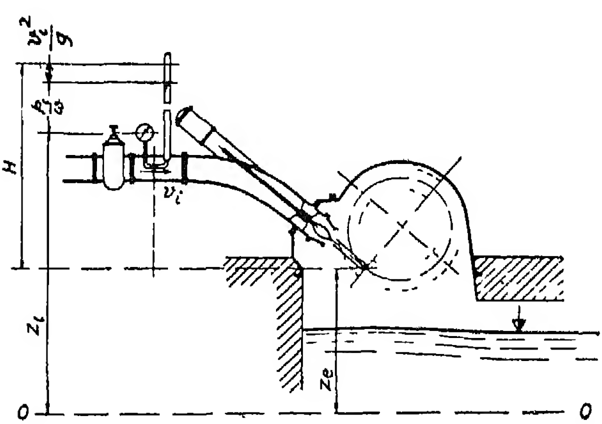


Fig 15 5 Impulse Turbine (Pelton)
Single Nozzle, Horizontal Shaft

$$\text{Net Head } H = z_i - z_e + \frac{p_i}{w} + \frac{v_i^2}{2g}$$

(For Net Head H given below See Fig 15 6 on page 497).

$$\text{Net Head } H = \frac{Q\left(z_{i_1} - z_{e_1} + \frac{p_{i_1}}{w} + \frac{v_{i_1}^2}{2g}\right) + Q_2\left(z_{i_2} - z_{e_2} + \frac{p_{i_2}}{w} + \frac{v_{i_2}^2}{2g}\right)}{Q_1 + Q_2}$$

b) **Pumps**—The total dynamic head H for different types of connections of pressure gauges fitted to the suction and delivery ends of pump piping is given in Fig 15.7. The precautions must be taken that *a*) The gauge connection pipes are full of water before the gauges are fitted as shown in Fig 15.7*a* and *b*,

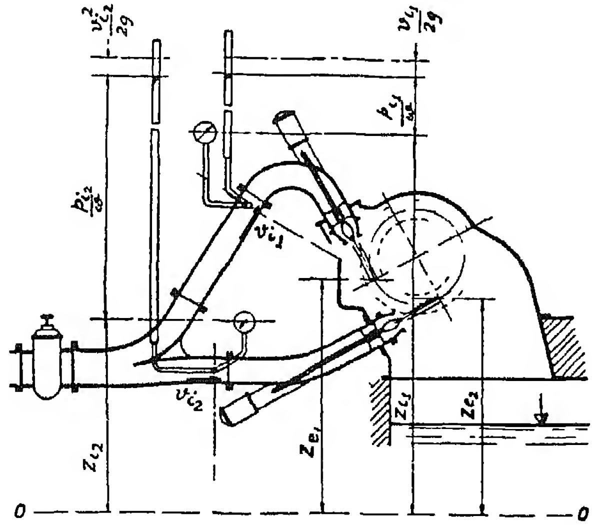


Fig 15 6 Impulse Turbine (Pelton)
Twin Nozzle, Horizontal Shaft (*For Net Head H See Page 496*)

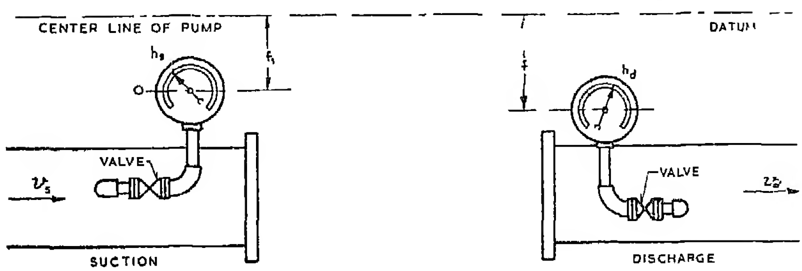


Fig 15.7 *a*) $H=h_d-h_s-f+f_1+\dfrac{v_d^2-v_s^2}{2g}$

Suction head at datum $h_s'=h_s-f_1$

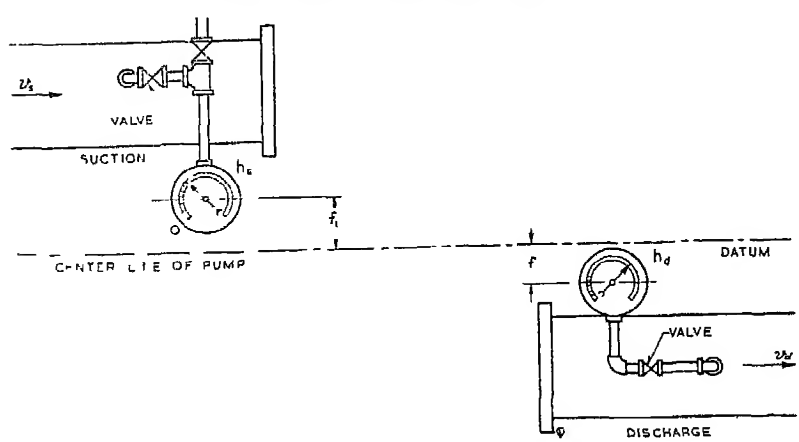


Fig 15 7 *b*) $H=h_d-h_s-f-f_1+\dfrac{v_d^2-v_s^2}{2g}$

Suction head at datum $h_s'=h_s+f_1$

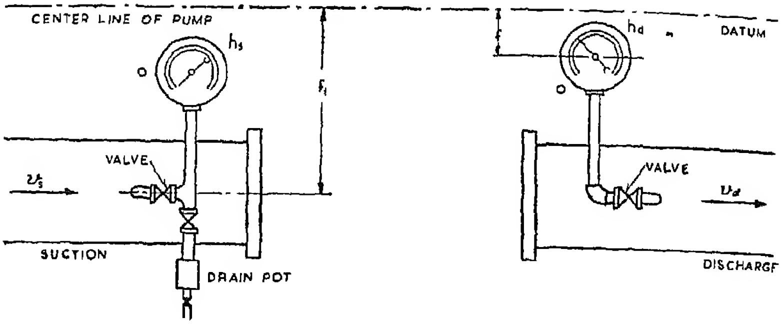


Fig 15.7 c) $H = h_d - h_s - f + f_1 + \frac{v_d^2 - v_s^2}{2g}$
Suction lift at datum $h_s'' = h_s + f_1$

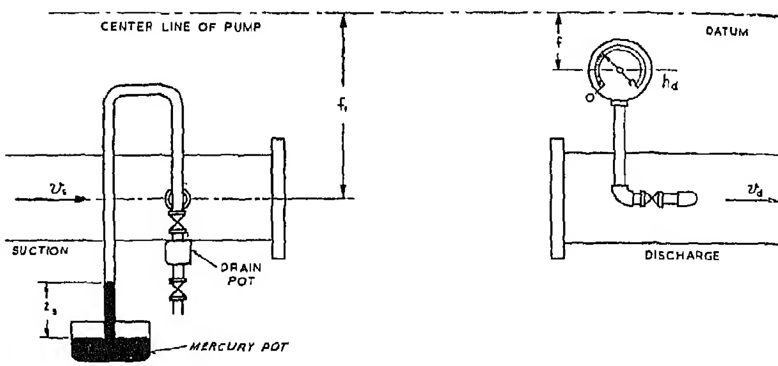


Fig 15.7 d) $H = h_d + s \cdot z_s - f + f_1 + \frac{v_d^2 - v_s^2}{2g}$
where s = sp gr of mercury
z_s = ft (or m) of mercury
Suction lift at datum $h_s'' = s \cdot z_s + f_1$

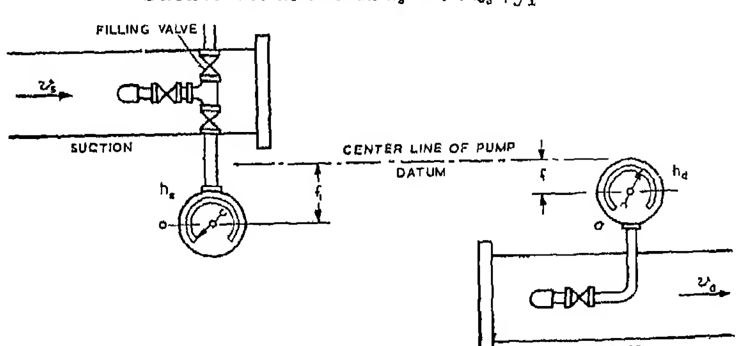


Fig 15.7 e) $H = h_d - h_s - f + f_1 + \frac{v_d^2 - v_s^2}{2g}$
Suction lift at datum $h_s'' = -h_s + f_1$

b) in Fig 15.7*c* discharge gauge connection pipes are full of water and suction gauge connection pipes are filled with air and open drain valve on suction connections occasionally,

c) in Fig 15.7*d* and *e*, the discharge gauge connection pipes are full of water.

15.11 Procedure of Testing for Turbines in a Laboratory— The turbines in the field must be supplied with natural flow of water (Q) under some head (H). However in a laboratory both Q as well as H are obtained by a suitable pumping unit. The pump draws water from a sump or underground reservoir of sufficient capacity, and delivers it under a specified pressure (H) to the turbine as explained above.

The pump is started as explained in Art. 15 15. The delivery valve 8 (see Fig. 15.1 aad 15.2) of the pump is opened fully in order to supply water to the turbine. The inlet valve to the turbine, if any, as well as the turbine nozzle (for Pelton turbine) or turbine gates (for reaction turbine) are opened. Let the turbine run for fifteen to twenty minutes before the readings are recorded.

Open the brake cooling water tap. Allowance for this water is made for the accurate calculations of turbine discharge.

The turbine is set for a number of inlet openings, atleast four, say 25, 50, 75 and 100%. For each inlet opening, the turbine is first run on no-load and then it is loaded slowly by means of dynamometer, till the turbine is at standstill. Not less than six different loadings for a single inlet opening, are to be recorded.

The head operating the turbine is kept constant as far as possible during testing. As explained in Art. 15.7 and 15.8 this is done by the help of air vessel (for Pelton turbine) or water tanks (for reaction turbine). In the absence of air vessel or water tanks, the head is kept constant by adjusting the delivery or regulating valve of the pump or by varying the speed of the driving motor.

15.12 Data to be Measured—

I. *The following data are measured while the experiment is being performed*—

a) **Turbine Gauge Pressure**—This is obtained from the pressure gauge 11 (see Fig. 15.1 and 15.2) connected to the turbine. The pressure gauge gives the values in ft (or m) of water or in lb/sq in. (or kg/cm^2).

b) **Brake Load** from the meter connected for the purpose. For the rope brake (see Art 15.20), measure the load from the spring balance.

c) **Turbine Speed** is measured by means of tachometer or RPM-counter (see Art 14.29) by inserting its knob in the centre hole made on the end of shaft. A tachometer may be connected to the turbine by means of belt.

d) **Head over Weir Crest** is measured by means of hook gauge (see Art 14.28 ii).

TABLE 15.1

Turbine Brake Test

Type of Turbine........................ Dated.......................

Turbine Specifications—

Manufacturers ....

Head H=......... Discharge Q=......... Output P_t=............, Working Speed N_o=..............

Overall Efficiency η_t=............ Specific Speed N_s=............, Runaway Speed N_r=..............

a) Internal diameter d of pipe at

Pressure Gauge Connection=	
Vacuum Gauge Connection=...	...

$\therefore \dfrac{v_i^2-v_e^2}{2g}=\dfrac{Q^2}{2g\times\left(\dfrac{\pi}{4}\right)^2}\left(\dfrac{1}{d_i^4}-\dfrac{1}{d_e^4}\right)=......Q^2$

b) Vertical Distance from Centre Line of Turbine to Gauge, Z_{gauge} (for pressure gauge)=..............,

Z_{gauge} (for vacuum gauge)=..............

$\therefore$ Effective Turbine Head $H=(H_{gauge}\pm H_{cor})+(H_{vac}\pm H_{cor})+\dfrac{v_i^2-v_e^2}{2g}+Z_{gauge\ (pres)}+Z_{gauge\ (vac)}$

[H_{vac}, $Z_{gauge\ (vac)}$ and v_e are for reaction turbine only]

c) Radius of Brake Arm l=..............

d) Type of Weir=................. Length of Weir Crest=.................

Crest Level of Weir=................. Q=................. (Write the formula used)

e) Any Other Type of Discharge Measurement

Inlet Opening=..............%

Point	Head in ft (or m)						Discharge in ft³/sec (or m³/sec)		Dynamometer (Brake)		Speed		Power in HP or Metric HP	
	Pressure Gauge			Vacuum Gauge			Head Over Crest							
	H_{gauge} ft (or m)	H_{cor} ft (or m)	$\frac{v^2}{2g}$ ft (or m)	H_{vac} ft (or m)	H_{cor} ft (or m)	H ft (or m)	h ft (or m)	Q ft³/sec (or m³/sec)	F lb (or kg) [For Spring Balance $F=W-S$]	Torque (if any) ft lb (or kgm)	Tacho-meter RPM	Correct RPM	P_a	P_t (BHP)

Tested by.......................... ...

II. *The following data to be measured are required for the calculations explained in the next Article 15.13*

a) **Internal Diameter** d of the pipe at a place where turbine pressure gauge is connected. For a reaction turbine internal diameter of the pipe at a place where vacuum gauge is connected, is also to be noted. These diameters are required to calculate the kinetic head of the flowing water in order to determine the effective head working on the turbine.

b) **Vertical Distance** Z_{gauge}, if any, from the centreline of the turbine inlet to the point where the pressure gauge is connected. This is also required to determine the turbine effective head. In case of reaction turbine the z_{gauge} distance for the vacuum gauge is also be taken into account.

c) **Radius of Brake Arm** l (see Fig. 15.8) from the centreline of the turbine shaft to the point of application of weight. This is required to calculate the torque applied on the turbine. Sometimes the mechanical dynamometer is such that it is fitted with a meter showing directly the torque to be calculated.

d) **Length of Weir Crest** B or L (See Fig 14.18 and 14.20) is measured if the weir used is rectangular or trapezoidal (*ie.* Cippoletti weir).

e) **Weir Crest Level** is measured by the hook gauge after the experiment has been performed and no more water is flowing over the weir.

III. *The following instruments are to be calibrated before they are used to perform the experiment—*

a) **Turbine Pressure Gauge and Vacuum Gauge,** the latter for reaction turbine only. It is required to calculate H_{cor} for the determination of effective head of the turbine.

b) **Tachometer or RPM-Counter** to determine the correct speed.

c) **Brake Load Meter, Torque Meter or Spring Balance.**

All the above data is tabulated for each inlet opening as shown in Table 15.1.

15.13 Calculated Data at *Constant* Head—

a) **Turbine Effective Head** H—

$$H=(H_{gauge}\pm H_{cor})+(H_{vac}\pm H_{cor})+Z_{gauge}\,(_{pres})+Z_{gauge}\,(_{vac})+\frac{v_i^2-v_e^2}{2g}$$

where H_{gauge}=Reading of pressure gauge in ft (or m) of water

H_{vac}=Reading of vacuum gauge in ft (or m) of water. This will be required in case of reaction turbine. The vacuum gauge is fitted to the outlet of turbine runner or inlet of draft tube.

H_{cor}=Correction factor for the gauge, if any. This will be known after the gauge has been calibrated.

v_i=Inlet velocity of water at the pipe cross-section where the pressure gauge is fitted.

v_e = Exit velocity of water at the pipe cross-section where the vacuum gauge is fitted. This is required for reaction turbine only.

$Z_{gauge\,(pres)}$ = Vertical distance from the centre line of turbine and the point where the pressure gauge is fitted.

$Z_{gauge\,(vac)}$ = Vertical distance from the turbine centre line to the point where vacuum gauge is fitted. This is required for reaction turbine only.

b) **Discharge** Q—The discharge is calculated by means of weir or Venturimeter or by one of the methods explained in Chapter 14. In case of weir, use the Eqns 14.14 to 14.20, depending upon the type of weir used.

c) **Torque or Turning Moment** M—

$$M = F \cdot l \quad \text{ft lb (or kg m)}$$

where F = Brake load in lb (or kg) from the meter connected to the brake. In case of rope brake, this is equal to difference of dead weight on brake and reading of the spring balance,

i.e. $F = W - S$

l = Radius of brake arm.

In some cases a meter fitted on the brake will give directly the value of moment M in ft lb (or kg m).

d) **Brake Horsepower (BHP) or Metric HP from Brake**— The turbine brake horsepower is determined as follows—

$$P_t = \frac{M \cdot \omega}{550} = \frac{F \cdot l \cdot \omega}{550} = \frac{F \cdot l \cdot 2\pi \cdot N}{60 \times 550} \text{ BHP}$$

$$\left[\text{or } P_t = \frac{F \cdot l \cdot 2\pi \cdot N}{60 \times 75} \text{ metric HP}\right]$$

where N = Speed of turbine shaft in rpm

e) **Available or Water Horsepower**

$$P_a = \frac{w \cdot Q \cdot H}{550} \text{ HP}$$

$$\left[\text{or } P_a = \frac{w \cdot Q \cdot H}{75} \text{ metric HP}\right]$$

f) **Turbine Overall Efficiency**

$$\eta_t = \frac{P_t}{P_a}$$

Tabulate the above results as shown in Table 15.2.

TABLE 15.2

Turbine Brake Test

Turbine Calculated Data at *Constant* Head

Outlet Opening=...... .. %   Head=........ ft (or m)

Point	H	Q	N	F	M	P_a	P_t	η_t
	ft (or m)	cusecs (or m³/sec)	rpm	lb (or kg)	ft lb (or kg/m)	HP (or metric HP)	BHP (or metric HP)	%

15.14 Calculated Data at *Unit* Head—It will be seen that for different inlet openings of the turbine, the constant head (See Table 15.2) has not got the same value. Therefore it is the usual practice to convert all the data given in Table 15.2 to a unit head which is then the constant head for all the values of inlet openings. Thus the different quantities given in Table 15.2 for the different inlet openings can then be easily compared with each other. To reduce the quantities to unit head, the following relations (See Art 3.13) are used

$$Q_1=\frac{Q}{\sqrt{H}}, \qquad N_1=\frac{N}{\sqrt{H}}, \qquad M_1=\frac{M}{H},$$

$$P_{a_1}=\frac{P_a}{H^{\frac{3}{2}}}, \qquad P_{t_1}=\frac{P_t}{H^{\frac{3}{2}}}$$

η_t has a non-dimensional value, therefore it will not change. Tabulating the results—

TABLE 15.3

Turbine Brake Test

Turbine Data at *Unit* Head

Inlet Opening=......%   Head=1 ft (or m)

Point	Q_1	N_1	F_1	M_1	P_{a_1}	P_{t_1}	η_t

The characteristic curves (See Art 15.30) are then drawn (See Fig 15.20 to 15.24).

15.15 Procedure of Testing for Pumps in a Laboratory—Before the pump is tested check the following :—

a) The shaft of pumping unit is not jammed—This is necessary so that the electric motor may not be burnt when it is switched on in case the shaft is jammed. It is checked by revolving the shaft-coupling by hand.

b) The delivery valve of the pump is closed, so that pressure can be built up by the pump.

c) The pump is primed properly—The pump will not build up its head if there is an air pocket in the pump impeller, volute or suction pipe. The priming is accomplished by opening the air valve and filling the pump with water until it overflows from the air valve. The pump is filled with water either by pouring it with buckets or from an overhead reservoir, through the delivery pipe.

Switch on the driving motor and bring it to speed gradually by moving the handle of the starter slowly. The air valve is kept open partially while the motor is brought to its required speed, so that any residual air may escape. In case there is air and the pump is not properly primed on opening the delivery valve, the delivery pressure (shown by the pressure gauge) will fall to a large extent and the suction pressure (shown by the vacuum gauge) will rise. On such occasions, it is advisable to switch the motor off and start it afresh after priming it properly.

Note the readings of the pressure gauge, vacuum gauge, tachometer, voltmeter, ammeter, power factor meter (for A.C. motors only) and head over weir crest. In case the electric motor is coupled to a mechanical dynamometer, note the readings on the spring balance and measure the radius of the arm. All readings must be taken simultaneously. The axis of tachometer must be kept horizontal with the help of a level gauge while measuring the speed of pumping unit. The reading of head over the weir should be taken after the water has travelled from the pump to the weir, say after about 3 minutes.

The delivery valve is opened gradually and several sets of all the above readings are taken at constant speed. The speed is kept constant by varying the voltage of the motor by moving the rheostat.

Change the speed of the motor and repeat the above process at constant speed. Take such four or five different sets of readings at constant speeds which are required to draw the various characteristic curves of the pump (See Fig 15.27 to 15.31).

15.16 Data to be Measured—

I. The following data are measured while the experiment is being performed—

a) **Head from Pump Gauges**—This is obtained from the pressure gauge and vacuum gauge connected to delivery side and suction side respectively, both measured in ft (or m) of water.

b) **Pump Speed** is measured by means of a tachometer or RPM-counter.

c) **Power Consumption by Electric Motor** by measuring voltage, amperage and power factor (for A C motors only). In case the driving motor is equipped with mechanical dynamometer (say rope brake), measure the load from the spring balance.

d) **Head Over Weir Crest** by hook gauge.

TABLE 15·4

Pump Brake Test

Type of Pump.........................

Pump Specifications :

Manufacturers.................................

Head H_{mano}=..............................Discharge Q=............................ Input SHP=..........................

Overall Efficiency $\eta_{overall}$=.......................... Speed N=....................... Specific Speed N_s=..............

Driving Motor Specifications : HP=......................, DC/AC, Type..................., Voltage=..............

---

a) Suction Pipe Dia d_s= ....

b) Delivery Pipe Dia d_d= ....

$$\therefore \frac{v_d^2 - v_s^2}{2g} = \frac{Q^2}{2g} \times \left(\frac{4}{\pi}\right)^2 \times \left(\frac{1}{d_d^4} - \frac{1}{d_s^4}\right) = \ldots\ldots\ldots\ldots Q^2$$

c) Vertical Distance between Points of Connections of Suction and Delivery Gauges, Z_{gauge}=....................

$\therefore$ Effective Pump Head $H_{mano} = (H_d \pm H_{cor}) + (H_s \pm H_{cor}) + Z_{gauge} + \dfrac{v_d^2 - v_s^2}{2g}$

d) Radius of Brake Arm of Mechanical Dynamometer l=.........................

e) Type of Weir.................................. Length of Weir Crest=...................................

Crest Level of Weir=...........................Q =....................................(Write the formula used)

f) Any other Type of Discharge Measurement

g) Efficiency Curve of Driving Motor to get η_{motor}—(This curve must be ready before the experiment is performed)

Speed=..................RPM (approx)

Point	Head in ft (or m)						Discharge in cusecs (or m^3/sec)		Speed in RPM		Motor Input in KW				Pump Input in HP		Dynamometer		
	Delivery Gauge		Suction Gauge				Head Over Weir Crest												
	H_d ft (or m)	H_{cor} ft (or m)	H_s ft (or m)	H_{cor} ft (or m)	$\frac{v_d^2-v_s^2}{2g}=\ldots\ldots\ldots Q^2$ ft (or m)	H_{mano} ft (or m)	h ft (or m)	Q cusecs (or m^3/sec)	Tach RPM	Correct RPM	Amp	Volt	Cos φ (for A.C)	P_{kw}	η_{motor}	SHP	F lb (or kg)	Torque (if any) ft lb (or kgm)	SHP

II. The following data to be measured are required for the calculations explained in the next article (Art. 15.17).

a) **Internal Diameters** of the pipes where the gauges are fitted.

b) **Vertical Distance** Z_{gauge} between the centre lines of the pressure and vacuum gauges.

c) **Radius of the Brake Arm** l from the center line of the pump shaft to the point of application of weight.

d) **Lengths of Weir Crest** (B or L) (see Fig 14.18 and 14.20).

e) **Weir Crest Level** is measured when no more water is flowing over the weir, after the experiment has been performed.

All the instruments employed for the experiment *e.g.* gauges, tachometer and electric meters, spring balance (for rope brake) must be calibrated before they are used.

All the above data is tabulated for each *constant* speed as shown in the Table 15.4.

15.17 Calculated Data at Constant Speed—

a) **Pump Effective Head** H as well as *b*) **Discharge** Q are determined as explained in Art 15.13 *a* & *b* for turbines.

c) **Input to Pump—**

$$\text{SHP}=\frac{A\,.\,V}{1{,}000}\times\eta_{motor}\ \text{KW} \quad \text{or} \quad \frac{A\,.\,V}{746}\times\eta_{motor}\ \text{HP}$$

where A=Amperage

V=Voltage

η_{motor}=Efficiency of the motor taken from the efficiency curve of motor supplied by the motor manufacturers or obtained earlier by performing the experiment on motor.

Input to the driving motor is also obtained by mechanical dynamometer by calculating the torque.

$$T=F\,.\,l \text{ ft lb (or kgm)} \qquad \text{(Also see Art 15.13 C)}$$

and in case of spring balance

$$F=W-S$$

where W=Dead Weight

and S=Reading of spring balance

$\therefore$ Output of driving motor or Pump Input

$$\text{SHP}=\frac{2\pi\,NT}{33{,}000}\text{HP}$$

(*d*) **Output of Pump**

$$\text{WHP}=\frac{w\,.\,Q\,.\,H}{550}\ \text{HP (FPS—units)}\ \left[\text{or WHP}=\frac{w\,.\,Q\,.\,H}{75}\ \text{metric HP}\right]$$

(e) Overall Efficiency of Pump

$$\eta_{pump}=\frac{\text{Output}}{\text{Input}}=\frac{\text{WHP}}{\text{SHP}}$$

Tabulate the above results as shown in Table 15.5

TABLE 15.5

Pump Brake Test

Pump Data at *Constant* Speed

N= ... .rpm=constant

Point	H_{mano}	Q		N	WHP	SHP	P_{Lw} (motor)	$\eta_{overall}=\frac{\text{WHP}}{\text{SHP}}$
	ft (or m)	cusecs (or m^3/sec)	GPM (or lit/sec)	rpm	HP	HP	KW	%

Problem 15.1 In the test of a centrifugal pump with water, the rate of discharge was found to be 3,400 gallons per min (or 0·257 m^3/sec). Pressure gauge reads 190 lb/sq in. (or 13·35 kg/cm^2), suction gauge reads 6 in. (or 152·4 mm) of *Hg* and the difference in elevation between the gauges was 2·6 ft (or 0 793 m). The diameters of suction and delivery pipes are same. Find the efficiency of pump if input HP is 600 (or 609 metric HP). (*AMIE—Nov 1956*)

Solution

Q=3,400 gpm (or 0·257 m^3/sec)

H_d=190 lb/sq in. (or 13·35 kg/cm^2)

H_s=6 in. (or 152·4 mm) of *Hg*

Difference in gauges elevation=2·6 ft (or 0·793 m)

HP=600 (or 609 metric HP)

$$H_d=\frac{190\times 144}{62\ 4}=438 \text{ ft of water}$$

$$\left[\text{or } H_d=\frac{13\ 35\times 100^2}{1,000}=133{\cdot}5 \text{ m of water}\right]$$

$$H_s=\frac{6\times 34}{30}=6{\cdot}8 \text{ ft of water}$$

$$\left[\text{or } H_s=\frac{152{\cdot}4\times 10\ 36}{760}=2{\cdot}075 \text{ m of water}\right]$$

h=2·6 ft (or 0·793 m)

∴ Total head supplied by the pump

$$H=H_d+H_s+h=438+6{\cdot}8+2{\cdot}6=447{\cdot}4 \text{ ft of water}$$

(or $H = 133{\cdot}5 + 2{\cdot}075 + 0{\cdot}793 = 136{\cdot}368$ m of water)

$$\text{Power delivered by the pump} = \frac{w.\ Q.\ H.}{550}$$

$$= \frac{62{\cdot}4 \times \left(\dfrac{3400 \times 10}{62{\cdot}4 \times 60}\right) \times 447{\cdot}4}{550}$$

$$= \frac{3{,}400 \times 10 \times 447{\cdot}4}{60 \times 550} = 461 \text{ HP}$$

$$\left[\text{or } P = \frac{1{,}000 \times 0{\cdot}257 \times 136{\cdot}368}{75} = 467 \text{ metric HP}\right]$$

$$\therefore \quad \text{Efficiency of pump } \eta = \frac{\text{output}}{\text{input}} = \frac{461}{600} \times 100$$

$$= 76{\cdot}8\% \quad \textit{Answer}$$

$$\left[\text{or } \eta = \frac{467}{609} \times 100 = 76{\cdot}8\% \quad \textit{Answer}\right]$$

B. Power and Efficiency Measurements

15.18 Different Methods for Measuring Power and Efficiency—There are several methods employed for the measurement of power output, and, hence, for the calculation of efficiency of turbine models during tests in the laboratory or of actual turbines during take-over tests on the site. These are classified under two heads :

Methods using mechanical means and

Methods using electrical means.

The following types of mechanical equipment are used for the measurement of power and efficiency—

a) Prony Friction Brake

b) Rope Brake

c) Tesla Fluid-Friction Brake

d) Froude Water Vortex Brake

e) Torsion Dynamometer.

15.19 Prony Friction Brake—The prime mover shaft is braked by means of friction and by measuring the *rpm* developed against a known frictional force, the output of power can be easily calculated.

Apparatus consists of a simple pulley keyed to the prime-mover shaft. Clamps held together by adjustable bolts and flynuts brake the pulley by means of mechanical friction through cork padding. To the clamp is attached a lever the other end of which is balanced against known weights as shown in Fig. 15.8. This is used for small values of N (*rpm*) and higher values of F. It is suitable for tests in the laboratory.

If, F=peripheral force on brake lever

T=tare

G=weight on the pan of the balance

then, $F = G - T$

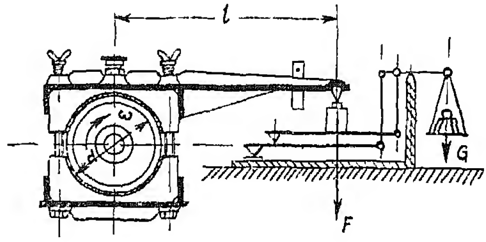


Fig 15 8 Prony Friction Brake

Let l=length of brake lever which is known for a particular brake then, torque$=F \cdot l$

Further, the angular velocity of pulley

$$\omega=\frac{2\pi \cdot N}{60}, \text{ where } N=\text{measured } rpm \text{ of shaft,}$$

and, Power, $P=(\text{Torque}) \cdot \omega=F \cdot l \cdot \frac{2\pi \cdot N}{60}$ ft lb/sec(or kgm/sec)

$$\therefore \qquad HP=\frac{P}{550}=\frac{F \cdot l \cdot 2\pi \cdot N}{60\times 550}$$

$$=\frac{F\ l \cdot 2\pi \cdot N}{33{,}000} \qquad \text{...(15 1)}$$

$$\left[\text{or } P=\frac{F \cdot l \cdot 2\pi \cdot N}{60\times 75} \text{ metric HP} \qquad \text{.. (15 1a)}\right]$$

Power in KW=Power in HP×0·746 . (15.1b)

or Power in KW=Power in metric HP×0 736 ...(15.1c)

Determination of Brake Test Curves—Power output expressed in HP,

$$P_{HP}=\frac{2\pi \cdot N \cdot F \cdot l}{33{,}000} \qquad \left[\text{or } P_{HP}=\frac{2\pi\ N \cdot F \cdot l}{60\times 75} \text{ metric HP}\right]$$

The factor $\frac{2\pi \cdot l}{33{,}000}$ $\left[\text{or } \frac{2\pi \cdot l}{60\times 75}\right]$ is a constant for a brake and

$$\therefore \qquad P_{HP}=\frac{F \cdot N}{K_1}$$

By adjusting the position of flynuts, mechanical friction on the pulley is varied and a series of values for F, *viz*, F_1, F_2, F_3 etc are chosen. Speed corresponding to each is measured with *rpm* counter Value of HP for each point is calculated. These are tabulated and a curve is drawn (See Fig 15 9). Also, since the power input P is known, efficiency η can be calculated.

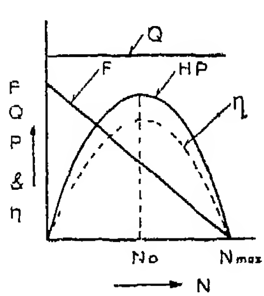


Fig 15 9 Brake Test Curves for Pelton Turbine (F, Q, P & η *vs* N)

The curve shown in Fig 15.9 belongs to a Pelton turbine. In this figure,

N_{max}=Runaway speed

and N_o = Speed for which the turbine is designed and where Power P would be maximum.

Problem 15.2 Find the values of F, the peripheral force on the lever on a Prony Brake, M the turning moment, P_t the turbine brake horsepower and η_t the total (overall) efficiency of the turbine from the following experimental data obtained from the characteristics of a Pelton wheel :

Total head = 80 97 m, Q = 55·24 lit/sec, N = 755 rpm
l = 1 43 m, Tare = 7·5 kg, Weights in pan = 39·9 kg

Solution

$F = G - T$

$= 39{\cdot}9 - 7{\cdot}5 = \mathbf{32\ 4\ kg}$ *Answer*

$M = F \,.\, l = 32\ 4 \times 1{\cdot}43 = \mathbf{46{\cdot}3\ kg\ m}$ *Answer*

$$P_t = \frac{2\pi \,.\, N \,.\, (F \,.\, l)}{60} = \frac{2\pi \times 755 \times 46{\cdot}3}{60} = \mathbf{3{,}660\ kg\ m/sec}\ \textit{Answer}$$

$P_a = w \,.\, Q \,.\, H$ where w = Specific weight of water,
= 1 kg per lit

$= 1 \times 55\ 24 \times 80{\cdot}97$

$= \mathbf{4{,}490\ kg\ m/sec}$ *Answer*

and $$\eta_t = \frac{P_t}{P_a} \times 100 = \frac{3{,}660}{4{,}490} \times 100 = \mathbf{81{\cdot}5\%} \quad \textit{Answer}$$

15.20 Rope Brake—Rope brake consists of a doubly wound rope over the rim of a pulley on a flywheel keyed on the shaft transmitting the power to be measured (See Fig 15.10). The rope is usually of cotton and for laboratory purposes its diameter is not more than ½ inch to 1 inch depending upon the power of the machine to be tested. A few U-shaped blocks are provided to keep the rope in position and to guard against its slipping off the pulley. Dead weights are suspended from the bottom end of the rope and a spring balance is connected to the upper end. A rod passing through slits in the weights is fixed to the frame of the prime-mover thus preventing the weights from flying off when the machine is being started or stopped.

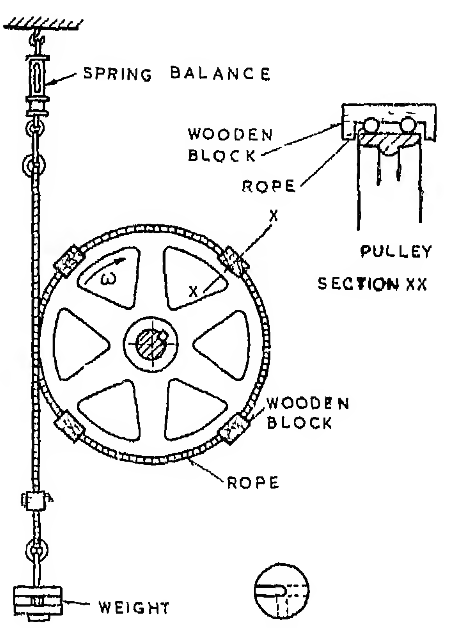


Fig 15.10 Rope Brake

A cooling arrangement may be provided if undue heat is being developed due to friction between the rope and the pulley. The pulley itself has a channel section with the flanges turned inside (see Fig 15.11). Cooling water supplied by a pipe is circulated through the channel and discharged by an outlet pipe with a flattened end which enables it to scoop the running water.

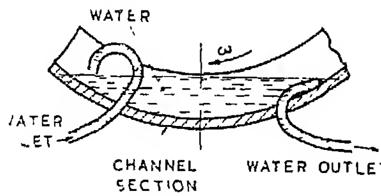


Fig 15.11 Cooling Arrangement in Rope Brake

Critical Speed of Pulley—In order to hold the water in the top portion of the channel, the centrifugal force acting on any particle should be greater than its weight. The resultant force would then act upwards and be equal to

Centrifugal force—weight of water$=F-w$

$$\text{Now,} \qquad F=\frac{w}{g}\cdot\omega^2\cdot R$$

where, R=mean radius at which water is revolving and ω, the angular velocity$=\frac{2\pi\cdot N}{60}$ (N is the speed of pulley in rpm)

$$\therefore \qquad F=\frac{w}{g}\cdot\left(\frac{2\pi\cdot N}{60}\right)^2\cdot R \qquad \text{.. (15.2)}$$

For critical speed, which is the minimum speed holding the water,

$$F-w=0$$

$$\text{or,} \qquad \left(\frac{2\pi\cdot N}{60}\right)^2\cdot\frac{R}{g}=1$$

$$\text{i.e.} \qquad N_{crit}=\frac{60}{2\pi}\sqrt{\frac{g}{R}} \qquad \text{...(15.3)}$$

If the speed falls below its critical value, water inside the rim would not rise to the top.

Problem 15.3—The rim of a braking wheel of a turbine is of channel section and its internal diameter is 18 in. Find the minimum speed in rpm at which the wheel will hold a layer of water one inch deep at the top of the rim.

Solution

$R=9$ in.

Depth of water layer at top$=1$ in.

$\therefore$ Minimum radius at top, $R=8$ in.

Now considering a water particle 8 inches vertically above the centre as a free body, for critical equilibrium :

Centrifugal Force = Gravitational Force

$$i.e. \quad \frac{w}{g} \cdot \left(\frac{2\pi \cdot N}{60}\right)^2 \cdot R' - w$$

$$\text{or} \quad N = \sqrt{\frac{60^2}{4\pi^2} \cdot \frac{g}{R'}} = \frac{30}{\pi} \sqrt{\frac{g}{R'}}$$

∴ Required minimum speed in rpm,

$$N = \frac{30}{\pi} \cdot \sqrt{\frac{32 \cdot 2}{1 \cdot 2}} = \mathbf{66 \cdot 4 \ rpm} \qquad \textit{Answer}$$

15.21 Tesla Fluid Friction Brake—The apparatus consists of a disc keyed to the shaft and moving with it, and a casing as shown in Fig. 15.12. Space between the casing and disc contains water or any other

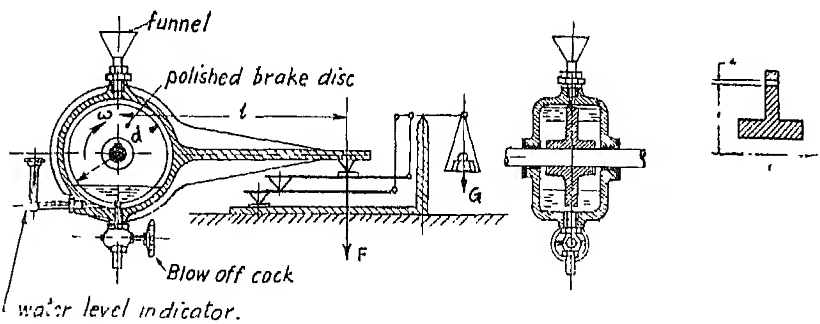


Fig 15.12 Tesla Fluid Friction Brake

liquid. Casing carries a lever, the end of which balances against weights and is, therefore, at rest. Relative motion between disc and casing causes viscous forces in the water which are utilised for braking the disc and the shaft.

$$\text{If shear stress, } f_s = \frac{F}{A} = \mu \cdot \frac{dN}{dy}$$

$$dF = da \cdot f_s$$

and area $da = 2 \cdot (2\pi \cdot r \cdot dr)$ considering both sides of disc,

$$\text{then,} \qquad dF = \mu \cdot \frac{dN}{dy} \times 2 \times (2\pi\, r \cdot dr)$$

Now, Turning Moment or Torque,

$$dM_t = dF \cdot r = \mu \frac{dN}{dy} \times 2 \times (2\pi \cdot r \cdot dr) \cdot r \qquad \ldots (15.4)$$

$$\therefore \qquad M_t = \int dM_t$$

$$\text{and} \qquad \text{Power} = M_t \cdot \omega$$

This arrangement is suitable for power measurement when the speed N is high and the force F is small.

15.22 Froude Water Vortex Brake—It consists of a rotating disc provided with several semi-elliptical blades forming brake chambers. The disc is keyed on the driving shaft of the turbine and revolves inside

a stationary casing equipped with counter blades projecting inside to form ring chambers. Fig 15.13 shows a cylindrical section through the

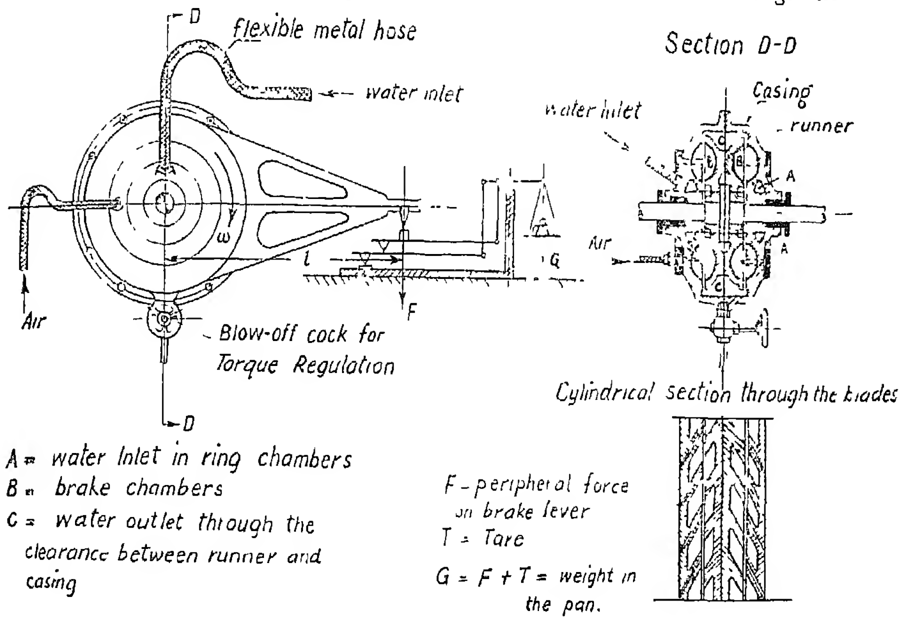


Fig 15.13 Froude Water Vortex Brake

blades of the runner and the casing. Water is supplied continuously through a flexible metal hose to the ring chamber of the casing at A and allowed to flow into the brake chambers of the disc, B. Air, which is supplied to the casing through a separate hose is also passed on to the runner. As the disc rotates, vortices and eddy-currents are set up in the water enabling the mechanical power to be dissipated as heat.

Heated water emerges at C, the clearance between runner and the casing (See Fig 15.13). The casing carries a lever balanced at the end against known weights as shown.

Power delivered to the brakes by the prime-mover

$$P_t = \frac{2\pi . N . F . l}{33{,}000} \text{BHP} \quad \left[\text{or} \quad P_t = \frac{2\pi . N . F . l}{60 \times 75} \text{ metric HP}\right]$$

$$\text{Heat generated per hour} = \frac{P_t \times 33{,}000 \times 60}{778} \; BTU$$

$$= 2{,}545 \; BTU \text{ per brake HP}$$

$$\left[\text{or} \quad = \frac{P_t \times 60 \times 75 \times 60}{427} \text{k cal}\right.$$

$$\left. = 633 \text{ k cal per metric HP}\right]$$

This is the amount of heat dissipated in water.

If the quantity of water supplied per BHP per hour in W lb (or kg), rise in temperature

$$= \frac{2{,}545}{W} \,{}^\circ F \quad \left[\text{or} \quad = \frac{633}{W} \,{}^\circ C\right]$$

Estimating the permissible rise in temperature the amount of water required per BHP (or metric HP) per hour is predetermined.

This arrangement is, generally, used for prime movers having more than 500 HP.

15.23 Torsion Dynamometer—This instrument is suitable for measuring power transmitted at high speeds but against a nearly constant load.

In principle, it is a load measuring coupling inserted between the prime-mover and the driven machine. Torque is transmitted by a steel bar capable of elastic deformation. The angle of twist of this bar, which is read by an ingenious stroboscope device, is a measure of the power transmitted. Accuracy depends largely on the uniformity of speed and torque.

In practice, the elastic bar *1* (See Fig 15.14) is rigidly connected at the ends of two flanges *3* and *4* which are bolted to similar flanges on the driver and driven shafts. The bar is carried inside a sleeve *2* which is integral with the flange *3* at one end but free at the other. When power

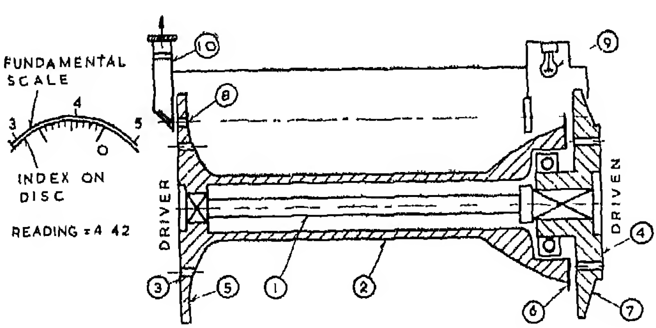


Fig 15.14 Amsler Torsion Dynamometer

is being transmitted, both the bar and the sleeve rotate with the same speed but the load is taken by the bar alone. There is, however, a clutch arrangement so that the sleeve can take the load when it exceeds the elastic limit set for the bar.

For measuring the angle of twist, three discs *5*, *6* and *7* are provided, *7* on the bar and *5* and *6* on the sleeve. The disc *7* carried by the bar is engraved with a scale known as fundamental scale which has widely spaced divisions. The disc *6* on the sleeve just adjacent to the former disc *7* carries an index divided in tenths of a fundamental scale division as shown The relative shift of the zero marks of the two scales *6* and *7* gives the angle of twist (see reading = 4·42 shown in Fig 15.14.) To enable this shift to be read the sleeve *2* carries another disc 5 with a slot *8* in the line of the zero marks of the two scales when at rest. If a fixed source of light *9* illuminates the rest position of the two scales and a fixed observation instrument *10* is placed in front of the rest position of the slot *8* on the third disc *5*, when the three discs rotate, the observer's eye will receive impulses of light of the same frequency as the rpm of torsion

meter. If this frequency is sufficiently high, persistence of vision will produce a static image and displacement of index 6 with respect to fundamental scale can be easily read.

One precaution is necessary. The critical speed for vibration of the meter or the system of rotating parts as a whole must not fall in the working range of the apparatus.

Let ϕ=angle of twist

l=length of shaft

I_p=polar moment of inertia of section of shaft

G=Modulus of rigidity $(=\frac{5}{13} \cdot E)$

Then M_t, the turning moment or twisting moment

$$=\frac{I_p \cdot G \cdot \phi}{l} \quad \text{...(15.5)}$$

(For a solid shaft of dia d, $I_p=\frac{\pi}{32} \cdot d^4$)

or $M_t=k \cdot \phi$

where $k=\frac{I_p \cdot G}{l}$ is a constant for a particular shaft.

Now, $\text{HP}=M_t \cdot \omega$

The value of k is determined by the *Gauss* method of least squares. If δ be the error,

$$(M_{t_1}-k \cdot \phi_1)^2=\delta_1^2 \quad \text{...(1)}$$

$$(M_{t_2}-k \cdot \phi_2)^2=\delta_2^2 \quad \text{...(2)}$$

$$(M_{t_n}-k \cdot \phi_n)^2=\delta_n^2 \quad \text{...}(n)$$

Now, $\sum_{i=1}^{i=n} (\delta_i)^2=f(k)$

and $\frac{\sum_{i=1}^{i=n} (\delta_i)^2}{\delta k}=0$

whence k can be calculated.

Problem 15.4 The horsepower of a turbine was found by observing that the angle of twist of a 20 ft long attached shaft at 480 rpm, was 1·75°. The shaft, which was solid, had a diameter of 7 inches, and it was known that the modulus of rigidity of the material of the shaft was 12,000,000 lb/sq in. Neglecting the effect of the end thrust, determine the brake horsepower of the turbine.

Solution

l=20 ft $\qquad N$=480 rpm

$\phi = 1{\cdot}75°$ $\qquad d = 7$ in

$G = 12{,}000{,}000$ lb/sq in.

$$\phi = 1\ 75 \times \frac{\pi}{180} = 0\ 0305 \text{ radians}$$

Polar moment of inertia,

$$I_p = \frac{\pi}{32} \cdot d^4 = \frac{\pi}{32} \times \left(\frac{7}{12}\right)^4 = 0\ 01135 \text{ ft}^4$$

$G = 12{,}000{,}000 \times 144$ lb/sq ft

$$\therefore \quad \text{Torque } M_t = \frac{I_p \cdot G \cdot \phi}{l} = \frac{0\ 01135 \times 12 \times 10^6 \times 144 \times 0{\cdot}0305}{20}$$

$$= 30{,}000 \text{ ft lb}$$

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 480}{60} = 50\ 2 \text{ rad/sec}$$

Neglecting the effect of end thrust, the power

$$P = M_t \cdot \omega = 30{,}000 \times 50{\cdot}2 \text{ ft lb/sec}$$

$$= \frac{30{,}000 \times 50\ 2}{550} = \mathbf{2{,}740\ HP} \quad \textit{Answer}$$

15.24 Measurement of Power with the Help of Electrical Epuipment—A generator directly coupled to the turbine converts the mechanical energy delivered to the turbine shaft into electrical power. The output of generator is measured with the help of electrical meters. The turbine power is evidently equal to the sum of the generator output and the losses in the generator.

Nowadays almost all electrical energy produced is AC 3-phase with delta or star connections. In India AC power is usually supplied at a frequency of 50 cycles per sec.

Measurement of Electrical Power—

DC Power—

$W = E \cdot I$ watts

where $\quad E$ = voltage

and $\quad I$ = current in amperes

AC Power—

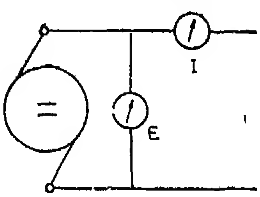


Fig 15.15 Connection for DC Supply

$$W = \sqrt{3} \cdot E \cdot I \cos \phi$$
(for 3-phase supply)

where $\quad E$ = line voltage

I = current

$\cos \phi$ = line power factor

Voltage and amperage for DC are measured by a voltmeter and an ammeter. Connections

for the same are shown in Fig 15 15. AC can be measured with either two-wattmeter or three-wattmeter methods. In practice the former is generally employed. For connections See Fig 15.16.

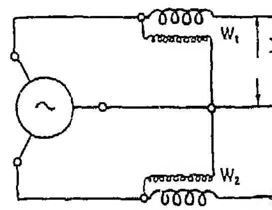


Fig 15 16 Connection of Two-Wattmeter of AC Supply

The electrical method is usually applied where power and efficiency of actual turbines have to be measured. The operation is carried out at the site after the turbo-units have been installed. The first tests after installation are known as take-over tests. The manufacturer fulfills the guarantees offered

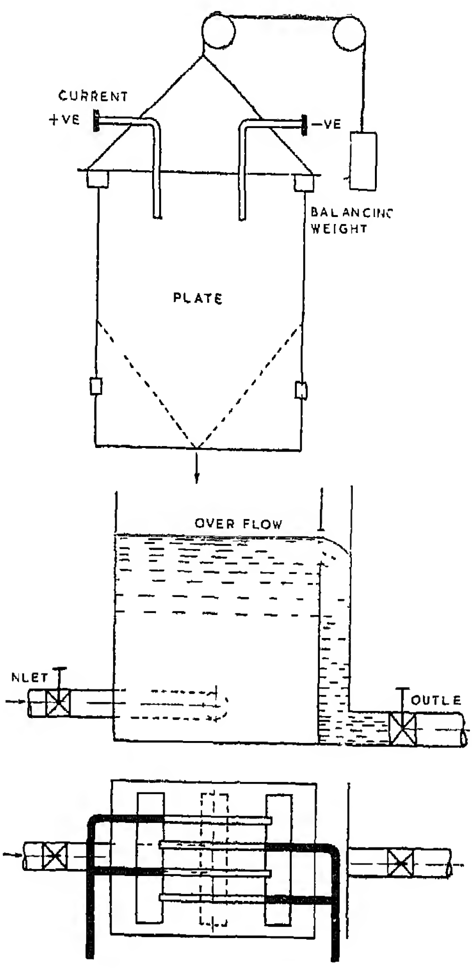


Fig 15 17 Water Resistance for DC Supply

to the customer at the time of submission of tenders. Since immediate consumers of electrical energy generated in take-over tests are not available, water resistance is employed as the load. Fig 15.17 shows the arrangement of water resistance for DC. Metal plate electrodes

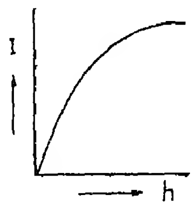


Fig 15.18 Characteristics of Water Resistance

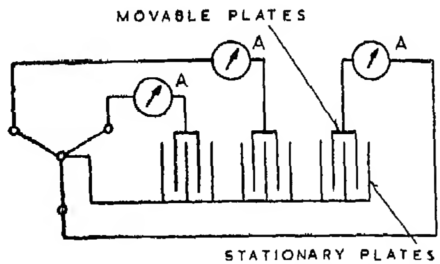


Fig 15 19 Connections for Y-Connected Three-Phase, AC Supply

are lowered in a large tank kept full of water by supply pipes. For a two inch (or 50 mm) separation between adjacent plates, the impressed current density is about 35 amp/sq ft (or 3.25 amp/m²) at 220 V. Characteristics of water resistance shown in Fig 15.18 is a curve obtained by plotting I as a function of h, where I is the current consumed and h is the depth of plate under water. Connections for water resistance for AC are shown in Fig 15.19.

15.25 Losses in a Generator—Power losses in a generator are classified as follows—

a) **Mechanical Losses—**

i) Bearing friction loss ΔP_{G_1}

ii) Air ventilation or windage loss ΔP_{G_2}

b) **Electrical Losses—**

iii) Excitation power in field coil, ΔP_{G_3}

iv) Copper loss, heating effect of current, ΔP_{G_4}

v) Iron losses or hysteresis loss, ΔP_{G_5}

Total generator loss,

$$\Delta P_G = \Delta P_{G_1} + \Delta P_{G_2} + \Delta P_{G_3} + \Delta P_{G_4} + \Delta P_{G_5}$$

$$\sum_{i=1}^{i=5} \Delta P_{G_i}$$

If the output of the generator be P_G and power supplied to it by the turbine is P_{G_t}, then

$$P_{G_t} = P_G + \sum_{i=1}^{i=5} \Delta P_{G_i}$$

Efficiency of generator, $\eta_G = \dfrac{\text{Output}}{\text{Input}} = \dfrac{P_G}{P_G + \sum\limits_{i=1}^{i=5} \Delta P_{G_i}}$ ...(15.6)

$$P_{G_i} = \frac{P_G}{\eta_G}$$

TABLE 15.6

Practical Data for Efficiency of Generator

P_G (KW)	100	500	1,000	5,000	10,000
η_G (percent)	93	94	95	96	97

Table 15 6 has been prepared on the assumption that power factor $\cos\phi = 1$. In practice ϕ` 0° and η_G is less than what is given above. If the generator is coupled directly to turbine, there are no transmission losses between the two, and power given by the turbine on its shaft,

$$P_t = P_{G_t}$$

15.26 Estimation of Turbine Efficiency—Power output of the turbine can be measured by any of the above methods. To determine its overall efficiency the rate of flow of water and total available head should be known. For methods of water measurement see Chapter 14A. Total head is the sum of the potential and kinetic heads. Potential head H' is equal to the height of the head race level above the tail race level. If v_i and v_d be the velocities of water at the head and tail races respectively,

kinetic head available $= \dfrac{v_i^2}{2g} - \dfrac{v_d^2}{2g}$

$\therefore$ Net head $H = H' + \left(\dfrac{v_i^2}{2g} - \dfrac{v_d^2}{2g}\right)$ ...(15.7)

If water is conveyed through penstocks, the head loss due to friction is to be subtracted to obtain the net head.

Gross available power,

$$P_a = w \,.\, Q \,.\, H$$

15.27 Turbine Losses—Energy losses in a turbine are divided under two heads :

a) **Mechanical losses**—

i) Bearing friction loss ΔP_{t_1}

ii) Ventilation and windage loss ΔP_{t_2}

b) **Hydraulic losses—**

i) Water head loss $\triangle P_{t_3}$

ii) Volumetric or water quantity loss $\triangle P_{t_4}$

Total power loss in the turbine,

$$\triangle P_t = \sum_{i=1}^{i=4} \triangle P_{t_i}$$

$\therefore$ Power put into the turbine shaft, $P_t = P_a - \triangle P_t$

Overall efficiency of turbine,

$$\eta_t = \frac{P_t}{P_a} = \frac{P_a - \triangle P_t}{P_a} = 1 - \frac{\triangle P_t}{P_a}$$

But the turbine looses $\triangle P_t$ cannot be directly estimated Therefore the generator power and its losses are measured, whence the turbine power,

$$P_t = \frac{P_G}{\eta_G}$$

and then $$\eta_t = \frac{P_t}{P_a} = \frac{P_G}{\eta_G \cdot P_a} \qquad \text{...(15.8)}$$

can be determined.

Thus in order to calculate η_t, the following must be measured :

$$P_G,\ \triangle P_G,\ Q,\ H',\ a_i \text{ and } a_d$$

From a_i and a_d, the velocities

$$v_i = \frac{Q}{a_i}, \quad \text{and } v_d = \frac{Q}{a_d}$$

can be calculated.

C. Characteristics of Water Turbines

15.28 Introduction—Machines are always designed to work under a given set of conditions or a limited range of conditions. A turbine may be designed for some particular data say H_o, Q_o, N_o and P_{t_o}, but in practice it may have to be used under conditions different from those for which it is designed. Therefore the exact behaviour of the prime-mover under varied conditions should be predetermined. This is graphically represented by means of curves known as *Characteristics* of the turbine. Practical data for these curves are obtained from experiments on models or actual sized machines. The model tests are conducted in research laboratories and actual tests on the site. Field tests performed at the time of customer taking the plant over from the manufacturers are known as "take-over" tests Performance of the machine

should be upto the standard specified in the tenders and all guarantees must be fulfilled.

15.29 Measurement of Characteristics Data—following data have to be obtained from the test :

1) Rate of flow of water,
2) Net head,
3) Available power,
4) Brake horse power,
5) Overall or final turbine efficiency.

i) **Rate of Flow** : See Chapter 14 for methods of water measurements.

ii) **Net Head** : (See Fig 15.3 to 15.6) Different methods are adopted according to the magnitude of head.

a) **Low Head Power Plants** : Potential head is determined by measuring simultaneously the head and tail race levels Kinetic head is equal to the difference between heads by virtue of the velocities of approach and discharge respectively. Net head is obtained by adding the kinetic head to the potential head and subtracting the losses if any. Transmission losses may be caused by friction if water is conveyed through a penstock.

b) **Medium and High Head Plants** : When these plants are closed-casing type, head can be measured with the help of a Piezometer tube, mercury manometer, Bourdon tube, pressure gauges etc.

iii) **Available Power** :

$$P_a = \frac{w \cdot Q \cdot H}{550} \text{ HP} \left[\text{or} \quad P_a = \frac{w \cdot Q \cdot H}{75} \text{ metric HP} \right]$$

where H is the net head.

iv) **Brake Horse Power** P_t : For different methods of measuring BHP see Chapter 15B.

v) **Overall efficiency** $\eta_t = \dfrac{P_t}{P_a}$.

15.30 Representation of Characteristics—The behaviour of a water turbine may be exhibited by the following curves :

i) Main characteristics (constant head curves),
ii) Operating characteristics (constant speed curves),
iii) Muschel curves (constant efficiency curves).

i) **Main Characteristics or Constant Head Curves**—The head is kept constant and speed varied by allowing a variable quantity of water to flow through the inlet opening. Thus a series of values of N and Q are obtained. For each value, brake horsepower is measured by braking mechanically or coupling with a generator. All the measured and calculated quantities are tabulated for each inlet opening as shown in Table 15.2.

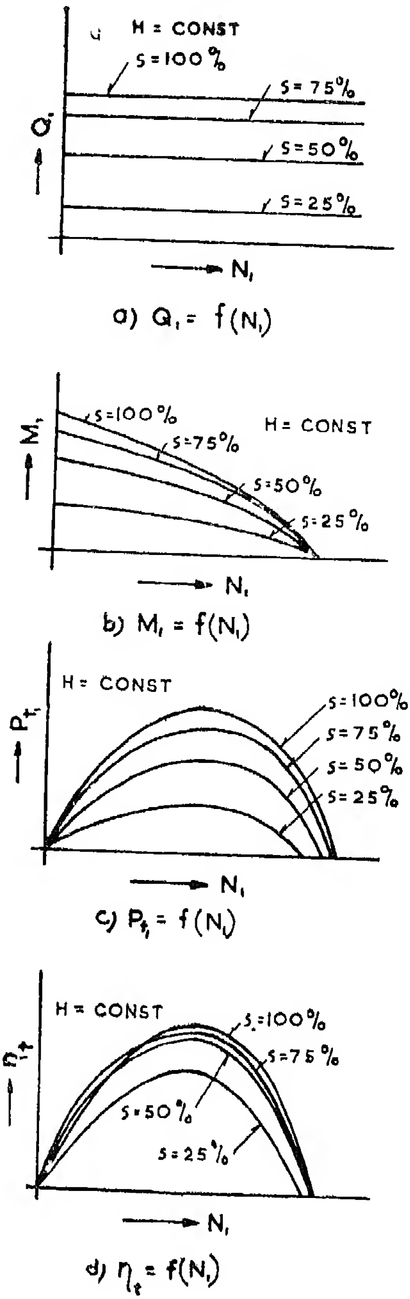


Fig 15.20 Typical Main Characteristics (Constant Head Curves) of a Pelton Turbine

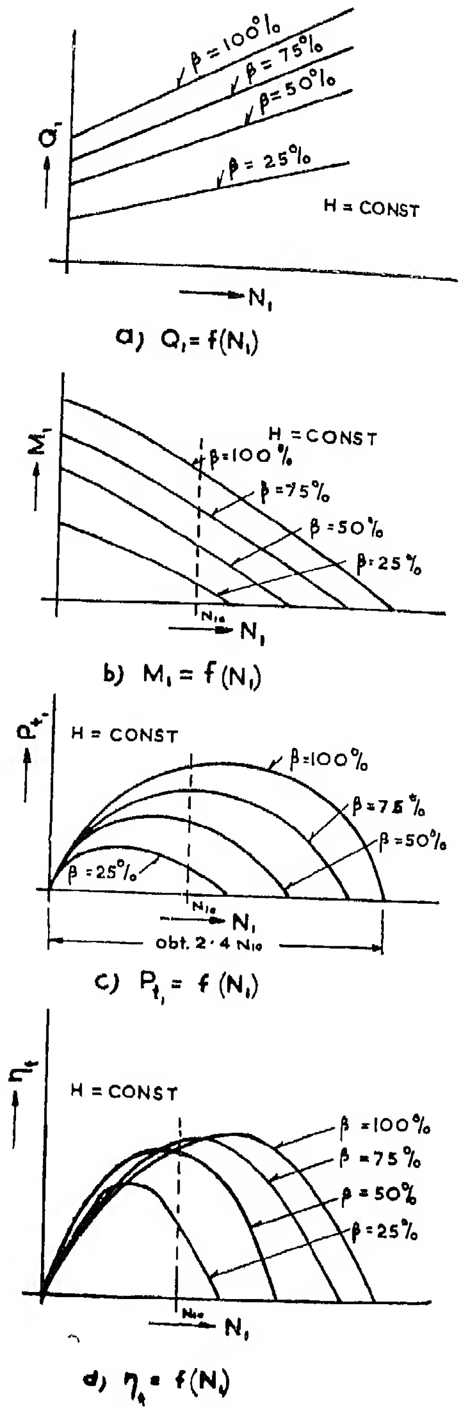


Fig 15.21 Typical Main Characteristics (Constant Head Curves) of a Reaction (Kaplan) Turbine

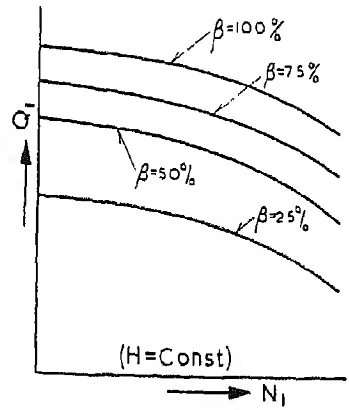


Fig 15.22 $Q_1 = f(N_1)$ for Francis Turbines

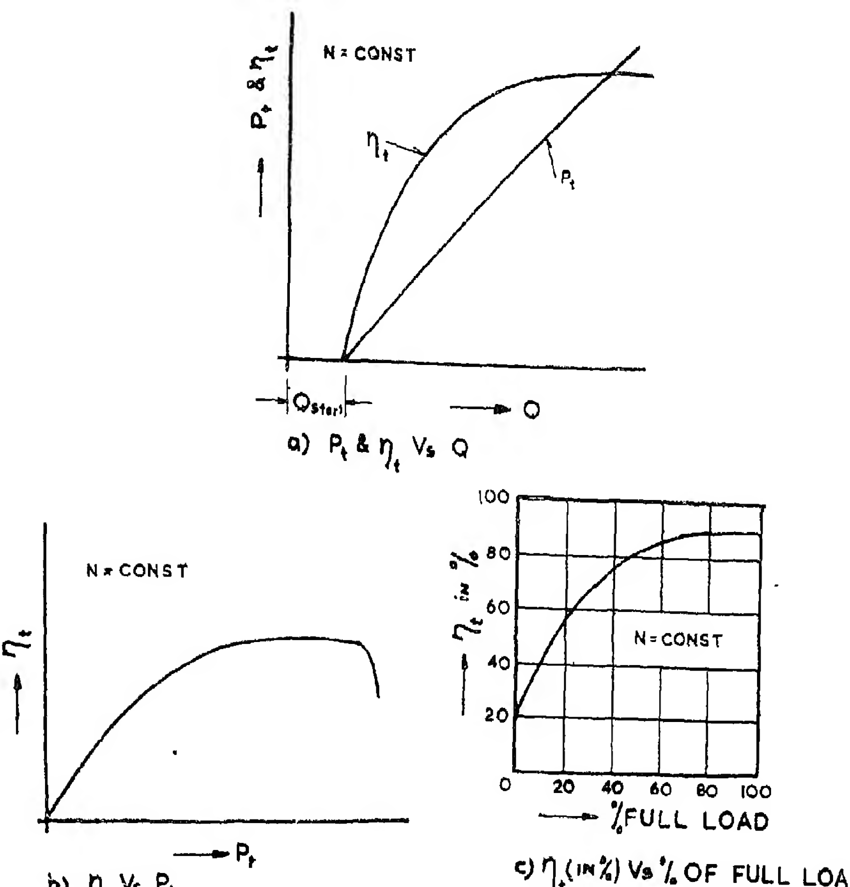


Fig 15.23 Typical Operating Characteristics (Constant Speed Curves) of a Water Turbine

The values of Q, N, M and P_t are reduced to unit head.

A new table containing these unit quantities is prepared for every inlet opening, as shown in Table 15.3.

The following curves are then drawn by plotting values from different tables :

$$Q_1=f(N_1), \qquad M_1=f(N_1),$$
$$P_{t_1}=f(N_1), \quad \text{and} \quad \eta_t=f(N_1).$$

Fig 15.20 and 15.21 show the typical main characteristics of Pelton and reaction (Kaplan type) turbines respectively. The main characteristics of Francis type reaction turbine remain same except $Q_1=f(N_1)$ curves, shown in Fig 15.23.

Curves can also be drawn showing Q_1, M_1, P_{t_1} and η_t in percentage of their maximum values, as functions of speed N_1 in percent of normal designed speed.

ii) **Operating Characteristics or Constant Speed Curves**— When a turbine is working for the generation of power its speed must remain constant. The other operating conditions *viz.*, Q and H may vary according to their availability.

Tables obtained from brake tests are used again and the following curves (See Fig 15.23) are drawn :

a) P_t, $\eta_t=f(Q)$

b) $\eta_t=f(P_t)$

c) $\eta_t=f(\%\ \text{full load})$

iii) **Constant Efficiency Curves** (Muschel Curves) Data for plotting these curves (See Fig 15.24) are obtained from the constant head and constant speed curves. The primary purpose is to find out

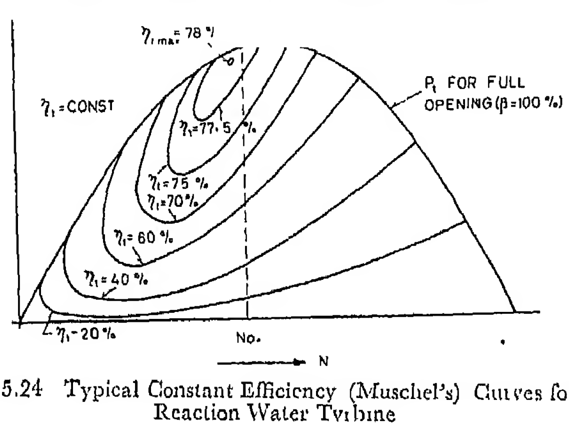


Fig 15.24 Typical Constant Efficiency (Muschel's) Curves for a Reaction Water Turbine

region of constant efficiency so that the turbine can always be operated with maximum efficiency. The curves are also helpful to the sales engineers in showing to the customers the exact performance of the machine at different efficiencies.

Problem 15.5 The following data are available from the main characteristics (constant head curves) of a Kaplan turbine :

Diameter of runner=3ft (or 0·914 m)

$P_{t_1}=7{\cdot}02$ HP (or 42 8 metric HP), $Q_1=60$ cfs (or 3 09 m³/sec),

$N_1=35{\cdot}1$ rpm (or 63·7 rpm)

Determine the runner diameter, the rate of flow and the speed of a geometrically similar turbine which works under a head of 90 ft (or 27·4 m) producing 2,700 HP (or 2,740 metric HP).

Solution

Given two turbines such that :

$D_a=3$ ft (or 0·914 m)	$D_b=?$
$(P_{t_1})_a=7{\cdot}02$ HP (or 42 8 metric HP)	$(P_t)_b=2{,}700$ HP (or 2,740 metric HP)
$(Q_1)_a=60$ cfs (or 3 09 m³/sec)	$(Q)_b=?$
$(N_1)_a=35{\cdot}1$ rpm (or 63·7 rpm)	$(N)_b=?$
$H_a=1$ ft (or 0.3048 m)	$H_b=90$ ft (or 27·4m)

Since the turbines are geometrically similar :

i) Specific powers are equal,

$$i.e.\quad \frac{(P_t)_b}{D^2_b\,.\,H_b^{\frac{3}{2}}}=\frac{(P_t)_a}{D^2_a\,.\,H_a^{\frac{3}{2}}}=\frac{(P_{t_1})_a}{D^2_a} \qquad \ldots(1)$$

ii) Specific flows are equal,

$$i.e.\quad \frac{Q_b}{D^2_b\,.\,\sqrt{H_b}}=\frac{Q_a}{D^2_a\,.\,\sqrt{H_a}}=\frac{(Q_1)_a}{D^2_a} \qquad \ldots(2)$$

iii) Speed ratios are equal,

$$i.e.\quad (K_{u_1})_a=(K_{u_1})_b \text{ or } \frac{D_b\,.\,N_b}{\sqrt{H_a}}=\frac{D_a\,.\,N_a}{\sqrt{H_a}}=D_a\,.\,(N_1)_a \qquad \ldots(3)$$

Now, from (1) by substituting the given values,

$$D_b=\sqrt{\frac{P_{t_b}}{(P_{t_1})_a}\cdot\frac{D^2_a}{H_b^{\frac{3}{2}}}}=\sqrt{\frac{2{,}700}{7{\cdot}02}\times\frac{9}{90^{\frac{3}{2}}}}=\mathbf{2\ ft}\ \textit{Ans.}$$

$$\left[\text{or}\quad D_b=\sqrt{\frac{2740}{42{\cdot}8}\times\frac{0{\cdot}914^2}{27{\cdot}4^{\frac{3}{2}}}}=\mathbf{0{\cdot}611\ m}\qquad \textit{Answer}\right]$$

Similarly, from (2),

$$Q_b=(Q_1)_a\,.\,\frac{D^2_b}{D^2_a}\,.\,\sqrt{H_b}=60\times\frac{4}{9}\times\sqrt{90}$$

$$=\mathbf{253\ cfs}\quad \textit{Answer}$$

[or $Q_b = 3{\cdot}09 \times \frac{0{\cdot}611^2}{0{\cdot}914^2} \times \sqrt{27{\cdot}4}$ = **7·19 m³/sec** *Answer*]

And, from (3),

$$N_b = (N_1)_a \cdot \left(\frac{D_a}{D_b}\right) \cdot \sqrt{H_b} = 35{\cdot}1 \times \frac{3}{2} \times \sqrt{90}$$

$$= \textbf{500 rpm} \quad \textit{Answer}$$

[or $N_b = 63{\cdot}7 \times \frac{0{\cdot}914}{0{\cdot}611} \times \sqrt{27{\cdot}4}$ = **500 rpm** *Answer*]

Problem 15 6 A Francis turbine gives the following performance :

$H = 17{\cdot}4$ ft $\qquad Q = 53{\cdot}85$ cfs,

BHP$=92{\cdot}77$, $\qquad$ RPM$=204{\cdot}3$

Diameter$=31$ in. and part of gate opening$=0{\cdot}873$

a) Compute the efficiency and the speed ratio,

b) Assuming the same wheel to operate at the same speed and same gate opening under a head of 50 ft, what will be the new RPM, Q and HP ? Check the results by recomputing from the efficiency and speed ratio.

Solution

a) $$P_t = \frac{w \cdot Q \cdot H}{550} \cdot \eta_t$$

$$\therefore \quad \eta_t = \frac{550 \times P_t}{w \cdot Q \cdot H} = \frac{550 \times 92{\cdot}77}{62{\cdot}4 \times 53{\cdot}85 \times 17{\cdot}4} = 0{\cdot}872$$

or **87·2%** *Answer*

Speed ratio $$K_{u_1} = \frac{\pi \cdot D_1 \cdot N}{60 \times \sqrt{2g\,H}}$$

$$= \frac{\pi \times \frac{31}{12} \times 204{\cdot}2}{60 \times 8{\cdot}02 \times \sqrt{17{\cdot}4}} = \textbf{0{\cdot}823} \quad \textit{Answer}$$

b) New speed under 50 ft head—

Specific speeds in both cases are equal,

$$\therefore \quad N_s' = N_s \quad \text{or} \quad \frac{N' \cdot \sqrt{P_t'}}{(H')^{\frac{5}{4}}} = \frac{N \cdot \sqrt{P_t}}{H^{\frac{5}{4}}}$$

$$\text{or} \quad N' = N \left(\frac{P_t}{P_t'}\right)^{\frac{1}{2}} \cdot \left(\frac{H'}{H}\right)^{\frac{5}{4}}$$

Also, unit powers in both cases should be equal, as the only factor which changes is head.

$$\therefore \quad \frac{P_t'}{(H')^{\frac{3}{2}}} = \frac{P_t}{H^{\frac{3}{2}}}$$

$$\text{or} \quad P_t' = P_t \cdot \left(\frac{H'}{H}\right)^{\frac{3}{2}} = 92{\cdot}77 \times \left(\frac{50}{17{\cdot}4}\right)^{\frac{3}{2}} = \mathbf{450\ HP} \quad \textit{Answer}$$

And substituting the values of P_t, P_t', H and H' in the above equation of N',

$$N = 204{\cdot}2 \times \left(\frac{92{\cdot}77}{450}\right)^{\frac{1}{2}} \times \left(\frac{50}{17{\cdot}4}\right)^{\frac{5}{4}} = \mathbf{346\ RPM} \quad \textit{Answer}$$

Further, since, unit discharges are equal,

$$\frac{Q'}{(H')^{\frac{1}{2}}} = \frac{Q}{H^{\frac{1}{2}}} \quad \text{or} \quad Q' = Q \cdot \left(\frac{H'}{H}\right)^{\frac{1}{2}} = 53{\cdot}85 \times \left(\frac{50}{17{\cdot}4}\right)^{\frac{1}{2}}$$

$$= \mathbf{91{\cdot}2\ cusecs} \quad \textit{Answer}$$

$$\textbf{Check}—P_t' = \frac{w \cdot Q' \cdot H'}{550} \cdot \eta_t = \frac{62{\cdot}4 \times 91{\cdot}2 \times 50}{550} \times 0{\cdot}872$$

$$= \mathbf{450\ HP} \quad \text{(correct)}$$

$$\text{and} \quad K_{u_1} = \frac{\pi \cdot D_1 \cdot N}{60 \times \sqrt{2gH}} = \frac{\pi \times 2\tfrac{1}{2} \times 346}{60 \times 8{\cdot}02 \times \sqrt{50}} = \mathbf{0{\cdot}823} \quad \text{(correct)}$$

Problem 15.7 The result of a test on water turbine operating at full gate opening under a head of 17·3 ft (or 5·28 m) are given as follows:

Unit Speed	FPS	36	40	42	44	46	48	50	52
	Metric	65·4	72·6	76·3	79·9	83·5	87·2	90·75	94·4
Unit Power	FPS	2 78	2 86	2·885	2 89	2 87	2 825	2·75	2 65
	Metric	16·8	17 25	17·42	17·45	17·32	17 06	16 6	16·0
Q	ft³/sec	132	129 3	127·6	125 8	124	122	120	117·5
	m³/sec	3·74	3·66	3 6	3 56	3·51	3·45	3·4	3·32

Plot graphs of unit power and efficiency against unit speed. If the turbine runs at a real speed of 200 rpm, find the power and efficiency. If the head is increased to 18 ft (or 5·5 m), the speed remaining the same, find the corresponding power and efficiency.

(*Punjab University—1949*)

Solution

Turbine output $P_t = P_{t_1} \cdot H^{\frac{3}{2}}$

Turbine available power $P_a = \dfrac{w \cdot Q \cdot H}{550}$ $\left[\text{or } P_a = \dfrac{w \cdot Q \cdot H}{75}\right]$

$$\therefore \quad \text{Turbine efficiency } \eta_t = \frac{P_t}{P_a} = \frac{550 \times P_{t_1} \, . \, H^{\frac{3}{2}}}{w \, . \, Q \, . \, H} = \frac{550 \times P_{t_1} \, . \, H^{\frac{1}{2}}}{w \, . \, Q}$$

$$\left[\text{or} \quad \eta_t = \frac{75 \times P_{t_1} \, . \, H^{\frac{1}{2}}}{w \, . \, Q}\right]$$

or $$\eta_t = \left(\frac{550 \times H^{\frac{1}{2}}}{w}\right)\left(\frac{P_{t_1}}{Q}\right) = \frac{550 \times 17.3^{\frac{1}{2}}}{62.4} \times \frac{2.78}{132} = \mathbf{0.771}$$

or **77.1%**

$$\left[\text{or} \quad \eta_t = \frac{75 \times 5.28^{\frac{1}{2}}}{1,000} \times \frac{16.8}{3.74} = \mathbf{0.771} \quad \text{or} \quad \mathbf{77.1\%}\right]$$

Similarly calculate the efficiency η_t for each unit power and Q, and tabulate the results are follows

N_1	FPS	36	40	42	44	46	48	50	52
	Metric	65.4	72.6	76.3	79.9	83.5	87.2	90.75	94.4
P_{t_1}	FPS	2.78	2.86	2.885	2.89	2.87	2.825	2.75	2.65
	Metric	16.8	17.25	17.42	17.45	17.32	17.06	16.6	16.0
Q	ft³/sec	132	129.3	127.6	125.8	124	122	120	117.5
	m³/sec	3.74	3.66	3.6	3.56	3.51	3.45	3.4	3.32
η_t		77.1	80.9	82.7	84	84.7	84.7	83.85	82.5

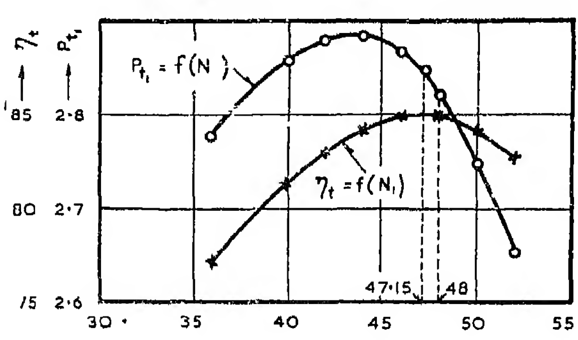


Fig 15.25 P_{t_1} & η_t vs N_1 (FPS units)

Draw P_{t_1} *vs* N_1 and η_t *vs* N_1 curves as shown in Fig 15.25.

Now, $N = 200$ rpm,

$$\therefore \quad N_1 = \frac{200}{17.3^{\frac{1}{2}}} = 48$$

$$\left[\text{or} \quad N_1 = \frac{200}{5.28^{\frac{1}{4}}} = 87\right]$$

Find the values of P_{t_1} and η_t against $N_1=48$ from Fig 15 25

$\therefore$ $P_{t_1}=2\,825$ or $P_t=2\,825\times 17\,3^{\frac{3}{2}}=$ **203 5 HP** *Answer*

[or $P_{t_1}=17\,07$ or $P_t=17\,07\times 5\,28^{\frac{3}{2}}=$ **207 metric HP** *Answer*]

and $\eta_t=$ **84 7%** *Answer*

If $H=18$ ft (or 5 5 m) and $N=200$ rpm,

$$N_1=\frac{200}{18^{\frac{1}{2}}}=47{\cdot}15$$

$$\left[\text{or}\quad N_1=\frac{200}{5\,5^{\frac{1}{2}}}=85\,25\right]$$

$\therefore$ P_{t_1} (from Fig 15.25) $=2{\cdot}845$ FPS units (or $17{\cdot}2$ metric units)

or $P_t=2\,845\times 18^{\frac{3}{2}}=$ **217 5 HP** and η_t (from Fig 15 25) $=$ **84 9%**

Answer

[or $P_t=17{\cdot}2\times 5\,5^{\frac{3}{2}}=$ **221 metric HP** and $\eta_t=$ **84 9%** *Answer*]

Problem 15.8 The head water surface and tail water surface levels in a water turbine installation are 170 ft and 150 ft above sea level respectively. The turbine is designed to run at a uniform speed of 75 rpm under all conditions.

The manufactures' tests under a head of 25 ft show that the turbine develops 2,000 HP at its maximum efficiency of 87%. The computed values of unit speed N_1 and unit power P_1 in percentage of normal values and the efficiency η in percentage of the maximum efficiency are tabulated below :

Units speed N_1 in % of normal value	60	70	80	90	100	110	120	130
Unit power P_1 in % of normal value	73·5	82 0	90 0	96·5	100	99 5	99 0	90 0
Efficiency η in % of max value	80	88	94	98	100	99	94	85

Calculate the HP and Q of the turbine under normal working conditions. *(UPSC—Jan 1953)*

Solution

$H_{test}=25$ ft $\qquad$ $H_{normal}=170-150=20$ ft

$P_{t(test)}=2{,}000$ HP $\qquad$ $N=75$ rpm (constant)

$\eta_{test}=87\%$

Designed speed $N=75$ rpm and tested head $=25$ ft

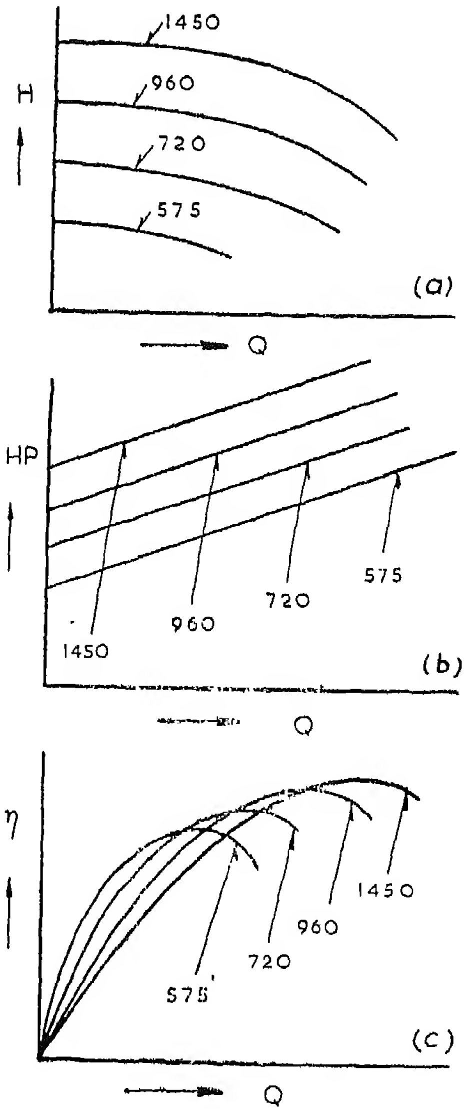


Fig 15.28 Main Characteristics of a Centrifugal Pump (*a*) *H* vs *Q*, (*b*) HP vs *Q* and (*c*) η vs *Q*

steam turbine. Pumps employed for irrigation purposes have to be installed in remote villages often having no electric power. Driving unit in such cases is a diesel engine. The pump may be coupled to the engine of a tractor which may be used for both ploughing and pumping. Smaller engines consume petrol while larger ones use diesel oil.

In such circumstances it is advantageous to know the performance of a pump at different speeds and this is best seen from the main characteristics.

A typical set of main characteristics is shown in Fig 15.28.

15.33 Operating Characteristics : During operation the pump must run constantly with the speed of the driving unit. Normally, this is the designed speed. That particular set of main characteristics which corresponds to the designed speed is mostly used in operation and is therefore known as the operating characteristics (See Fig 15.27).

These curves are said to be rising, flat or falling characteristics respectively according as the head increases, remains more or less constant, or decreases with increase in rate of flow. It depends upon the outlet vane angle β_2. Ordinarily pumps have falling characteristics (See Fig 15.29).

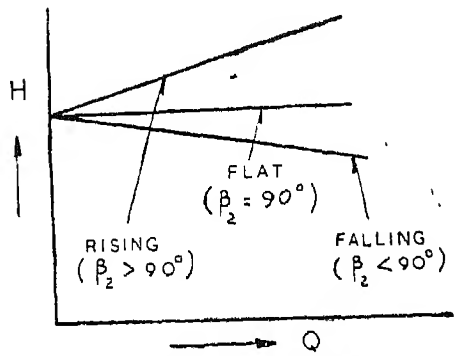


Fig 15.29 Rising, Flat or Falling H vs Q Curves of a Centrifugal Pump

15.34 Muschel Curves or Constant Efficiency Curves—With the help of data obtained from the above curves a series of constant efficiency curves can be drawn (See Fig 15.30). They facilitate the job of the salesman and enable the prospective customer to see directly the range of operation with a particular efficiency. They serve as a suitable basis for a comparison of pumps, specially from a commercial point of view.

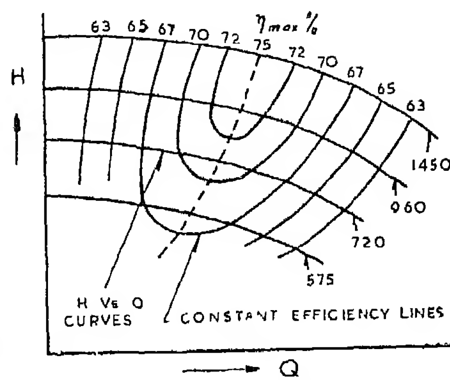


Fig 15.30 Constant Efficiency or Muschel Curves of a Centrifugal Pump

15.35 Constant Head and Constant Discharge Curves—It is quite possible that a pump may be required to deliver water at a certain height, in which case H is fixed. If for some reason the speed varies, discharge will also be affected. In order to predetermine the performance of the pump under such conditions, it is necessary to draw a constant H curve by plotting Q vs N (See Fig 15.31).

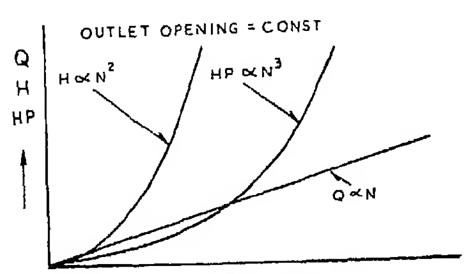


Fig 15.31 Q, H and HP vs N Curves of a Centrifugal Pump

Similarly, to determine the speeds required to discharge a certain quantity at different pressures or to find the variation of H with N, it is convenient to draw constant Q curves showing H against N (See Fig 15.31).

UNSOLVED PROBLEMS

A. Testing of Turbines and Pumps

15.1 What are the purposes of testing of the turbines and pumps ?

15.2 What are testing codes ? What do they contain and for what purposes are they used ?

15.3 What are acceptance or take-over tests ? What is the necessity of their performance ? What steps are taken to carry out such tests ?

15.4 Define model tests ? What is the general size of a model turbine ? Describe briefly the procedure to carry out such tests for turbines and pumps.

15.5 What is the necessity of equipping the manufacturers' works with a test bed ? Why are different test beds needed for each type of turbine and pump ?

15.6 Explain briefly "Aerodynamic Test Bed" and "Cavitation Test Bed." What is the time of exposure for taking cavitation photographs ?

15.7 Describe briefly the lay out of a test bed for—

a) Pelton Turbine

b) Reaction turbine

c) Centrifugal pump.

15.8 What is the correct factor which is applied for calculating the net head for a pump, if the suction pipe diameter is bigger than the delivery pipe diameter ?

15.9 Why are D.C. motors used for driving the test pumps ?

15.10 State the precautions taken while connecting gauges to the suction and delivery pipes.

15.11 The discharge through an experimental Pelton wheel installed in a laboratory is to be accurately measured. What type of meter or meters would you propose for the purpose ? If the discharge is to be 600 gallons per minute, give an approximate size of the apparatus that you would recommend.

Give a neat sketch of the layout of an experimental Pelton wheel for a laboratory. (*Madras University—Sept 1952*)

15.12 A test bed for testing of centrifugal pumps for their characteristics at different speeds, heads and discharge, has to be fitted up with the necessary drive, measuring instruments etc. The ranges of speed, head and discharge are 600 to 1600 rpm, 20 to 100 ft and 200 to 600 gallons/min respectively.

Make a neat sketch of the test bed you would propose with all the apparatus and give, in a tabular form, the list of equipment and measuring apparatus you consider necessary.

Give the size and capacity of each of these items so that the bed will be able to take all pumps within the above range.

(*Madras University—March 1952*)

15.13 Describe the procedure of testing a water turbine in a laboratory.

15.14 Which are the data required to be measured while the experiment is being performed ? Give the information in a tabular form.

15.15 Which are the data required to be measured before the experiment is started ? Explain how each of such information is used for calculating various other data.

15.16 Name the instruments which should be calibrated before the experiment is performed.

15.17 How is the effective or working head on the turbine determined ? State the difference in calculations if the turbine is reaction or impulse.

15.18 State how is the overall efficiency of water turbine determined by brake test.

15.19 Why are the turbine data finally calculated at unit head ?

15.20 Explain the procedure of Testing for pumps in a laboratory.

15.21 Name the items to be checked before the pump is tested.

15.22 Explain how is the overall efficiency of a centrifugal pump determined. Draw a table for such a pump test.

B. Measurement of Power and Efficiency

15.23 What are the different equipments used to find the power and efficiency of a water turbine ?

15.24 How would you measure the power and the efficiency of a turbine with the help of Prony brake ?

15.25 Where do you employ torsion dynamometer and rope dynamometer to measure the power of prime movers ?

15.26 Sketch and describe one form of torsion dynamometer and explain how the HP transmitted is calculated.

15.27 Which one of the two methods for power measurement is more accurate—*a*) Electrical or *b*) Mechanical.

15.28 Describe briefly how will you measure the power and the efficiency of power unit by means of electrical generator. Where would you use this method in practice ?

15.29 A Prony brake of the type shown in Fig 15.8 was used to measure the power output of a turbine running at 180 rpm. The gross load on the scales during the test was 58 lb and the tare weight 12 lb. *a*) What is the brake constant ? *b*) What is the HP output of the turbine ? The brake pully has a diameter of 20 in. and leverage is 21·7 in. (0·000342, 2·84 HP)

15·30 A test of power and efficiency for a water turbine is made by connecting the prime mover to an electrical dynamometer which operates as a generator. The frame or stator of the dynamometer is mounted independently of its rotor in bearings which are co-axial with those of the rotor. In this way a direct measurement of the torque on the rotor developed by the generation of electric current

may be made. The electrical energy generated is converted into heat by passing the current through electrical resistors. In the particular test involved the speed of the turbine is 500 rpm. The force measured during test is 22 lb and that due to the unbalanced weight of the stator alone with the motor disconnected is 4 lb. The turbine consumes 0 294 cusecs while working under a head of 110 ft. The leverage of the dynamometer is 18 in. The turbine is directly connected to the dynamometer. Find the HP and the efficiency of turbine. (2·57 HP, 70%)

15 31 A Pelton wheel works under a head of 394 ft and the mean diameter of the wheel is 3·8 ft. During a brake test in which the needle valve is kept fully open it is found that when the wheel is brought completely to rest, the torque on the shaft is 2,170 ft lb. Estimate the speed and power of the wheel when running under its normal working conditions. (*Madras University—Sept 1957*)

15.32 In a Pelton wheel the water is turned in the buckets through an angle of 165°, the diameter of the wheel is 2·75 feet and the velocity of the jet 160 feet per second. The Pelton wheel is tested with the needle valve fully open and at speeds varying from 0 to 200 RPM.

Plot graphs showing the relation between (*a*) the wheel speed and the thrust on the buckets and (*b*) the wheel speed and the efficiency, taking at least three different speeds within the range.

Calculate the speed and the power of the wheel when working under the condition of maximum efficiency.
(*Madras University—Sept 1951*)

C. Characteristics of Water Turbines

15.33 Draw the main characteristics of a reaction water turbine.

15 34 What is the difference between the main and the operating characteristics of a water turbine ? Draw a set of such curves for a Pelton turbine.

15.35 Draw the following characteristics curves of a Kaplan turbine :

i) P_{t_1} vs N_1 *ii*) P_1 vs Q (*AMIE—May 1955*)

15.36 What are the different characteristic curves of a water turbine? Draw the following typical curves for a Pelton turbine ?
i) P_t vs Q *ii*) η_t vs Q (*Jadavpur University—1955*)

15.37 Describe the method of preparing the characteristic diagram for a turbine when the co-ordinates are 'unit speed' and 'unit power'. What is the use of such a diagram ? (*AMIE—May 1954*)

15.38 What are constant efficiency curves of a water turbine and where are they employed ?

15.39 What is the difference between $Q_1=f(N_1)$ curves of Francis and Kaplan turbines ?

15.40 In a series of brake tests of a small Pelton wheel the following tabulated results were obtained :

W	6·5	5·7	5·4	4·7	4·4	3·5	2·9	2·1	1	0
N	960	1,360	1,480	1,800	1,900	2,240	2,520	2,800	3,280	3,580

Where W= effective load in lb on brake lever at 12 in. from the axis of wheel, and N= speed of wheel in rpm. The weight of water used in each test was 41·5 lb/min, and the pressure of water was 700 lb/sq in. in the pipe behind the orifice. The diameter of orifice was 0·0835 in. Complete the above table by adding BHP and the efficiency in percent. Plot HP and efficiency on a speed base.

Scale HP 2 in. = 1 HP

η 1 in. = 20%

N 1 in. = 500 rpm

State the maximum HP and maximum efficiency.

(1·6 HP ; 79%)

15.41 *a*) When an impulse turbine runs at *constant* nozzle opening under *constant* head, its performance will be affected by the *speed* in rpm. Show these variations, for idealized conditions, by sketching graphs between rotational speed and (*i*) torque, (*ii*) efficiency, and rate of flow of water, for a range of speeds from zero to maximum.

b) In an actual Pelton wheel installation, the turbine gives a maximum efficiency of 86% when running at a speed of 500 rpm, under a head of 800 ft. Assuming the speed and nozzle opening to remain unchanged, and the efficiency curve to be a parabola over the speed-ratio range considered, make a close estimate of turbine efficiency when the head is (*a*) 600 ft and (*b*) 1,000 ft.

(84 and 85·1%) (*Jadavpur University—1960*)

[*Hint* : The efficiency-unit speed curve being a parabola, the maximum efficiency occurs at half the runaway speed.]

Operating unit speed

$$=N_1=\frac{N}{\sqrt{H}}=\frac{500}{\sqrt{800}}=17{\cdot}65$$

Unit speed at which the efficiency is required

$$=\frac{500}{\sqrt{600}}=20{\cdot}4 \quad \text{and} \quad \frac{500}{\sqrt{1{,}000}}=15{\cdot}8$$

at points C and D in Fig 15.32.

Equation of parabola : $y^2=4\ ax$

At point C, $y=CP=20{\cdot}4-17{\cdot}65=2{\cdot}75$

$\therefore\ 2{\cdot}75^2=(2\times 17{\cdot}65)x \qquad (\because\ AB=4a=2\times 17{\cdot}65)$

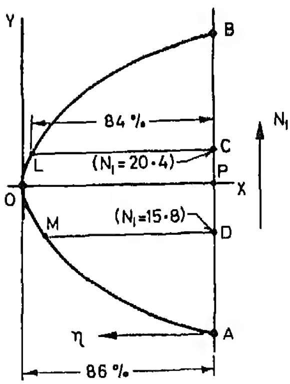


Fig 15.32 Efficiency vs Unit Speed (Parabola) curve of Pelton Turbine

or $x=0{\cdot}201$

$$\therefore\ CL=\frac{2\times 17\ 65}{4}-0{\cdot}201=8{\cdot}624$$

$$\text{but } OP=\frac{2\times 17\ 65}{4}=8{\cdot}825 \text{ units}$$

which corresponds to 86%

$\therefore$ Efficiency at 600 ft head

$$=\frac{CL}{OP}\times 86\%$$

$$=\frac{8{\cdot}624}{8{\cdot}825}\times 86=\mathbf{84\%}$$

Similarly find DM and then the efficiency at 100 ft head.

D. Characteristics of Pumps

15.42 What are the characteristic curves of a centrifugal pump ?

15.43 Draw the operating characteristics of an ordinary centrifugal pump.

15 44 What are the constant efficiency curves of a centrifugal pump ? Are there similar curves for the turbines too ? For what purpose are such curves used ? (*Jadavpur University—1954*)

15.45 What is the use of constant head and constant discharge curves ?

15.46 Show on a sketch of the constant speed characteristics of a centrifugal pump how an increase in the friction head due to the formation of scale in a long delivery pipe affects the performance of the pipe. [*AMI Mech E* (*Lond*)—*Oct 1958*]

15.47 A test on a single stage centrifugal pump when running at 1,000 rpm gave the following results :

Discharge	0	1,000	2,000	2,500	3,000	3,500	4,000	gpm
Total head	122	117	111	106	96	82	62	ft
Shaft input	75	80	93	98	105	110	112	HP

Plot curves of total head and efficiency to a base of discharge and find the specific speed at the best efficiency.

It pumps water from a river to reservoir through a pipe 12 in. diameter and 1,000 ft long, having co-efficient of friction 0·006. The difference of levels between the river and the reservoir is 60 ft.

Plot the curve of total head of the main for the above static head against discharge base and so find the head, discharge and efficiency at the operating point.

[Efficiency 0, 44·3, 72·4, 82, 83·1 (max), 79, 67·1 HP]

$N_s = 2{,}860$

$$H_{total} = H_{stat} + H_{L_f} + \frac{v^2}{2g}$$

Q	500	1,000	2,000	3,000	4,000	5,000	gpm
H_{total}	63·95	75 81	123·4	174·24	313	455	ft

This Q vs H_{total} curve intersects the first H vs Q curve at the operating point where $Q = 1{,}830$ gpm, $H = 112{\cdot}5$ ft and $\eta = 70\%$)

(*Jadavpur University—1956*)

15.48 A pumping installation consists of two identical constant speed centrifugal pumps working in parallel and forcing water through a single pipeline against a static or dead head of 56 meters. The head discharge characteristic curve of each pump can be represented by the equation :

$$H = 128 + 14Q - 108Q^2$$

Where H is the manometric head in meters and Q is the discharge per pump in m^3/sec.

The pipeline is 826 meters long and 0 6 meter in diameter ; its frictional co-efficient 'f' can be taken as 0·0045.

Estimate the discharge through each pump and through the pipe.

(*Note* : 'f' $= gdh/2lv^2$, $g = 9{\cdot}81$ m/sec^2)
(0·691 and 1·382 m^3/sec) [*AMI Mech E (Lond)—Oct 1958*]

15.49 Explain the head-discharge characteristics of two similar centrifugal pumps with different impeller blade angles at outlet only.

In a pumping station there are two units whose head-discharge characteristics at their constant working speeds can be represented by $H = 132 + 15Q - 116Q^2$, where H and Q have their significances. The pumps deliver into a single pipe line in which the static head

is 46 meters ; the pipe is 0 6 m diameter and 890 m long and the value of the pipe co-efficient is 0 005. Compute the total discharge through the pipe, if the pumps were arranged—

i) In parallel,

ii) In series. Neglect velocity head.

(*Madras University—March 1955*)

SECTION V

Hydraulic Systems

CHAPTER 16

HYDRAULIC SYSTEMS

16.1 Hydraulic System—Hydraulic System is a circuit in which force and power are transmitted through a fluid, generally an oil. Hydraulic systems may be divided into two groups, the hydrostatic and hydro-kinetic.

Hydrostatic System is that in which the primary function of hydraulic fluid is the transmission of force and power by pressure. A hydrostatic system consists essentially of a pumping unit, a hydraulic motor and connecting leads which form a circuit between the two main components. The pumping unit transmits fluid pressure and is, therefore, known as *transmitter*. The hydraulic motor which receives force and power by means of fluid pressure is known as *receiver*.

Example : A pumping unit operating a hydraulic press. In this case the pumping unit is a transmitter and hydraulic press is a receiver. The work done by the pump is utilised for displacement of oil against a force which arises from resistance to motion of the plunger in the hydraulic press.

Hydro-kinetic System—The purpose of a hydro-kinetic system is to transmit power and the required effect is obtained primarily by virtue

of changes in velocity of flow of the working media and changes of pressure are avoided as far as possible.

A hydro-kinetic transmitter consists essentially of a centrifugal pump or impeller mounted on the driving shaft and an oil turbine or runner mounted on the driven shaft. Power is transmitted from the driving to the driven shaft through circulation of oil between the impeller and the runner.

Example : Hydraulic coupling and torque converter.

16.2 Rotary Positive Displacement Pumps for Hydrostatic Systems—Reciprocating, centrifugal and other types of pumps dealt with in Chapters 11, 12 and 13 are usually employed as water lifting devices where large volumes have to be handled. In the oil hydraulic field, however, rotary pumps are used, for they develop very high pressure.

A rotary pump is a positive displacement pump with a circular motion. In outward appearance, it resembles a centrifugal pump. But it differs from a centrifugal pump in action. While it continuously scoops the liquid out of the pump chamber, the latter only imparts a velocity to the stream of fluid.

16.3 Classification of Rotary Pumps—

- Rotary Pump
  - Constant Delivery Pump
    - Gear Pump
      - External Type
      - Internal Type
    - Screw Pump
    - Vane Pump
    - Radial Piston Pump
  - Variable Delivery Pump
    - Sliding or Rotary Piston Pump
    - Seated Valve or Radial Piston Pump

16.4 Constant and Variable Delivery Pump—Constant delivery pump is one which has a continuous discharge of liquid at a uniform rate of flow whereas a variable delivery pump has a continuous discharge of liquid at rates of flow varied as required. Selection of pump depends upon the nature of duty of the hydraulic receiver or motor. When the receiver or motor operates continually at varying speed or with movement varying in extent, variable delivery pump is usually employed. The construction of variable delivery pump is complex and as such these pumps are more costly than constant delivery pumps. Therefore, constant delivery pumps are used to give variable delivery by the provision of variable speed control gear for the driving motor, or by regulating the flow of liquid by means of valves independent of pump.

Constant Delivery Pumps

16.5 External Type Gear Pump (See Fig 16.1*a*, *b*)—A single-stage external type gear pump consists of two identical intermeshing spur

wheels working with a fine clearance inside a suitably shaped casing. One gear is keyed to the driving shaft of a motor and the other revolves idly. Oil is entrained in the spaces between the teeth and the casing and is carried round between the gears from the suction port to the discharge port. It cannot slip back into the inlet side due to the meshing of gears.

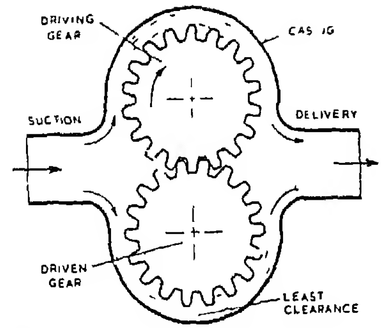


Fig 16 1 (*a*) Line Diagram of Gear Pump

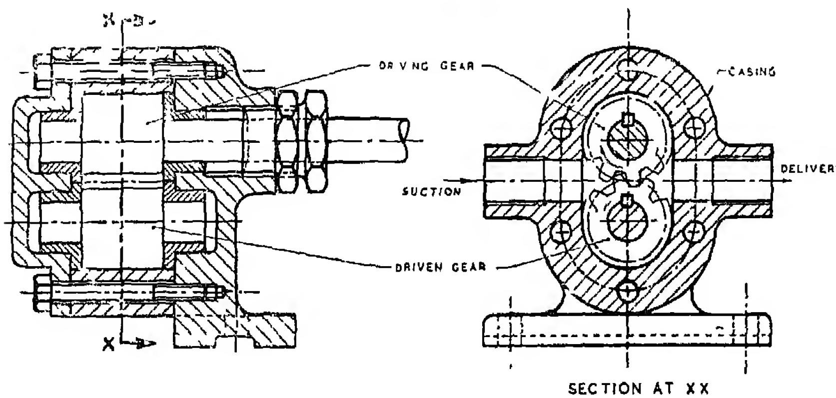


Fig 16.1 (*b*) External Type Gear Pump—Sectional View

If a=area enclosed between two adjacent teeth and casing,
l=axial length of teeth,
n=number of teeth in each pinion

and N=speed in rpm,
then, volume of liquid pumped in one revolution$=2 \cdot a \cdot l \cdot n$

$\therefore$ Total or ideal discharge$=\frac{2 \cdot a \cdot l \cdot n \cdot N}{60}$

If η_Q be the volumetric efficiency, the actual discharge,

$$Q=\frac{2 \cdot a \cdot l \cdot n \cdot N}{60} \cdot \eta_Q \qquad \text{...(16.1)}$$

The thicker the fluid, the higher is the efficiency η_Q.

If it is difficult to determine a, the area enclosed between the teeth and the casing, then the following empirical relation could be used for finding the volumetric displacement of pump per revolution :

$$q=0\ 95\ \pi \cdot c \cdot (D-c) \cdot l \qquad \text{.. (16.2)}$$

where c=centre to centre distance between axis of gears,
and D=outside diameter of gears.

Further, n the number of teeth in the gear is related to ratio $\frac{D}{c}$.

TABLE 16.1

n	7	10	13	18
$\frac{D}{c}$	1·28	1·21	1·13	1·12

Problem 16.1 A gear pump works against a total pressure of 90 lb/sq in. delivering 85 gpm of oil of specific gravity 0·92, when running at 720 rpm. The volumetric efficiency of the pump is 95% and its overall efficiency is 50%. The length of gear wheels is 1·5 times their outside diameter. Find the outside diameter of wheel and the horsepower input of the pump. Assume each gear has 12 teeth.

Solution

$p=90$ lb/sq in. $\qquad Q=85$ gpm

Sp gr $=0·92$ $\qquad N=720$

$\eta_Q=0·95$ $\qquad \eta_{overall}=0·5$

$l=1·5\,D$ $\qquad n=12$

Take the value of ratio $\frac{D}{c}=1·18$ from Table 16·1

$$q \text{ per revolution}=\frac{85\times 10}{62·4\times 0·92\times 720}=0·00205 \text{ cu ft}$$

or $\quad 0·00205\times 1{,}728=35·6$ cu in.

$$\therefore \quad 35·6=0·95\times\pi\,.\,c\,.\,(D-c)\,.\,l$$

$$=0·95\times\pi\times\frac{D}{1·18}\times\left(D-\frac{D}{1·18}\right)\times 1·5\,D$$

or $\quad D^3=\dfrac{35·6\times 1·18\times 1·18}{0·95\times\pi\times 0·18\times 1·5}=61·6$

or $\quad D=3·96\approx$ **4 in.** *Answer*

A single gear pump can build up pressure as high as 2,000 lb/sq in. (or 140 kg/cm²). However, due to the possibility of internal leakage it is generally used for a delivery pressure upto 400 lb/sq in. (or about 30 kg/cm²) only. Gear pump finds its application in machine tools drive using oil at relatively low pressure, for combined lubrication and hydraulic control systems of water turbines, steam turbines and other machines.

Fig 16 1 shows a pump having straight spur wheels, which make noise because the oil trapped between intermeshing teeth cannot easily escape. To avoid this difficulty single helical or double helical teeth gears are used. They run quieter at high speeds.

The gears of such pumps are generally external; however, internal gears are also used in exceptional cases.

16.6 Internal Type Gear Pump—Fig 16.2 shows an internal spur gear pump. The idler which is an internal gear meshes with the driving gear. The space between the outside diameter of driving gear and the inside diameter of idler is sealed by a crescent shape projection which form a part of the cover.

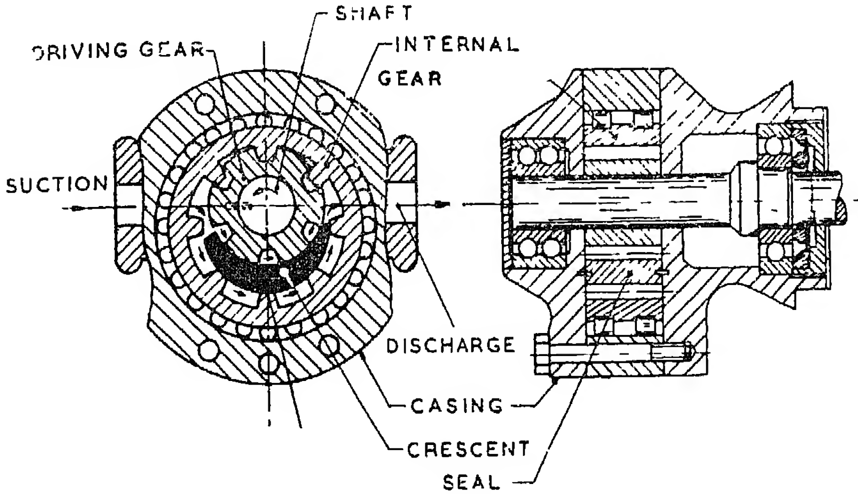


Fig 16.2 Internal Type Gear Pump

The driving gear gets its motion from the driving motor. As the teeth come out of the mesh, there is an increase in volume which creates a partial vacuum. Liquid is forced into this space by atmospheric pressure and stays in the spaces between the teeth of the driving gear and idler until the teeth mesh when, the liquid is forced from these spaces and out of the pump. The pump can be reversible if a provision is made for swinging the crescent through 180°.

16.7 Screw Pump (Fig 16.3)—Screw pump consists of a rotor or screw to which the source of power is directly connected. Unlike

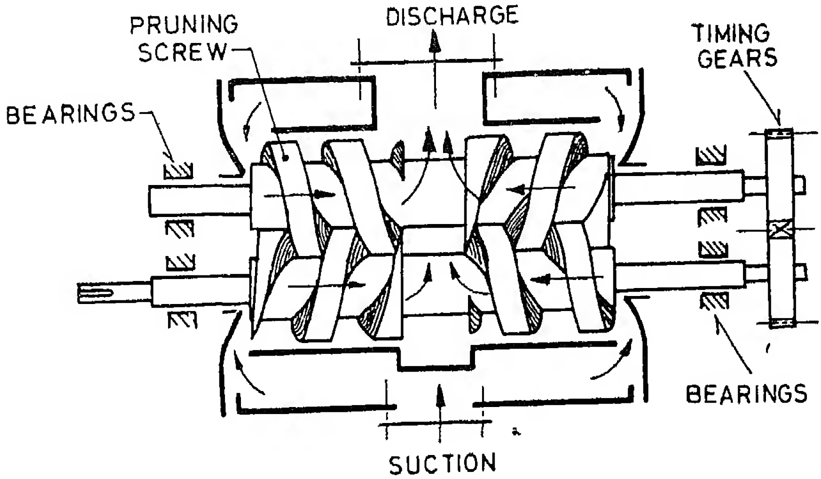


Fig 16.3 Screw Pump

gear pump the rotors of screw pumps are extended and are of smaller diameter. The screw may be double helical (right and left hand) or single helical (one hand only). The advantage of double helical is that they are balanced axially. The fluid is carried forward to the discharge along the rotor in pockets formed between teeth and the casing.

There may be one, two or three screws. In a 2-screw pump one is a rotor and the other is idler. In a 3-screw pump there are two idlers on either side of the rotor. Two idlers act as seals to the power rotor and are driven by fluid pressure and not by metallic contact with rotor. The liquid entering the inlet passage divides and flows to the two ends of the rotor where it is trapped in pockets formed by the threads and is carried towards the discharge, like a nut on the power screw.

Advantages :

a) Screw pump operates upto a continuous working pressure of 2,000 lb/sq in. (or 140 kg/cm^2)-

b) The screw pump is free from turbulence and pulsation.

c) The rotors of a screw pump are balanced dynamically, the pump is silent and free from vibration while running.

d) Screw pump is especially suitable for operation at high speed and thus can be directly driven by an electric motor or steam turbine running at 3,000 to 4,500 rpm.

16.8 Vane Pump—Vane pump consists of a rotor disc having a number of slots into which fit sliding blades or vanes (See Fig 16.4).

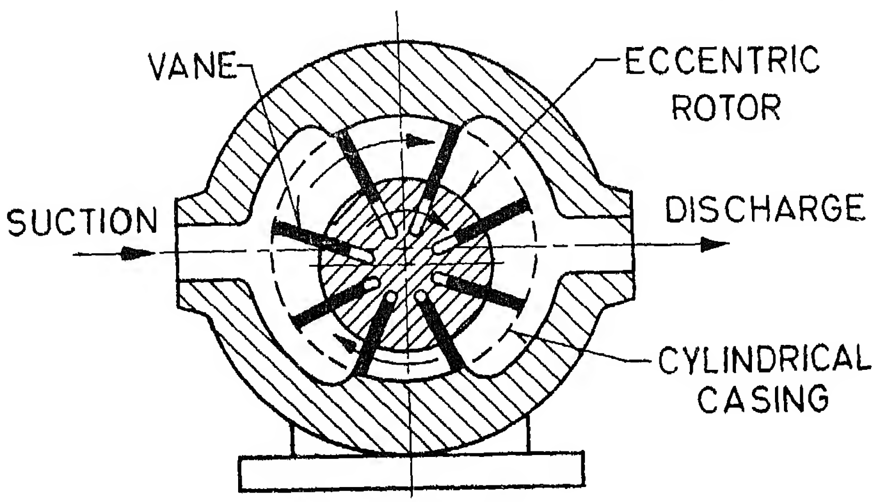


Fig 16.4 Vane Pump

The rotor turns in a casing which is bored eccentric to the driving shaft the vanes are free to slide radially with the help of springs and thus form the required seal between the suction and discharge connections. When the rotor turns, the liquid flows in between its side and casing and is trapped by the vanes, which force it from inlet to discharge.

Fig 16.5 shows a modified form of vane pump. The rotor fitted with vanes is supported by bronze bushes. These are integral with plates

constraining the rotor and vanes axially. Drive is by means of the splined shaft carried on ball bearings. The sleeve, machined oval internally, surrounds the vanes. The suction and delivery connections respectively, communicate with two ports, diagonally opposite in the plate valve. Thus the rotor is hydraulically balanced, and the pockets between the vanes undergo two cycles of expansion and contraction during each revolution. Therefore this type of vane pump will handle twice the amount of fluid handled by simple vane pump shown in Fig 16.4.

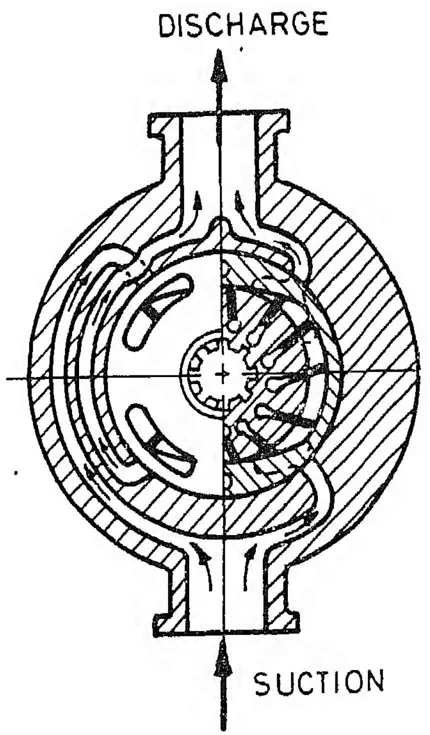


Fig 16.5 Vane Pump

Advantages :

a) Single stage vane pump can be used for oil pressure of 250 to 1,000 lb/sq in. (or 17·5 to 70 kg/cm^2) continuous rating. Two-stage pump can operate at 2,000 lb/sq in. (or 140 kg/cm^2).

b) Wide range of application, especially for machine tools. For industrial equipment the vane pump is possibly the most used at the present time.

16.9 Radial Piston Pump—Radial piston pump consists of a number of cylinders disposed radially about an eccentric on the driving shaft. The rotation of shaft causes reciprocation of the pistons. The demand for operating pressures above 1,000 lb/sq in. (or 70 kg/cm^2) was responsible for the design of such a pump. Operating pressure upto 6,000 lb/sq in. (or 420 kg/cm^2) could be produced with a very high speed and volumetric efficiency. This is mainly due to the spherically seated valve used in the pump.

Fig 16.6 shows a radial piston pump manufactured by Automotive Products Co Ltd. The oil enters the pump through inlet connection *A* and passes through inlet port *B* to reach an annulus in the end cover which forms a part of the pump chamber. The cylinder blocks *C* are bolted to the end cover and have inlet ports on the opposite side. Oil flooding the pump chamber reaches the inlet ports by way of annular clearance space inside the casing *D*. Each outlet port in the high pressure end cover leads to a spring-loaded non-return valve *E* communicating with the outlet connection *G*. The piston is moved through its suction stroke by two retaining rings which engage flats on the piston driving pins. These fit in slippers which transmit thrust to and from a bearing ring mounted on needle rollers on the eccentric. Two roller bearings carry the shaft. Output is about 70 cu in./min (or 1150 cm^3/min) at 3,000 rpm.

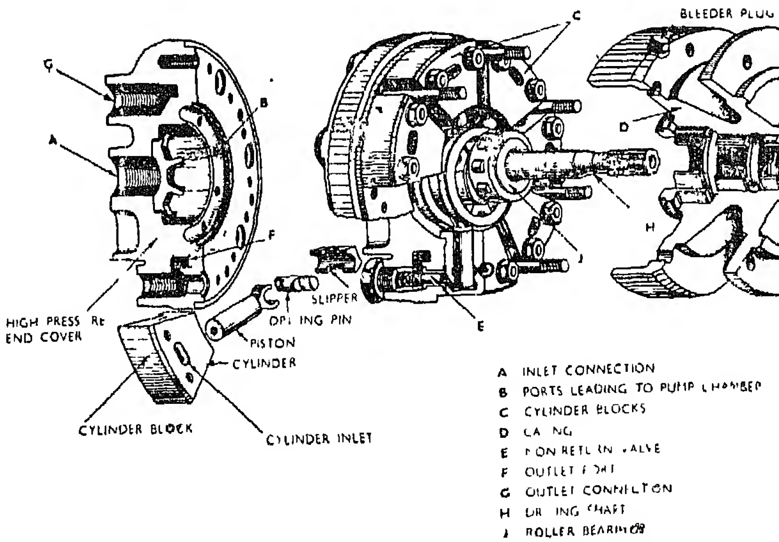


Fig 16.6 Radial Piston (Constant Delivery) Pump
(Manufactured by Automotive Products Co Ltd)

VARIABLE DELIVERY PUMPS

16.10 Rotary Piston Pump—Fig 16.7(*a*) shows a rotary piston pump. It consists of a cylinder body *A* having a number of radial pistons (in this case 5). The cylinder body rotates about a fixed control cylindrical valve axis *D*. The valve has suction and delivery openings. The pistons are connected to a floating ring *B* which is driven by the shaft *C*. The valve axis *D* is fixed to a swing arm *E* and can be adjusted to the piston corresponding to the desired output with hand wheel *F*. As in a vane pump, the fluid volume pumped depends upon the degree of eccentricity.

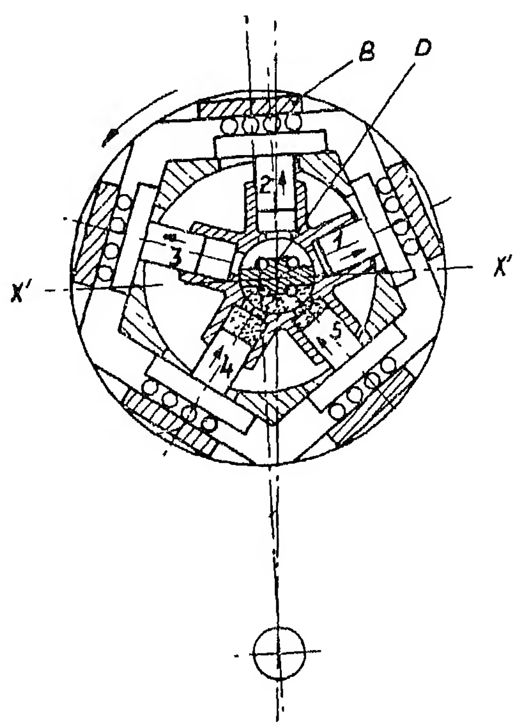


Fig 16.7 (*b*) Rotary Piston Pump

Working Principle—The floating ring *B* is driven over the shaft *C* in the direction shown in Fig 16.7 (*b*). As the axis *D* is made eccentric, the pistons move to and fro. Piston 1 moves outward as shown by an arrow and sucks the oil. Suction takes place in all

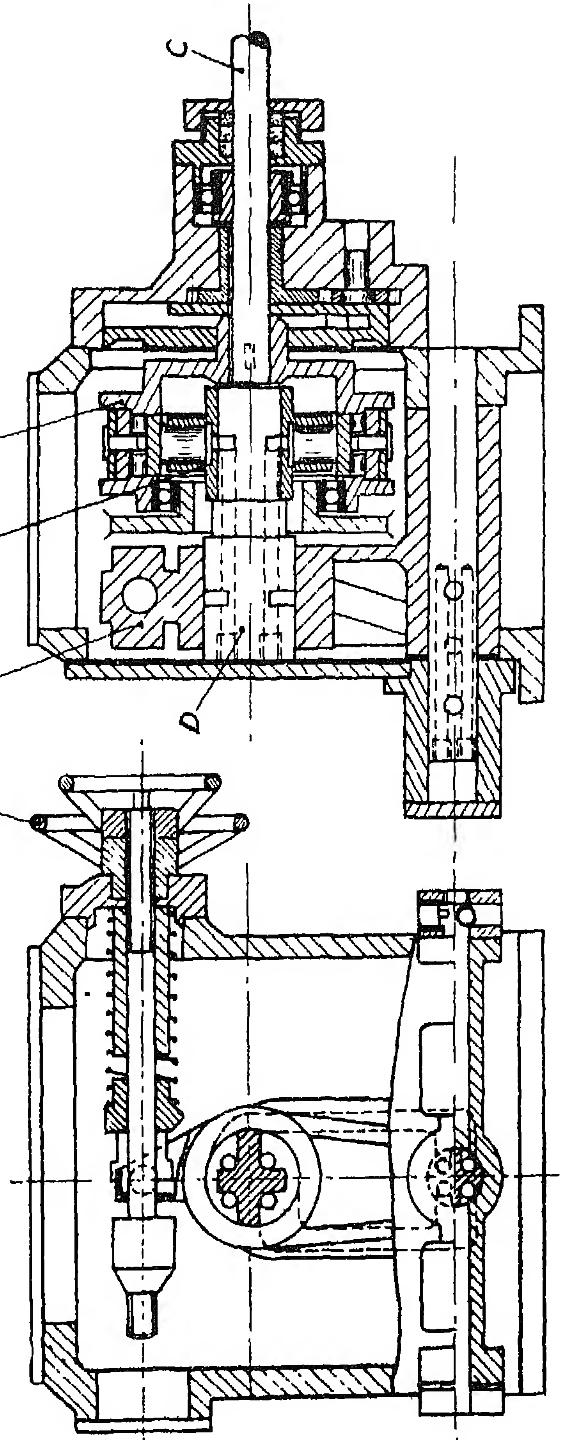


Fig 16 7 (a) Rotary Piston Pump

pistons above the centre line $X'X'$. Below the line $X'X'$, delivery takes place for the pistons 4 and 5 moving inward. The pressure generated is about 1,000 lb/sq in. (or 70 kg/cm^2) when running at 860 rpm.

Advantages—The oil flow can be regulated in both positions positive as well as negative (reverse direction), which is very useful in machine tools.

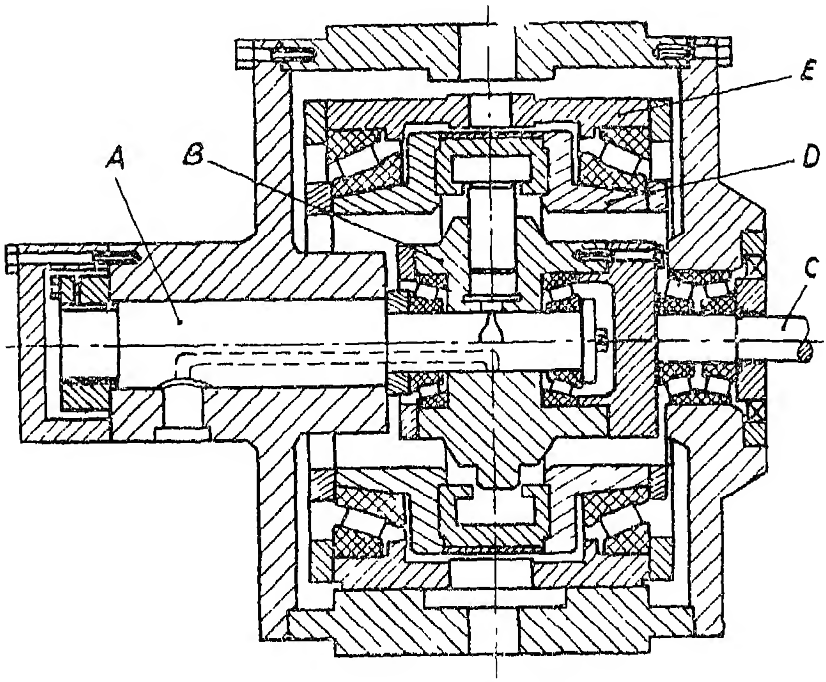


Fig 16.8 (*a*) Radial Piston (Variable Delivery) Pump

16.11 Radial Piston Pump—Fig 16.8 (*a*) shows a radial piston pump for variable discharge. In this case the regulation of discharge is not effected by shifting the valve axis as in the case of rotary piston pump (Art 16.10) but, by the floating ring with which the pistons move eccentrically. The valve axis *A* is fixed to the casing and carries the cylinder body *B*. The suction and delivery pipes are bored axially in the valve axis.

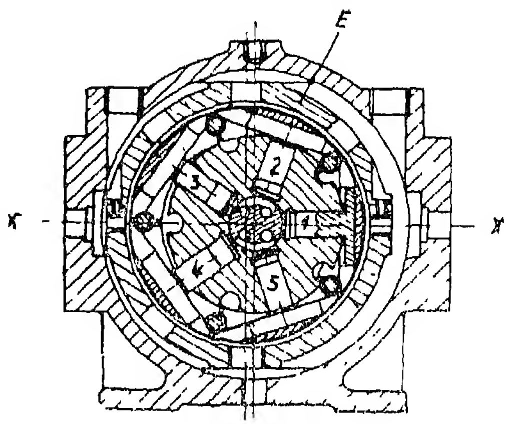


Fig 16.8 (*b*) Radial Piston (Variable Delivery) Pump

Working Principle: The cylinder body *B* is driven by the shaft *C*. The floating ring rotates in the outer ring which can be shifted horizontally in the casing to obtain the required eccentricity for the variation of discharge.

. In Fig 16.8 (*b*), piston 1 has finished its delivery stroke and would move outwards by further rotating the floating ring (see piston positions 2 and 3) which means it would commence sucking. Below the centre line XX the motion is reversed and the piston movement is inward (see piston positions 4 and 5). The direction of flow depends upon the piston of outer ring E.

The maximum pressure generated would be about 2,600 lb/sq in. (or 180 kg/cm^2). The speed of the pump is 900 to 1,800 rpm according to its capacity.

HYDROSTATIC SYSTEMS

16 12 Functions of Hydrostatic System—There are mainly three functions of a hydrostatic system :

a) Actuation of mechanical control,

b) Force multiplication and

c) Transmission and control of power.

a) **Actuation of Mechanical Control**—Control of the motion of driving and driven mechanisms is achieved by means of transmission by the fluid of power and thrust. The development of power and force is of less importance than the nature of the motion imparted to driven unit. An example of an application of this type is the use of a hydraulic relay. An arrangement in which oil forms a power-driven pump is used for control purposes constitutes a hydraulic relay. Prime movers such as water turbines and steam turbines are controlled by means of hydraulic relays. The control of governing of water turbines by hydraulic relay is described in Chapter 8.

b) **Force Multiplication**—In this system, transmission of thrust is the important consideration. Transmission of power is also involved, example of application—Hydraulic Press (see Art 16.14.)

c) **Transmission and Control of Power**—Power is transmitted for producing movement at a distance. The hydrostatic system always includes the control of movement and speed of driven unit. Transmission of thrust also occurs, but is not so important as in case (*b*). Hydraulic drives in machine tools described in Art. 16.21 are applications of this kind.

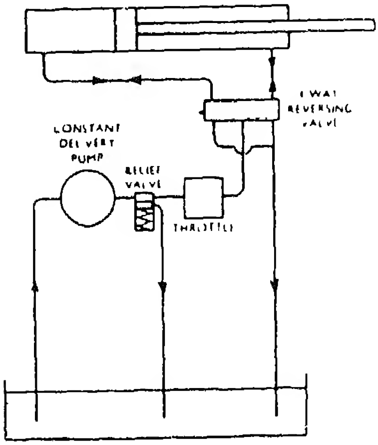
16.13 Methods of Control—The motion of hydraulic motor or receiver is governed by the flow of oil. Two methods are used for this purpose depending on the design of the pump :

a) Constant delivery system and

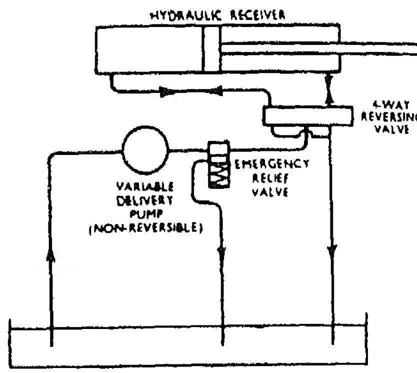
b) Variable delivery system.

a) **Constant Delivery System**—The discharge of the pump is constant and the flow in the circuit is controlled by means of regulating valves. The system consists of a throttle valve fitted in the circuit between the motor and the receiver. As the pump discharges at a constant rate, a by-pass or relief valve is provided to return the excess oil to the tank, as well as to protect against overloads. The flow of oil passes through a four-way reversing valve which delivers the oil to the two sides

of the piston as shown in Fig 16.9 (*a*). Constant delivery system is cheaper than the variable delivery system but, the delivery of excess oil back to the tank represents a loss of power.



a) Constant Delivery System



b) Variable Delivery System

Fig 16.9 Methods of Control

b) **Variable Delivery System**(See Fig 16.9 *b*)—The delivery of pump is variable and the flow in the circuit is controlled by the variable pump output. Variable delivery pumps are complex and costly ; however, there is no wastage of power by diversion of excess oil back to the tank. The pump discharge can be varied or direction of flow reversed by employing a rotary piston pump described in Art 16.7 (*a*). In this system the pressure, against which the pump discharges, is governed by resistance to motion of the driven unit, and not by the rate of delivery of oil by the pump. It is possible to arrange for the delivery rate to vary automatically and independently of back pressure. A relief valve is employed to protect the system against overload.

16.14 Hydraulic Press—Hydraulic press was first built in 1785 by Ernest Bramah. Since then, hydraulic power has been in use for heavy operations in cranes, lifts and capstans. With the development of electric power, the hydraulic applications were replaced until more recent times when hydraulic power has come into prominence again. As in a modern machine shop individual drive has replaced the line shaft, similarly in case of modern hydraulic drive the accumulator system has been replaced by self-contained hydraulic systems, using a separate pump for each machine.

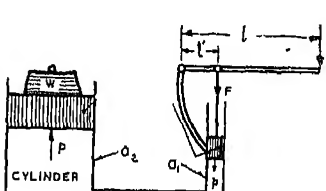


Fig 16.10 Principle of Hydraulic Press

Principle—Mechanical advantage is obtained by a practical application of Pascal's Law of transmission of fluid pressure. Two pistons of different size operate inside two cylinders suitably connected with a pipe so that pressure in each is the same (See Fig 16.10). If p be the pressure and a_1 and a_2 the cross-sectional areas of cylinders, then a force F_1 applied to the

smaller plunger will make available a large force F_2 at the large plunger.

$$p=\frac{F_1}{a_1}=\frac{F_2}{a_2} \qquad ...(16.3)$$

$$\therefore \quad \frac{F_2}{F_1}=\frac{a_2}{a_1} \qquad ...(16.3a)$$

If the volume of liquid is constant, the displacement of large piston will be proportionately smaller.

$\therefore$ Mechanical advantage of

$$\text{press}=\frac{a_2}{a_1} \qquad ...(16.4)$$

If force in the smaller piston is applied by a lever which has a mechanical advantage $\frac{l}{l'}$ then total mechanical advantage of machine

$$=\frac{l}{l'}\cdot\frac{a_2}{a_1} \qquad ..(16.4a)$$

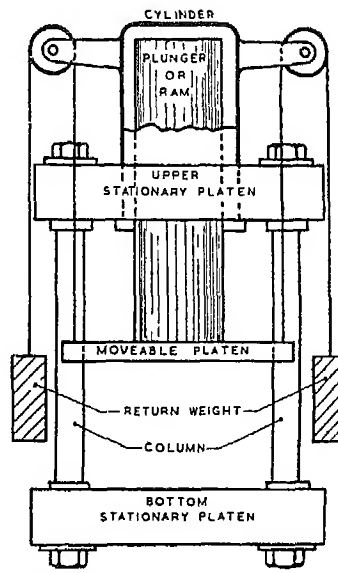


Fig 16.11 Elementary Inverted Press

Essentials of Hydraulic Press—A hydraulic press consists of two stationary platens and a movable platen (See Fig 16.11). The latter is carried by a plunger or ram which passes through the upper stationary platen and moves in the cylinder. The upper and lower stationary platens are joined by columns. Under hydraulic pressure, which is supplied by the pumps, the ram moves down and applies tremendous pressure upon any material placed between the moveable and stationary platen. Fig 16.12 shows a hydraulic press used to compress plates. The pumping unit is shown on the right front. This can exert a force of 4,500 tons. Fig 16.13 shows an autoclave vulcanising press.

Hydraulic press is employed to perform many odd jobs requiring tremendous pressures. Some examples are given below :

i) Metal Press Work : Employed to press sheet metal to any desired shape (Fig 16.12),

ii) Bakelite Press : Used to prepare moulds and castings of bakelite,

iii) Autoclave Vulcanising Press, (See Fig 16.13)

iv) Packing Press,

v) Cotton Press,

vi) Metal Pushing Press,

vii) Filter Press,

viii) Forging Press,

ix) Plate Press,

x) Tablet making machine and

xi) Drawing and Pushing rods, bending and straightening any metal piece.

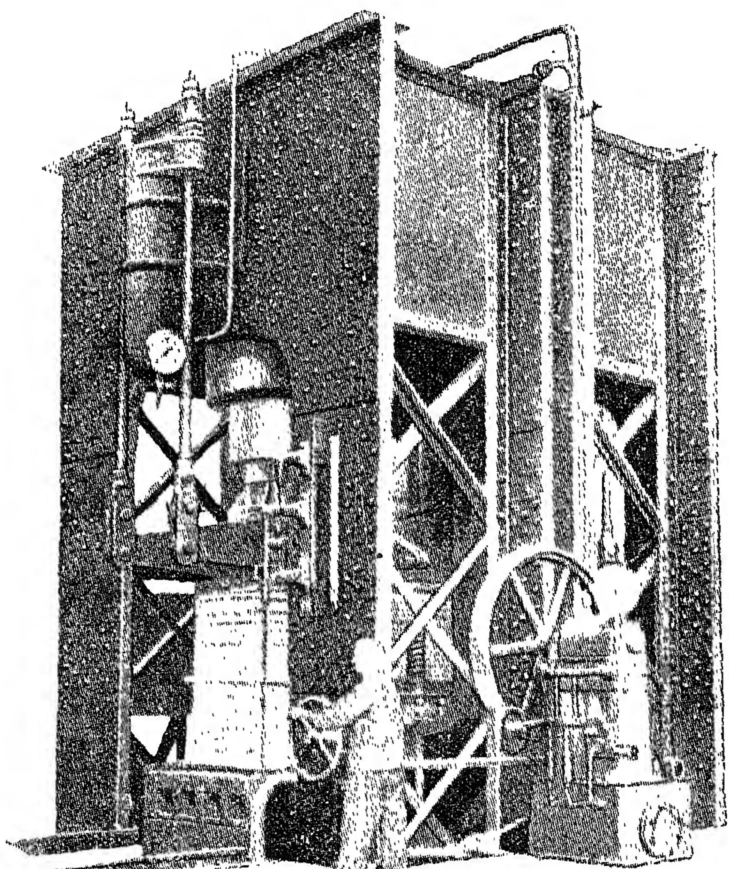


Fig 16.12 Hydraulic Plate Press for 4,500 Tons Pressure with Pumping Unit. Total Weight 3,20,000 lb

Manufactured by Theoder Bell & Co Ltd, Kriens (Switzerland)

These presses are generally worked by electrically driven pumps supplying oil under pressure.

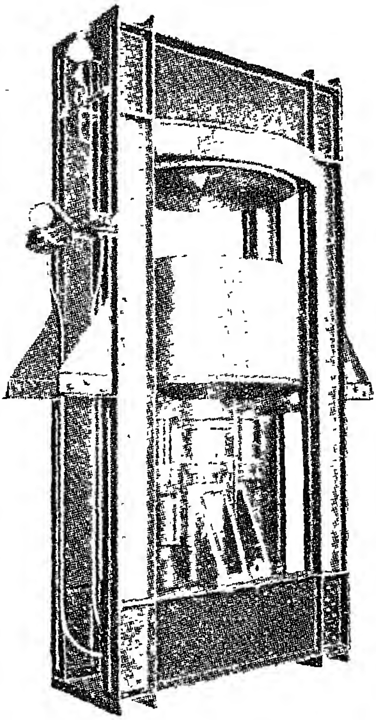


Fig 16.13 Autoclave Vulcanising Press for 150 Tons Pressure. Manufactured by Theoder Bell & Co Ltd, Kriens (Switzerland)

16.15 Hydraulic Crane (See Fig 16.14) consists of two main parts *jigger* and crane itself. The jigger attached to the most of cranes, is made up of a cylinder and a ram, both having a set of pulleys, at their ends as shown in Fig 16.14. The water under pressure is forced into the cylinder, which pushes the ram vertically up. The ram descends by opening an outlet valve. The set of pulleys fixed to the cylinder remain stationary while the other set fixed to the ram moves vertically up or down. A wire rope with one of its ends fixed to a movable pulley and passing over all the pulleys, is stretched over the jib of the crane, down to the load W. The velocity ratio of crane hook to ram, depends upon the number of set of pulleys, thus a six-sheaf pulley block system will have a velocity ratio of 6 : 1, which means the load suspended to the hook of the crane will move six times the speed of the ram.

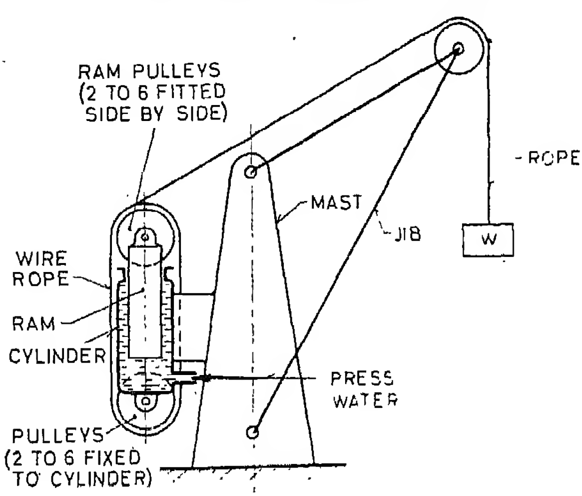


Fig 16.14 Hydraulic Crane

The crane consists of a vertical mast to which is attached a jib. The mast is supported on a pedestal, and can be revolved with its vertical axis. The jib swings with the mast and can be raised or lowered together with the load by means of wire rope when the ram moves in the cylinder.

Hydraulic crane finds its use in docks and warehouses. It can lift a load upto 250 tons. However a hydraulic crane can hardly compete with the modern electric crane.

16.16 Hydraulic Lift (See Fig 16.15)—A simple form of hydraulic lift consists of a cylinder and a ram. The water under pressure is forced into the cylinder, thus pushing the ram vertically upwards. A platform or cage is fitted to the top end of the ram. The platform can be made to stay, siding with the floors, so that one can walk over it. The platform or cage should move between guides.

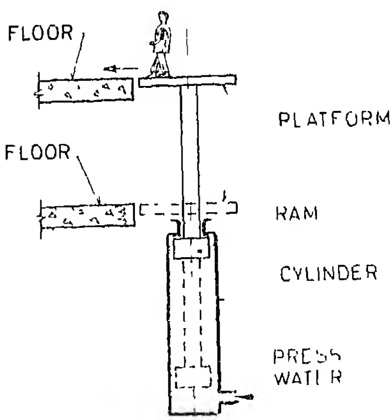


Fig 16.15 Hydraulic Lift

The modified form of hydraulic lift is fitted with a jigger explained in Art 16.15, attached to hydraulic crane. The weight of cage in which the persons or load ascend, is balanced by balance weights which also moves in guides.

Electric lifts have become quite common now-a-days.

16.17 Hydraulic Jack—This is based on the same principle as the hydraulic press. It is short-stroke hydraulic lift which is fed from hand pump. The hydraulic jack may be portable. This is extensively used for lifting automobiles usually to facilitate cleaning and repair.

16.18 Hydraulic Riveter (See Fig 16.16)—It was very popular for making rivet heads but has lately been replaced by pneumatic riveters. A hydraulic riveter can exert a thrust of 50 tons on the rivet.

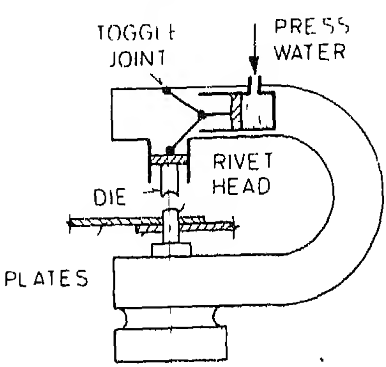


Fig 16.16 Hydraulic Riveter

At the time of closing, the cross-sectional area of rivet decreases which means a large pressure will then be required. This is supplied by means of toggle joint.

16.19 Pressure Accumulator—It is a device to accumulate liquid under pressure delivered by the pump when it is not required by the machine. The pressure can be later supplied to the machine as and when needed.

The various presses enumerated above require separate pumping units to furnish liquid at the desired pressure. Such pumps are known

as *Press Pumps.* Normally, the pressure generated by these pumps ranges from 750 lb/sq in. to 1 ton/sq in. (or about 50 to 160 kg/cm^2) and is uniform throughout the supply period. However, the demand for liquid and its required pressure are variable. At some instants the machine may not be doing any work at all, and to deal with such difficulties an arrangement to receive and store the liquid, being constantly supplied by the pump, is necessary. The device should be able to deliver the liquid back to the machine when needed. In some cases it may be desirable to obtain the stored liquid at a pressure higher than that provided by the pump itself. All this is done by the pressure accumulator or the hydraulic accumulator.

The hydraulic accumulator (See Fig 16.17) consists of a cylinder and a plunger generally known as a ram. One side of the cylinder is connected to the press pump and other to the hydraulic machine. Either the cylinder or the ram may be fixed. Generally, the cylinder is fixed and the ram moves up and down to accomodate a variable quantity of liquid inside by cylinder.

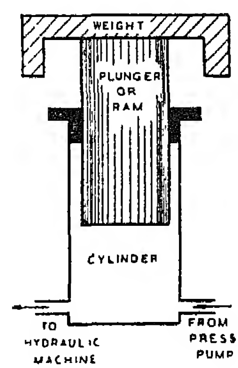


Fig 16.17 Hydraulic Accumulator

Accumulators may be dead load type or variable load type. In the former, dead weights are employed to press the plunger in, while the latter employs steam pressure. Main advantage of this type is that the pressure may be varied at will but, it is handicapped by the need of a boiler to supply steam. It can be used, however, on ships if steam is readily available.

The accumulator also serves the purpose of a pressure regulator. A suitable arrangement can be easily made to switch off and start the motor after a certain travel of the ram.

Capacity of Accumulator is the maximum amount of energy which is stored by it. The storage capacity is equal to the potential energy of the lifted ram together with weight.

Let d = diameter of ram

s = stroke or lift of ram

p = intensity of pressure of water supplied

Total moving weight or weight of ram $W = \frac{\pi}{4} d^2 \cdot p$

Work done in lifting ram or capacity of accumulator

$$= W \cdot s = \frac{\pi}{4} d^2 \cdot p \cdot s \qquad \text{...(16.5)}$$

Volume of accumulator $= \frac{\pi}{4} d^2 \cdot s$

$\therefore$ Capacity of ram $= p \times$ volume ...(16.5*a*)

Problem 16.2 The weight of a 14 in. plunger of an accumulator is 10,000 lb. What additional weight is to be placed upon it to develop a hydraulic pressure of 600 lb/sq in.?

Solution

$d=14$ in. $\quad p=600$ lb/sq in. $\quad w_1=10{,}000$ lb

Cross-sectional area of plunger $a=\frac{\pi}{4}\times 14^2=154$ sq in.

Total weight to balance the hydraulic pressure

$$W=w_1+w_2$$

where $w_2=$ (weight to be added)

Now $W=p\,.\,a$

$$=600\times 154=92{,}400 \text{ lb}$$

$\therefore$ $w_2=W-w_1=92{,}400-10{,}000$

$$=\mathbf{82{,}400\ lb} \quad \textit{Answer}$$

Problem 16.3 Find the length of stroke required for an accumulator having a displacement of 25 gallons (or 113·5 litres). The diameter of the plunger is 14 in. (or 356 mm).

Solution

Displacement of accumulator is the volume displaced by the plunger per stroke.

$$\text{Displacement}=25 \text{ gallons}=\frac{25\times 10\times 1{,}728}{62{\cdot}4}$$

$$=25\times 277=6{,}925 \text{ cu in.}$$

$$\left[\text{or Displacement}=113{\cdot}5 \text{ litres}=\frac{113{\cdot}5}{1{,}000}=0{\cdot}1135 \text{ m}^3\right]$$

$$\text{Cross-sectional area of plunger}=\frac{\pi}{4}\times 14^2=154 \text{ sq in.}$$

$$\left[\text{or area}=\frac{\pi}{4}\times 0{\cdot}356^2=0{\cdot}0994 \text{ m}^2\right]$$

$$\therefore \text{ Length of stroke}=\frac{\text{displacement}}{\text{area of plunger}}$$

$$=\frac{6{,}925}{154}=45 \text{ in.}$$

$$=\mathbf{3{\cdot}75\ ft} \quad \textit{Answer}$$

$$\left[\text{or Length of stroke}=\frac{0{\cdot}1135}{0{\cdot}0994}=\mathbf{1{\cdot}142\ m} \quad \textit{Answer}\right]$$

Problem 16.4 An accumulator has a ram 12 in. in diameter, an effective stroke of 20 ft and is loaded with a total weight of 50 tons. If the friction of ram amounts to 3% of the total load, find the total horsepower delivered to the hydraulic machine if the ram falls steadily

through its full stroke in 2 minutes, while at the same time the pump delivers 100 gpm.

(Delhi University—1957)

Solution

$d = 12$ in. $= 1$ ft $\qquad S = 20$ ft

Load $\quad W = 50 \times 2240$ lb $\qquad$ Friction loss $= 3\%$

$\therefore$ Net load $= 50 \times 2240 \times 0{\cdot}97$ lb

Time of ram fall $= 2$ min

$Q_{pump} = 100$ gpm

Work is supplied to hydraulic machine by the pump which is working continuously as well as by the accumulator ram which is falling steadily through its weight.

i) Work supplied by accumulator ram per minute

$$= (50 \times 2240 \times 0{\cdot}97) \times \frac{20}{2} = 1{,}087{,}000 \text{ ft lb}$$

ii) Intensity of pressure in accumulator

$$p = \frac{W \times 0{\cdot}97}{\frac{\pi}{4} \times d^2} = \frac{50 \times 2240 \times 0{\cdot}97}{\frac{\pi}{4} \times 1^2}$$

$$= 1{,}38{,}500 \text{ lb/sq ft}$$

Head H due to this pressure

$$= \frac{p}{w} = \frac{138{,}500}{62{\cdot}4} = 2{,}220 \text{ ft of water}$$

Work supplied by the pump per minute

$$= (w \,.\, Q)\ H = (100 \times 10) \times 2{,}220$$

$$= 2{,}220{,}000 \text{ ft lb/min}$$

$\therefore$ Total work supplied to hydraulic machine

= work supplied by ram + work supplied by pump

$= 1{,}087{,}000 + 2{,}220{,}000 = 3{,}307{,}000$ ft lb/min

or Total HP supplied to hydraulic machine

$$= \frac{\text{Work done/min}}{33{,}000} = \frac{3{,}307{,}000}{33{,}000} \approx \textbf{100 HP} \quad \textit{Answer}$$

Problem 16.5 In an installation of 6 hydraulic cranes, the working cycle of each of which takes 90 seconds in hoisting and lowering, each crane is fed with water at a pressure of 50 kg per sq cm and is required to lift a load of 50 tons at a speed of 18 meters per minute, through total height of 12 meters, the jigger system giving a velocity ratio of 6. Estimate the stroke and diameter of the rams assuming an efficiency of 60%.

It is assumed that all the six cranes are making the working stroke at the same time. Calculate the minimum capacity of the pump feeding the installation, and that of the accumulator.

(*Madras University—March 1955*)

Solution

No. of cranes$=6$ $\qquad t=90$ seconds

$p=50$ kg/cm^2 $\qquad W=5$ tons$=5000$ kg (assuming metric tonne)

$v=18$ m/min $\qquad h=12$ meters

velocity ratio$=6$ $\qquad \eta=60\%$

Load on each ram $\times$ velocity of ram $\times\,\eta$

$=$Load lifted $\times$ velocity of lifting load

$$\therefore \text{ Load on each ram}=\frac{W\times\text{vel. ratio}}{\eta}$$

$$=\frac{5000\times 6}{0{\cdot}6}=50{,}000 \text{ kg}$$

$$\text{Area of ram}=\frac{\text{Load on ram}}{\text{pressure } p}=\frac{50{,}000}{50}=1{,}000 \text{ cm}^2$$

$$\therefore \quad \text{Dia of ram}=\sqrt{\frac{1{,}000}{\frac{\pi}{4}}}=\mathbf{35{\cdot}63\ cm} \quad \textit{Answer}$$

$$\text{Stroke of ram}=\frac{\text{Distance } h}{\text{velocity ratio}}=\frac{12}{6}=\mathbf{2\ m} \quad \textit{Answer}$$

Volume swept by ram$=$area of ram $\times$ stroke

$=1{,}000\times(2\times 100)$

$=2\times 10^5$ cm^3

Since there are 6 cranes, volume of water supplied by the pump

$=2\times 10^5\times 6$ cm^3

$=12\times 10^5$ cm^3

$\therefore$ Minimum capacity of feeding pump

$$=\frac{\text{volume of water}}{\text{time taken}}=\frac{12\times 10^5}{90}\times\frac{1}{1{,}000}$$

$=13{\cdot}33$ lit/sec

$$\text{Operating time of cranes}=\frac{h}{v}=\frac{12}{18}\times 60=40 \text{ seconds}$$

$\therefore$ Idle time of crane$=90-40=50$ seconds

But the pumps go on feeding the accumulator

$\therefore$ Accumulator Volume$=13{\cdot}33\times 50$

$=$**666·5 litres** *Answer*

Accumulator Capacity $=p\times$Volume

$=50\times(666{\cdot}6\times 10^3)$ kg-cm

$$\text{or} \qquad \frac{50\times 666{\cdot}6\times 10^3}{100}=\mathbf{3{\cdot}33\times 10^5\ kg\text{-}m} \quad \textit{Answer}$$

Problem 16 6 An accumulator maintains a pressure of 900 lb per sq in. (or 63·3 kg/cm^2) in a 2 in. (or 50 8 mm) diameter hydraulic main. A hydraulic crane situated at a distance of 800 ft (or 244 m) from the accumulator is supplied with pressure water from this main. The ram of the hydraulic crane is 9 in. (or 228·6 mm) diameter. Velocity ratio of crane hook to ram is 4 : 1. A pressure of 40 lb per sq in (or 2·81 kg/cm^2) may be assumed on the ram to account for mechanical friction of ram, pulleys etc.

Assume a co-efficient of friction for the hydraulic main as 0·01. Calculate the load lifted when it is raised with a speed of 2 ft sec (or 0 61 m/sec). *(Poona University—1955)*

Solution

Diameter of hydraulic main	=2 in. (or 50·8 mm)
Diameter of ram	=9 in. (or 228·6 mm)
Pressure in the accumulator	=900 lb per sq in. (or 63·3 kg/cm^2)
Pressure lost due to mechanical friction	=40 lb per sq in. (or 2 81 kg/cm^2)
Length of hydraulic main	=800 ft (or 244m)
Velocity ratio of crane hook to ram	=4 : 1
Co-efficient of friction for hydraulic main	=0 01
Velocity of crane hook	=2 ft per sec (or 0·61 m/sec)
Velocity of ram	$=\frac{2}{4}=0{\cdot}5$ ft per sec (0 1525 m/sec)

Let v=velocity of water in 2 in. (or 50·8 mm) main.

As quantity of water per second flowing through main equals quantity per second in the ram cylinder, then

$$v\times\frac{\pi}{4}\times(2)^2=0{\cdot}5\times\frac{\pi}{4}\times(9)^2$$

$$\left[\text{or}\quad v\times\frac{\pi}{4}\times(0{\cdot}0508)^2=0\ 1525\times\frac{\pi}{4}\times(0{\cdot}2286)^2\right]$$

or $\qquad v=10{\cdot}125$ ft per second

[or $\qquad v=3{\cdot}085$ m/sec]

Head of water in accumulator $=\dfrac{900\times144}{62{\cdot}4}=2{,}077$ ft of water

$$\left[\text{or}\qquad =\frac{63{\cdot}3\times100\times100}{1{,}000}=633 \text{ m of water}\right]$$

Head lost in friction in main $h_f=\dfrac{4\times0{\cdot}01\times800}{\frac{2}{12}}\times\dfrac{(10{\cdot}125)^2}{2\times32{\cdot}2}$

$$=308 \text{ ft of water}$$

$$\left[\text{or}\qquad h_f=\frac{4\times0{\cdot}01\times244}{0\ 0508}\times\frac{3\ 085^2}{2\times9{\cdot}81}=93{\cdot}6 \text{ m of water}\right]$$

Head lost due to mechanical friction of ram, pulley etc.

$$=\frac{40\times144}{62{\cdot}4}=92\ 5 \text{ ft of water}$$

$$\left[\text{or} \qquad = \frac{2.81 \times 100 \times 100}{1{,}000} = 28{\cdot}1 \text{ m of water}\right]$$

Net head available on the ram $=2{,}077-(308+92{\cdot}5)$

$=1676{\cdot}5$ ft of water

$$[\text{or} \qquad =633-(93{\cdot}6+28{\cdot}1)=511{\cdot}3 \text{ m of water}]$$

$\therefore$ Net intensity of pressure on ram $= \dfrac{1675{\cdot}5 \times 62{\cdot}4}{144}$

$=727$ lb per sq in.

$$\left[\text{or} \qquad = \frac{511{\cdot}3 \times 1{,}000}{100 \times 100} = 51{\cdot}13 \text{ kg/cm}^2\right]$$

Load on the ram $=727 \times \dfrac{\pi}{4} \times (9)^2 \times \dfrac{1}{2{,}240} = 20.6$ tons

$$\left[\text{or} \qquad =51{\cdot}13 \times \frac{\pi}{4} \times (22{\cdot}86)^2 \times \frac{1}{1{,}000} = 20{\cdot}95 \text{ tonnes}\right]$$

$\therefore$ Load lifted by the crane hook $= \dfrac{20.6}{4} =$ **5.12 tons** *Answer*

$\left[\text{or} \qquad = \dfrac{20{\cdot}95}{4} = \right.$ **5·24 tonnes** *Answer* $\left.\right]$

16.20 Pressure Intensifier—Pressure Intensifier, sometimes known as differential accumulator, is a device to multiply the pressure supplied by the pump to suit the requirements of a high-pressure machine. Often a fluid pressure machine requires a high pressure at a particular stage in its operation. It can be easily provided by the intensifier.

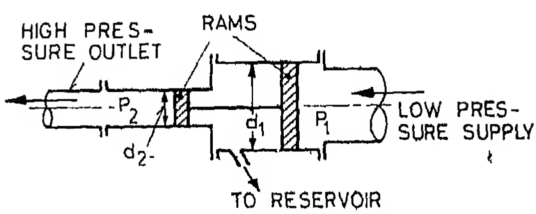


Fig 16.18 Pressure Intensifier

Normally, a simple intensifier (See Fig 16.18) consists of two coaxial rams or pistons moving in cylinders as shown in Fig 16.18. Low pressure liquid is admitted to ram or piston of large cross-sectional area which then transmits force to a small ram or piston by a rod connecting the two rams or pistons. The piston on left hand side being of smaller cross-sectional area than the piston on the right hand side, the intensity of pressure of the liquid coming out will be high. The volume between the two pistons must be vented.

Let d_1 and d_2 be the diameters of the two rams and p_1 and p_2 the respective pressures of the liquid inside them. Then, if the ram moves slowly,

$$p_1 \cdot \frac{\pi}{4} \cdot d_1^2 = p_2 \cdot \frac{\pi}{4} \cdot d_2^2$$

or
$$p_2 = p_1 \cdot \frac{d_1^2}{d_2^2} \qquad \ldots(16.6)$$

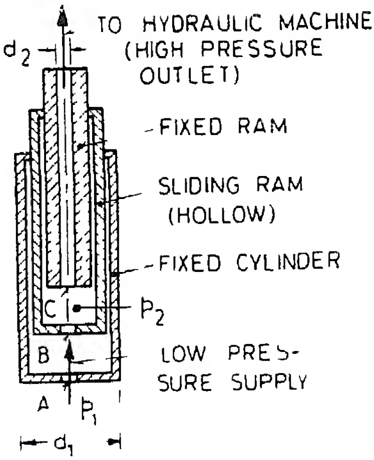


Fig 16.19 Modified Form of Intensifier

Fig 16 19 shows a modified form of intensifier which consists of two coaxial rams inside a cylinder. The cylinder and the outer ram are fixed. A, B and C are the valves provided for fixed cylinder, sliding ram and fixed ram respectively To start with, the hollow sliding ram is full with liquid and valve A and C are open. The liquid having a low pressure p_1 enters from the mains through A and forces the sliding ram upwards, so that the liquid inside the sliding ram goes out through valve C at a greater pressure p_2. When the sliding ram has reached its top position, valve B opens while the valve A and C close. The liquid which is now entrapped between the fixed cylinder and sliding ram enters inside the hollow sliding ram which then comes back to its starting position. This completes one cycle of intensifier.

Sometimes, compressed air is supplied to the larger cylinder in place of low pressure hydraulic supply, in which case the intensifier is known as a **Hydro-Pneumatic Accumulator or Intensifier.**

Steam can also be supplied to the larger cylinder in place of low pressure hydraulic supply or compressed air. In such a case it is called **Steam Intensifier.**

Problem 16.7 The diameters of small and large plungers of a pressure intensifier are 3 in. and 10 in. respectively. Find the pressure in the small cylinder, if the pressure in the large cylinder is 600 lb/sq in.

Solution

$$d_1 = 10 \text{ in.} \qquad d_2 = 3 \text{ in.} \qquad p_1 = 600 \text{ lb/sq in.}$$

$\therefore$ Pressure in the small cylinder

$$p_2 = p_1 \cdot \frac{d_1^2}{d_2^2} = 600 \times \frac{10^2}{3^2} = \mathbf{6{,}667 \text{ lb/sq in.}} \quad \textit{Answer}$$

16.21 Fluid Drives for Machine Tools—Fluid pressure is widely employed for obtaining different types of movements of modern machine tools. Pressurised oil is supplied by one or more independent pumps. Rapid progress has been made in this field in recent years. The main advantages of fluid drive in machine tools are :

i) Speed can be varied within wide limits, uniformly and gradually. With the conventional equipment speed variation in steps was unavoidable.

ii) Mechanism is more durable. Life of machine tool using hydraulic drive is about 50% more than with mechanical drive.

iii) Quick and shockless reversal of motion is made feasible.

iv) Economy in power consumption.

v) Fluid system is quiet and noiseless.

vi) Simplicity and flexibility of design.

The movements which are obtainable by oil pressure are the following :

a) Straight line movement—tables and slides.
b) Rotary movement—spindles.
c) Auxiliary movement.
a) Straight line drives are employed in :

i) Broaching machines in horizontal or vertical positions where draw head slide is traversed at the required cutting speed.
ii) Shaping machines—to move the tool.
iii) Planing machines—to move the tables.
iv) Grinding wheel—to move horizontal or vertical spindle.
v) Drilling, boring and reaming machines—to impart power and speed to spindle heads.
vi) Milling machine—to move the tables.
vii) Lathe—for sliding and surfacing motion.

b) Rotary drive is mainly employed for spindles of lathes, cutter spindles and milling machines, rotary movement of honing machine spindle and for rotary tables on surface grinding machines. The drive however is not so common as straight line drive.

c) Auxiliary movement :
i) Clamping the work in chuck, etc.
ii) Motion of tail-stock centre in lathes.
iii) Tool slide advancement in lathes.

Some of the simple hydraulic circuits employed in machine tool operations are given in the following articles.

16.22 Hydraulic Shaper—The hydraulic circuit for the operation of a shaper is shown in Fig 16.19. It consists of a variable delivery pump for pressure generation. If in place of a variable delivery pump, a constant delivery pump is used, a control valve is to be fitted in the circuit to obtain the required speeds. For low pressure requirements upto 4 to 5 HP, a constant delivery pump is generally economical and a variable delivery pump is recommended above 10 to 12 HP. For horse powers between 5 to 10, any of the two types can be used. A relief valve is provided in the circuit to take care of the pump and the system against possible overloads.

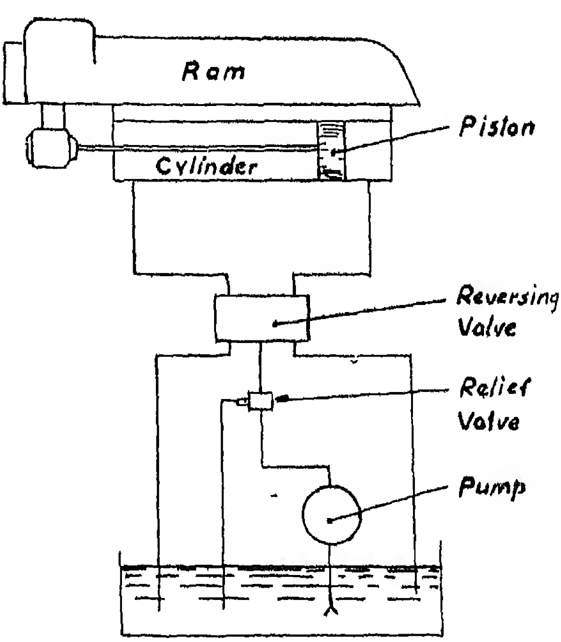


Fig 16.19 Hydraulic Shaper Circuit

16.23 Surface Grinder—The hydraulic circuit for the operation of the table of Albwerk surface grinder is shown in Fig 16.20. A is the gear pump supplying oil through valve B to double piston valve C. The latter distributes oil to cylinder F through pipes D and E. G is the throttle valve which controls the table speed. Part of the oil goes through the pipe H to control valve J which is operated either by hand lever L or automatically.

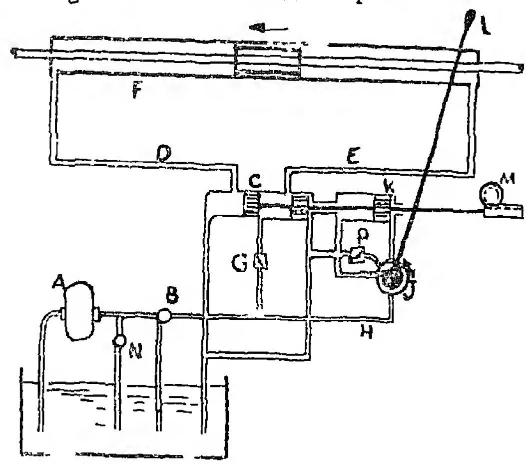


Fig 16.20 Circuit for Albwerk Surface Grinder

It controls the auxiliary piston K which is coupled with piston valve C. The piston K controls the automatic transverse motion of the cross slide through the rack and pinion arrangement M. Throttle valve P is adjusted to provide shockless reversal of the

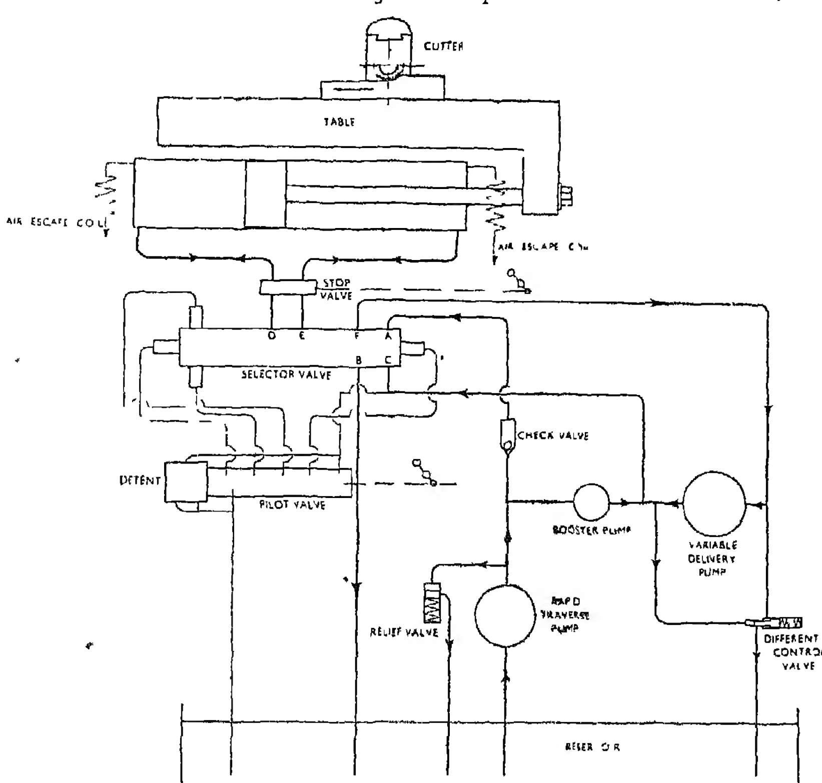


Fig 16.21 Locked Hydraulic Circuit of Hydromatic Milling Machine (Cincinnati)

table by regulating the flow of oil expelled during the movement of piston K, and consequently the speed at which the piston moves. Excess oil is by-passed through a relief valve N.

16.24 Milling Machine—Fig 16.21 shows the hydraulic circuit of a hydraulic Cincinnati make milling machine. The variable delivery pump is placed between the forward and back pressure lines. By setting this pump, feed water is controlled. A booster pump is connected to the forward pressure line to make up for leakage losses, and to maintain pressure on both forward and back pressure lines. The differential relief valve determines the relationship between the forward and backward pressures. Sum of forward and backward pressures remains constant and a variation of one is compensated for by the other. A large capacity pump is meant for rapid traversing in either direction. The selector valve controls the direction and rate of travel and it is adjusted by a pilot valve.

Finally, there is a start-stop by-pass valve from where two pipe lines are connected to the cylinder having a piston moving to and fro inside.

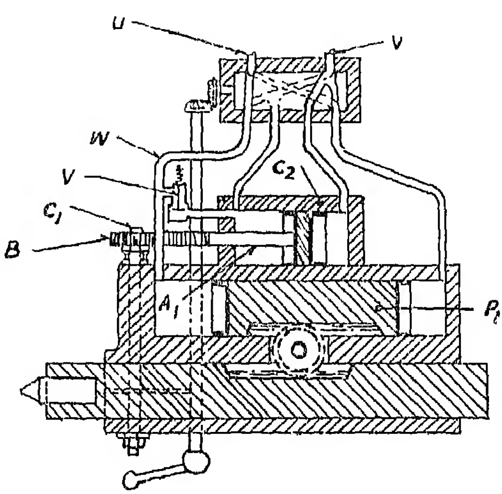


Fig 16.22 Diagrammatic Section of Tailstock with Hydraulically Operated Centre

16 25 Tail Stock—Fig 16 22 shows the hydraulic tail stock operation employed on Drummond make lathes. The centre and barrel are moved towards and away from the job by means of double acting piston P_1. Rack teeth are cut on the piston P_1 as well as the barrel and a common pinion works both of them. Either of the two ends of the cylinder C_1 are connected to the pressure supply pipe by a rotary reversing valve operated by hand lever L through bevel gearing. The opposite end is connected to exhaust. When the barrel is moved to the left in order to engage the centre with the job, momentarily, pressure is built up in pipe W. When a predetermined pressure is reached, the spring loaded valve V is opened and pressure is admitted in cylinder C_2 causing the piston P_2 (in cylinder C_2) to move towards right. The rack teeth are cut on the piston rod A_1 which meshes with pinion B. The pinion B serves as a nut to the clamping screw C_1 and the rotation of pinion clamps or releases the tail stock barrel according to the direction of movement of the piston P_2 (in cylinder C_2).

16.26 Some More Applications of Hydrostatic Transmission—There are so many applications of hydrostatic transmission that it is difficult to present a general picture which will adequately cover them all but, perhaps the following machines are worth mentioning in addition to what has already been explained in this chapter.

i) **Hydraulic Deck Machinery**—Hydraulic power is transmitted to deck machines through pipes by means of constant delivery pumps generally situated in the ship engine room, driven either by auxiliary diesel engines or by constant speed electric motors. The deck machines workable by hydraulic pressure are cargo winches, anchor windlasses, capstans and warping winches, deck cranes, mast winches and dredgers.

ii) **Agricultural Machinery**—The hydraulic power transmission system in agricultural machinery is employed in tractor external fitments through which the required services are applied to the implements. Mostly the hydraulic applications are found in connection with bull-dozers and scrapers. However, there is a great scope for future developments in the extension of hydraulic services to control the tractor or implement components. Another line of development is the transmission of power from the engine to the rear wheel of the tractor.

iii) **Road and Rail Traction**—Experiments were carried out in the past in connection with the use of hydraulic transmission of power for road and rail vehicles which achieved a fair degree of technical success, but their use was not made properly. Recently, there has been a revival of interest in the applications of past experimental results.

iv) **Lifting Equipment**—Though hydraulic driven cranes and lifts have been replaced by electric ones, yet modern types of transmission are used in a large number of comparatively novel lifting and tilting applications such as hydrostatic lifting and titling gear for fork-lift trucks, tilting furnaces, X-ray tilting table etc. In electric cranes the hydraulic control is applied to give stable creeping speeds when required, by automatically applying brakes to the required extent. Winches of helicopters and flight refuelling gear are often worked by hydraulic motors. Operation of penstock sluices and bulk-head doors are some more applications of the same.

HYDRO-KINETIC SYSTEMS

16.27 Hydro-Kinetic System and its Applications—The hydro-kinetic system is based upon changes in velocity of working fluid whereas the hydrostatic system is based upon differences of pressure. Hydro-kinetic system is used to transmit power. It is simply a combination of a centrifugal pump and a turbine. The former acts as driver and the latter as driven. The two distinct elements are build into a single unit with a closed hydraulic circuit. As there is no mechanical connection between the driver and the driven, impulsive shocks and periodic vibrations are prevented by using a fluid coupling.

The principle of hydro-kinetic system can be easily demonstrated by means of two ordinary electric fans which are set facing each other. If one of them is switched on, air currents will be generated moving the blades of the other which is not connected to electric mains.

Hydro-kinetic transmissions are of two types—hydraulic coupling and hydraulic torque convertor. The coupling provides for power transmission with the same torque on driving and driven shaft, whereas the converter provides for transmission with torque multiplication and could be compared with reduction gearing. Both the coupling and the converter are outgrowths of the original Foettinger transformer of Vulcan Co (Hamburg, W. Germany), a torque converter developed before World War I for marine use as a reduction gear between the steam turbine and propeller.

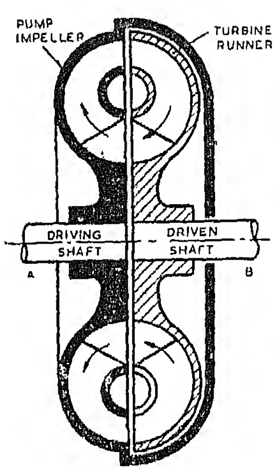


Fig 16.23 Fluid or Hydraulic Coupling

16.28 Fluid or Hydraulic Coupling—Fluid coupling consists of a radial pump impeller keyed to a driving shaft A (Fig 16.23), and a reaction (radial type) turbine runner keyed to driven shaft B. There is no mechanical or rigid connection between the driving and the driven shafts. The impeller and turbine runner together form a casing completely filled with oil as the fluid with which the shaft A and B are supposed to be coupled. The fluid is mostly ordinary mineral lubricating oil. Now if the shaft A is made to revolve slowly, the oil due to a forced vortex, will flow out from the impeller and will strike the turbine runner blades. After sufficient head has been built up by increasing the speed of A, the fluid will drive the turbine runner and thus set the shaft B in motion. When at full speed, the two shafts are supposed to be rotating at the same rate, but in practice, owing to slip, the driven shaft speed is about 2% less. Thus, the efficiency of the coupling would be 98%. If both driver and follower rotate at the same speed, the circulation of oil cannot take place. It is the difference of centrifugal forces set up in the driver and the follower which causes oil circulation. The necessary reduction in speed of the driven thus maintains the continuous flow of oil from impeller to the turbine runner. The blades of impeller and runner are generally of straight radial type.

Use : Fluid couplings are employed in rail road and automobiles to transmit power from the internal combustion engine to the moving wheel. Also, they are widely used for power driven excavators.

Fluid couplings having sizes from 1 to 36,000 HP have been manufactured. The larger couplings are built for the diesel engine driven warships.

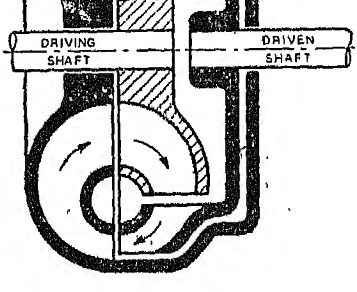


Fig 16.24 Fluid or Hydraulic Torque Converter

16.29 Fluid or Hydraulic Torque Converter—Fluid torque converter differs from the fluid coupling in that a third member generally known as a reactionary member (See Fig 16.24) is introduced between the pump impeller and the turbine runner. The function of the reactionary member is to change the direction of the fluid. It consists of a series of fixed guide vanes. As the fluid flowing out of the impeller rim strikes the turbine runner and blades mounted on the stationary housing, it releases its energy in the form of torque and speed. As a result of the fluid reacting upon the turbine and stationary blades, the input or engine torque is increasingly multiplied, and the speed of the output falls. Thus the stationary blades by redirecting the fluid flow multiply the

torque delivered by the engine and form a torque converter. Efficiency of a torque converter is very high at low speeds and may be as high as 85 to 87%. Fig 16.24 shows a fluid torque converter having a fixed speed ratio. However, with several stages of turbine runners and stationary vanes, a variable speed ratio can be obtained when a converter having a variable speed ratio is fitted to a road or rail vehicle driven by an internal combustion engine, it acts as a variable gear box. The required gear ratio is obtained by opening the throttle. This principle has also been applied in motor car drives. The converter provides for smooth starting and absorbs torsional vibrations and shocks.

16.30 Fluid Transmission of Power in Automobiles—With the advent of fluid couplings and torque converters, the traditional system of power transmission by means of clutch, gear box and rear axle has been dispensed with in modern cars. Now-a-days a pump or a series of small pumps enclosed in a casing attached to the engine have replaced the above mentioned gear drive. Engine drives the pumps which transmit high pressure oil to the rear wheels where it is used to cause rotation on the turbine principle. To obtain neutral gear, oil may be suitably by-passed without driving the rearwheels. Different gear ratios are achieved with the aid of two sets of fluid drives, one on each rear-axle shaft and three rotary sliding vane pumps of different capacities. To obtain different speeds and power, different combinations of three pumps and two drives may be used.

Modern automatic systems employ fluid torque converters in conjunction with three or four speed gear boxes. Mechanical pedal clutch may be dispensed with. Automatic control is effected by accelerator and foot brake pedal.

Fluid Brakes in Automobiles—Liquid medium is used to transmit and multiply the force applied for braking the wheels. The essential parts of the brake-system are the following :

i) Master cylinder with reserve tank,

ii) Brake or wheel cylinder,

iii) Brake-pedal link mechanism and

iv) Piping for liquid transmission.

When the brake pedal is depressed, it moves the piston inside the master cylinder and displaces liquid to the four wheel cylinders which brake their respective wheels. As the pressure on foot pedal is released, brake shoes return to their original positions by means of releasing springs.

UNSOLVED PROBLEMS

16.1 What are hydraulic systems ? Describe briefly a hydrostatic and hydro-kinetic system and cite examples.

16.2 What is the difference between a constant delivery and a variable delivery pump ? Describe one from each type.

16.3 What are rotary pumps and how are they classified ?

16.4 What is a gear pump and state the difference between an internal and an external type of such pump.

16.5 Describe with sketches the working of

a) Hydraulic Jack *c*) Hydraulic Crane
b) Hydraulic Press *d*) Hydraulic Lift

(*Madras University*)

16.6 Describe with the aid of neat sketches the working of a hydraulic intensifier. (*Jadavpur University—1954*)

16.7 Write brief notes on—

a) Intensifier *b*) Hydraulic Accumulator

c) Application of fluid pressure in machine tools.

16.8 What are the advantages of hydraulic drive in machine tools? Describe one such drive.

16.9 What is the difference between a hydraulic coupling and a hydraulic torque converter? Where are they used in practice?

16.10 A gearwheel type of positive-rotary pump has two identical rotors or gearwheels each 6·30 in. long with a pitch diameter of 3·15 in. When running at a speed of 500 rpm, it delivers 43·9 gallons of oil per minute against a total head of 125 ft.

Draw graphs, approximately to scale, showing how this pump would behave as follows: (*i*) between speed in rpm and discharge in gal/min, for the range 300 to 800 rpm against a constant head of 125 ft (*ii*) between discharge in gal/min and total head in ft, when running at a steady speed of 500 rpm.

Then estimate the size of a pump geometrically similar to the specified pump, such that it would deliver 147 gallons of oil per minute against a head of 49 ft when running at 360 rpm. The rotor pitch diameter and length should be stated.

(*AMI Mech E* (*Lond*)*—Oct 1958*)

16.11 The ram for a hydraulic crane has diameter of 6 in. and the ratio between the movement of the load and ram is 6 to 1. Water is supplied through a $1\frac{1}{2}$ in. diameter pipe having a length of 1,600 ft, the pressure at the inlet end of the pipe being 1,100 lb/sq in. The co-efficient of friction for the pipe is 0·01. A pressure of 85 lb/sq in. on the ram is required to overcome mechanical losses.

Determine (*a*) the maximum speed with which a load of 2,500 lb can be lifted and (*b*) the load and speed of lifting which correspond to the maximum power obtained from the crane.

(4·45 ft/sec, 3,180 lb, 3·7 ft/sec) (*Lond University—July 1949*)

16.12 An intensifier has a ram diameter of 6 in. and sliding cylinder or piston diameter of 30 in. Calculate the pressure of water on the low pressure side of the intensifier if the pressure of water on the high pressure side is to be 3,000 lb per sq in. The loss due to friction at each of the packings of the intensifier is 5% of the total pressure on each piston.

(132·3 psi) (*Madras University—Sept 1953*)

16.13 Describe the working of an accumulator. The ram of an accumulator is loaded with 150 tons. It supplies power through a 4 in. diameter pipe, 5,000 ft long. The loss in the pipe is estimated to be 2% of the power supplied by the accumulator. The co-efficient of friction in the pipe is 0·01. The diameter of the ram, working frictionless is $2\frac{1}{2}$ ft. What is the maximum horse power it can supply? (16·33 HP) (*Poona University—1958*)

16.14 230 HP is to be conveyed to several hydraulic machines at a distance of 2 miles from the accumulator by means of pipes 6 in. diameter. The water pressure in the accumulator is 700 lb per sq in. Find the least number of pipes if the efficiency of transmission is not to be less than 92%. Take $f=0.007$ for the pipes.
(2) (*Aligarh University—1953*)

16.15 Calculate the displacement of the ram of a hydraulic crane, whose efficiency is 50% in order that with a water pressure of 700 lb per sq in. it may raise a load of 25 tons. Find the diameter of the ram, if the stroke is 6 times the diameter.
(7·93 cu ft ; 1 ft 2·28 in. Dia) (*Annamalai University—1953*)

16.16 *a*) Describe with sketches any type of hydraulic accumulator or intensifier.

b) An accumulator has a ram 1 ft diameter and a stroke of 16 ft. It is loaded with 80 tons but friction of the gland consumes 1·5% of this load. If the ram falls steadily in 2 minutes while the pumps are delivering 200 gpm what HP is being delivered to the machinery by the hydraulic system ?
(261·5 HP) (*Bombay University—1957*)

16.17 A hydraulic lift raises a load of $8\frac{3}{4}$ tons at a speed of 1·8 ft/sec through a height of 45 ft once in 1·6 minutes, being worked from an accumulator which is fed by a pump having an efficiency of 80%. If the pressure of the water is 500 lb per sq in. and the efficiency of the lift 70%, find the HP required to drive the pump and the minimum capacity of the accumulator.
(29·85 HP ; $9{\cdot}33 \times 10^5$ ft lb) (*Madras University—1956*)

16.18 An intensifier of 4 in. diameter ram and 42 in. diameter piston is connected to a press having a ram 12 in. diameter. Water is supplied to the intensifier from a tank 40 ft above the intensifier through 2 in. diameter pipe, 200 yds long, $f=0.008$. Calculate the speed of advance of the press ram in inches per min when exerting a force of 60 tons.
(58·2 in./min) (*Madras University*)

16.19 A hydraulic crane lifts 20 tons when working at a pressure of 800 lb per sq in. The rate of lifting is 60 ft per minute and total lift is 30 ft. The crane works once every two minutes. The efficiency of the crane is 0·8. Calculate the minimum HP of the pump and minimum capacity of the accumulator which is also provided. What is the volume of the crane cylinder ?
(25·45 HP ; 1,260,000 ft lb, 11·65 cu ft) (*Gujrat University—1953*)

16.20 Describe with the aid of suitable sketches the principle of working of a hydraulic crane. Under what circumstances would you recommend the use of such a crane ?

A hydraulic crane has a ram 9 in. diameter and water is supplied to the crane at 660 lb per sq in. at the rate of 0 03 cusecs. The pulley system of the crane has a velocity ratio of 6. If the height to which the load is to be lifted is 45 ft, find the load that can be lifted and the time taken to lift the same.

Assume an overall efficiency of 71%.
(4,970 lb, 110 sec) (*Roorkee University—1958*)

16.21 Describe with sketches the working of the hydraulic accumulator.

Certain machinery is worked from a weight loaded accumulator through a pipe line 3,000 ft long and 4 in. diameter. The accumulator has a ram of 15 in. diameter and 12 ft stroke. It is loaded with 47 tons and is supplied with water by a three throw pump having a diameter of 4 in., stroke 8 in. and speed 50 rpm. The slip may be estimated at 3% and the pipe co-efficient 0·007. If the machinery absorbs 60 WHP, calculate the longest period during which it may be operated continuously.

(One Minute) (*Bombay University—1957*)

16.22 In an installation of hydraulic cranes, the cranes are supplied with water at a pressure of 50 kg per cm². Each crane is required to raise a load of 5 tons (5000 kg) at a speed of 18 meters per minute through a total height of 12 meters. The system of ropes and pulleys provides a velocity ratio of 6. Assuming an efficiency of 60%, estimate the diameter and stroke of the crane hydraulic motor.

There are five cranes in the installation and the working cycle of each crane occupies 90 seconds. If it be assumed that all the five cranes be making their working strokes simultaneously, calculate the minimum capacity in *litres* per second of the pump feeding the installation and the minimum capacity in litres of the accumulator required.

(35·7 cm dia ; 2 m stroke ; 11·11 lit/sec ; 555·5 litres)

(*Bombay University—1957*)

16.23 In a testing machine the force is applied to the specimen by hydraulic pressure on a ram of 10 in. diameter. The maximum force required is 100 tons, and the frictional resistance at the ram may be taken as an additional 5 tons. The water is supplied from an accumulator consisting of a vertical cylinder with a loaded plunger, 4 5 in. diameter. Find the necessary load on the plunger if friction there causes a resistance of 2 tons to its motion. If the water for the accumulator is supplied by a pump with an efficiency of 0 85 and the friction force at the plunger has same value when the accumulator is receiving water, calculate the overall efficiency of the arrangement. Find this efficiency when the force required is 50 tons. (23·26 tons, 0 681 and 0·34)

16.24 The ram of a hydraulic accumulator is 18 inches (or 457·2 mm) in diameter. The stroke is 23 ft (or 7 02 m) and the water pressure is 1,120 lb/sq in. (or 78 8 kg/cm²). If the useful work given by the accumulator during one full downward stroke is utilized in raising W tons to a height of 45 ft (or 13 7 m) by means of a hydraulic crane, whose efficiency is 60%, find the value of W. If this work is done in 3 minutes, what is the gross horsepower of the crane ?

[87,400 lb (or 39,000 kg) ; 66 2 HP (or 67 2 metric HP)]

(*Utkal University—1958*)

APPENDIX I

Conversion Factors

(See also Chapter on Introduction to Metric Units)

Measure	FPS Unit	FPS → To Metric	Metric Unit	Metric → to FPS
Length (or Linear)	inch (")	× 25·4	mm $= \frac{1}{1000}$ m	× 0·03937
	foot (')=12"	× 0·3048	m	× 3·28
	yard =3'	× 0·9144	m	× 1·09361
	mile =1,760 yd	× 1·6093	km=1,000 m	× 0·621
	nautical mile	× 1·852	km=1,000 m	× 2·510
Area (or square)	sq in.	× 6·451	cm²	× 0·1550
	sq ft	× 0·0929	m²	× 10·764
	acre	× 0·4047	hectare	× 2·471
Volume (or cubic)	cu in.	× 16·3871	cm³	× 0·0610
	Imp gallon (=1·2 US gallon)	× 4·546	litre=(dm)³	× 0·220
	Imp gallon (=10lb)	× 0·004546	m³	× 220
	US gallon	× 3·785	litre	× 0·2642
	Petroleum barrel =42 US gallons	× 1·59	hl	× 0·63
	cu ft	× 28·316	litre	× 0·0353
	cu ft	× 0·0283	m³	× 35·38
Weight	grain $= \frac{1}{7000}$ lb	× 0·0648	gr	× 15·43
	ounce $= \frac{1}{16}$ lb	× 28·35	gr	× 0·035
	pound=1 lb	× 0·4536	kg	× 2·205
	cwt=112 lb	× 50·802	kg	× 0·0197
	short ton or US ton =2,000 lb	× 0·907	tonne	× 1·102
	long ton=2,240 lb	× 1·016	tonne	× 0·98421
Pressure and Head	lb/sq in. $\left(= \frac{\text{ft of water}}{2·31}\right)$	× 0·0703	kg/cm² $\left(= \frac{\text{m of water}}{10}\right)$	× 14·22
	lb/sq in.	× 0·068	atm	× 14·7
	long ton/sq in.	× 157·5	kg/cm²	× 0·00635
	in. mercury $\left(= \frac{\text{ft of water}}{1·13}\right)$	× 245	mm of water	× 0·0029
	in. mercury	× 25·4	mm of mercury	× 0·03937
	1 atm. (=34 ft of water=14·7 lb/sq in.)	× 1·033	1 atm. (=10 m of water or 1 kg/cm²)	× 0·967

Measure	FPS Unit	FPS → to Metric	Metric Unit	Metric → to FPS
Density	lb/cu ft	× 16 02	kg/m³	× 0·0625
	lb/cu ft	× 0·01602	kg/dm³ or kg/litre	× 62 424
	lb/in.³	× 27 6799	kg/cm³	× 0 0362
Moment of Inertia	lb ft² (WR^2)	× 0 1686	kg m² (GD^2)	× 5·933
Velocity	ft/sec	× 0 3048	m/sec	× 3 28
	ft/min	× $\frac{5{\cdot}08}{1,000}$	m/sec	× 196 85
Viscosity	ft units $\left(=\frac{\text{lb sec}}{\text{ft}^2}\right)$	× 479	Poise $=\left(\frac{\text{dyne . sec}}{\text{cm}^2}\right)$	× $\frac{2{\cdot}083}{1,000}$
Kinematic viscosity	ft units (= ft²/sec)	× 929	Stokes (= cm²/sec)	× $\frac{1\ 076}{1,000}$
Temperature	°F	× $\frac{5}{9}$(°F - 32)	°C	× $\frac{9}{5}$ °C+32
Rate of	cu ft/sec (cusec)	× 0 0283	m³/sec	× 35·38
Discharge	cu ft/sec (cusec)	× 28·316	lit/sec (= dm³/sec)	× 0 0353
	cu ft/min	× 0 472	lit/sec (= dm³/sec)	× 2 22
	gal/min (gpm)	× 0·0757	lit/sec (= dm³/sec)	× 13 22
	gal/min (gpm)	× $\frac{0{\cdot}757}{10,000}$	m³/sec	× 13,220
Energy	ft lb	× 0 1382	m kg	× 7 23
(or Power)	HP (= 550 ft lb)	× 1 014	HP (= 75 kgm/sec)	× 0·986
	1 KW (= 1 341 HP)	× 1	1 KW (= 1·36 HP)	× 1
Heat	B T U	× 0·252	k cal	× 3 968
	B T U	× 107·6	m kg	× $\frac{9\ 29}{1,000}$
	B T U/sq ft	× 2·7126	k cal/m²	× 0·3687

APPENDIX II

Equivalents for Hydraulic Calculations

1 Imperial gallon	=10 lb
	=0 16 Cubic Foot
	=277 cubic inches
	=1 2 US gallons
	=*4 56 litres*
1 U S. gallon	=8 33 lb
	=0 83 Imperial gallon
	=0 133 cubic foot
	=231 cubic inches
	=*3 8 litres*
1 Cubic Foot of Water	=6 23 Imperial gallons
	=7 48 U.S. gallons
	=*28 375 litres*
	=*0·0283 cubic metre*
	=62·4 lb fresh water
	=64 lb salt water
	=0·557 cwt
	=0 028 ton
1 Cusec	=1 cubic foot per second
	=375 gallons per minute
	=*0 0283 cu m per sec*
1 Cubic Metre of water	=*1 kilogram*
	=*1,000 litres*
	=220 Imperial gallons
	=264 US gallons
	=35·3 cubic feet
	=1 ton (approx)
1 Ton (=2,240 lb)	=36 cubic feet (fresh water)
	=224 Imperial gallons (fresh water)
	=268·8 US gallons (fresh water)
	=35 cubic feet (salt water)
	=218 Imperial gallons (salt water)
	=*1,000 litres (approx)*
	=*1 cubic metre (approx)*
1 Litre of water	=2·2 lb
	=0 22 Imperial gallon
	=0 264 US gallon
	=0·0353 cubic foot
	=61 cubic inches
1 Atmosphere	=14 7 lb per square inch
	=30 inches of mercury
	=34 ft head of water
1 Metric Atmosphere	=*1 kilogram per square centimetre*
	=14·22 lb per square inch
	=*735 millimetres mercury*
	=29 inches mercury
	=*10 metres head of water*
	=32·8 feet head of water
1 Horsepower	=550 ft lb per second
	=33,000 ft lb per minute
1 Metric Horsepower	=*75 kilogram metres per second*
	=0·9863 British horsepower
1 Gravitational Acceleration (g)	=32·2 feet per second per second.
	=*9·81 metres per second per second*

APPENDIX III

Mensuration of Surfaces and Solids

Circumference of a circle	$= \pi \times$ diameter
Area of a circle	$= \frac{\pi}{4} \times (\text{diameter})^2$
Area of sector of circle	$= \frac{\text{radius}}{2} \times$ length of arc
Area of square, parallelogram, rectangle rhombus or rhomboid	$=$ base $\times$ height
Area of trapezium	$= \frac{1}{2}$ sum of two parallel sides $\times$ height
Area of regular polygon	$= \frac{1}{2}$ radius of inscribed circle $\times$ length of one side $\times$ number of sides
Area of triangle	$= \frac{1}{2}$ base $\times$ altitude
Area of parabola	$= \frac{2}{3} \times$ base $\times$ altitude
Area of ellipse	$= \frac{\pi}{4} \times$ major axis $\times$ minor axis
Area of cycloid	$= 3 \times$ area of generating circle
Surface area of sphere	$= \pi \times (\text{diameter})^2$
Volume of sphere	$= \frac{\pi}{6} \times (\text{diameter})^3$
Volume of parabolid	$= \frac{1}{2}$ volume of circumscribing cylinder.
Volume of prism	$=$ area of base $\times$ altitude
Volume of cylinder	$= \frac{\pi}{4}$ (diameter)$^2 \times$ height
Volume of pyramid	$= \frac{1}{3} \times$ area of base $\times$ height
Volume of cone	$= \frac{1}{3} \times \frac{\pi}{4} \times$ (diameter of base)$^2 \times$ height
Length of arc	$=$ Radius of circle $\times$ angle subtended at the centre in radians $(= r \cdot \theta)$
One radian	$= 57{\cdot}5°$

APPENDIX IV

Fractional Sub-divisions of an Inch to Decimal of an Inch

$\frac{1}{64}$	0·015625	$\frac{21}{64}$	0·328125	$\frac{41}{64}$	0·640625
$\frac{1}{32}$	0·03125	$\frac{11}{32}$	0·34375	$\frac{21}{32}$	0·65625
$\frac{3}{64}$	0·046875	$\frac{23}{64}$	0·359375	$\frac{43}{64}$	0·67185
$\frac{1}{16}$	0·0625	$\frac{3}{8}$	0·375	$\frac{11}{16}$	0·6875
$\frac{5}{64}$	0·078125	$\frac{25}{64}$	0·390625	$\frac{45}{64}$	0·703125
$\frac{3}{32}$	0·09375	$\frac{13}{32}$	0·40625	$\frac{23}{32}$	0·71875
$\frac{7}{64}$	0·109375	$\frac{27}{64}$	0·421875	$\frac{47}{64}$	0·734375
$\frac{1}{8}$	0·125	$\frac{7}{16}$	0·4375	$\frac{3}{4}$	0·75
$\frac{9}{64}$	0·140625	$\frac{29}{64}$	0·453125	$\frac{49}{64}$	0·765625
$\frac{5}{32}$	0·15625	$\frac{15}{32}$	0·46875	$\frac{25}{32}$	0·78125
$\frac{11}{64}$	0·171875	$\frac{31}{64}$	0·484375	$\frac{51}{64}$	0·796875
$\frac{3}{16}$	0·1875	$\frac{1}{2}$	0·5	$\frac{13}{16}$	0·8125
$\frac{13}{64}$	0·203125	$\frac{33}{64}$	0·515625	$\frac{53}{64}$	0·828125
$\frac{7}{32}$	0·21875	$\frac{17}{32}$	0·53125	$\frac{27}{32}$	0·84375
$\frac{15}{64}$	0·234375	$\frac{35}{64}$	0·546875	$\frac{55}{64}$	0·859375
$\frac{1}{4}$	0·25	$\frac{9}{16}$	0·5625	$\frac{7}{8}$	0·875
$\frac{17}{64}$	0·265625	$\frac{37}{64}$	0·578125	$\frac{57}{64}$	0·890625
$\frac{9}{32}$	0·28125	$\frac{19}{32}$	0·59375	$\frac{29}{32}$	0·90625
$\frac{19}{64}$	0·296875	$\frac{39}{64}$	0·609375	$\frac{59}{64}$	0·921875
$\frac{5}{16}$	0·3125	$\frac{5}{8}$	0·625	$\frac{15}{16}$	0·9375
				$\frac{61}{64}$	0·953125
				$\frac{31}{32}$	0·96875
				$\frac{63}{64}$	0·984375

APPENDIX V

a) Bernoulli's Theorem with regard to Absolute Motion

This important theorem is proved and derived in several ways. The following proof is based on consideration of momentum.

Consider a small element of fluid of length ds, breath b and thickness l and $(l+dl)$ at the two ends (See Fig (V.1). This element is a part of a stream tube.

Forces on the element are the fluid pressure and gravitation.

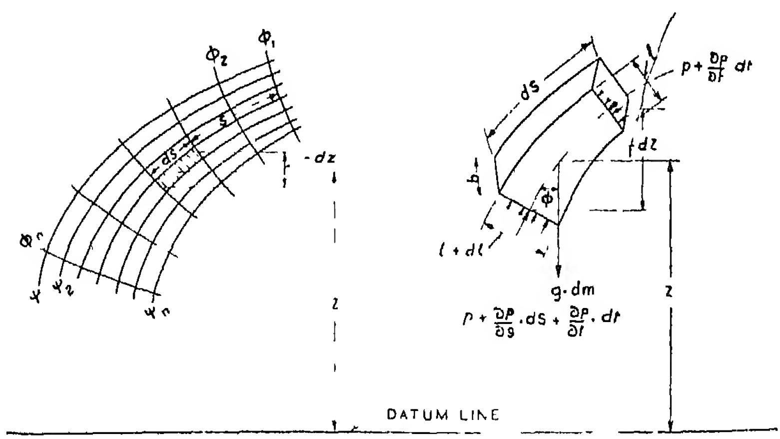


Fig V.1 Absolute Motion of a Fluid Particle

If p be the initial pressure at the right end at any given time, then after a time dt,

Pressure at right end $=p+\frac{\partial p}{\partial t}\,.\,dt$

Pressure at left end $=p+\frac{\partial p}{\partial s}\,.\,ds+\frac{\partial p}{\partial t}\,.\,dt$

For steady flow, $\frac{\partial p}{\partial t}=0$,

$\therefore$ Pressure at right end $=p+0=p$

Pressure at left end $=p+\frac{\partial p}{\partial s}\,.\,ds+0$

$=p+\frac{dp}{ds}\,.\,ds=p+dp$

Acceleration of fluid may be easily calculated

Let, $v=f(t, s)$

Then, $dv=\frac{\partial v}{\partial t} \cdot dt+\frac{\partial v}{\partial s} \cdot ds$

$$\therefore \quad \frac{dv}{dt}=\frac{\partial v}{\partial t}+\frac{\partial v}{\partial s} \cdot \frac{ds}{dt}$$

$$=\frac{\partial v}{\partial t}+v \cdot \frac{\partial v}{\partial s}$$

For steady flow, $\frac{\partial v}{\partial t}=0$

$$\therefore \quad \frac{dv}{dt}=v \cdot \frac{\partial v}{\partial s}=v \cdot \frac{dv}{ds}$$

Net external force on the element in the direction of motion

$$=w \cdot b \cdot l \cdot ds \cdot \cos\phi+p \cdot b \cdot l-(p+dp) \cdot b \cdot (l+dl)$$

$$=w \cdot b \cdot l \cdot ds \cdot \cos\phi-dp \cdot b \cdot l \quad \ldots\ldots\left(\text{neglecting } \frac{dl}{l}\right)$$

This must be equal to the rate of change of momentum = mass × acceleration

$$=\frac{w}{g} \cdot (b \cdot l \cdot ds) \cdot v \cdot \frac{dv}{ds}$$

$$\therefore \quad \frac{w}{g} \cdot b \cdot l \cdot ds \cdot v \cdot \frac{dv}{ds}=w \cdot b \cdot l \cdot ds \cdot \cos\phi-dp \cdot b \cdot l$$

or $\frac{v \cdot dv}{g}=ds \cdot \cos\phi-\frac{dp}{w}$

$$=-dz-\frac{dp}{w} \quad \ldots\ldots(\text{since } ds \cdot \cos\phi=-dz)$$

Integrating,

$\frac{v^2}{2g}+\frac{p}{w}+z=c$ (Assuming that w is constant, i.e. fluid is incompressible). ...(V.I)

This holds good for potential flow of ideal fluid. In practice, frictional losses have to be considered, and then

$$\frac{v_2^2}{2g}+\frac{p_2}{w}+z_2=\frac{v_1^2}{2g}+\frac{p_1}{w}+z_1-H_{L(1-2)} \quad \ldots(\text{V.I}a)$$

(b) Bernoulli's Equation for Relative Motion

Consider the motion of fluid inside a turbine runner or an impeller of a centrifugal pump. Let its angular velocity ω be constant, and let relative velocity be w.

Again take an element similar to the one chosen in the previous case.

Let the angles which the directions of vertical gravitational force and radial centrifugal force make with the tangent to the stream line at the point under consideration be denoted by ϕ and ψ respectively.

Let the weight density *i.e.*, specific weight of fluid be denoted by w_d to distinguish it from relative velocity w and the angular velocity ω.

For steady flow,

$$\frac{\partial p}{\partial t}=0 \; ; \qquad \frac{\partial w}{\partial t}=0$$

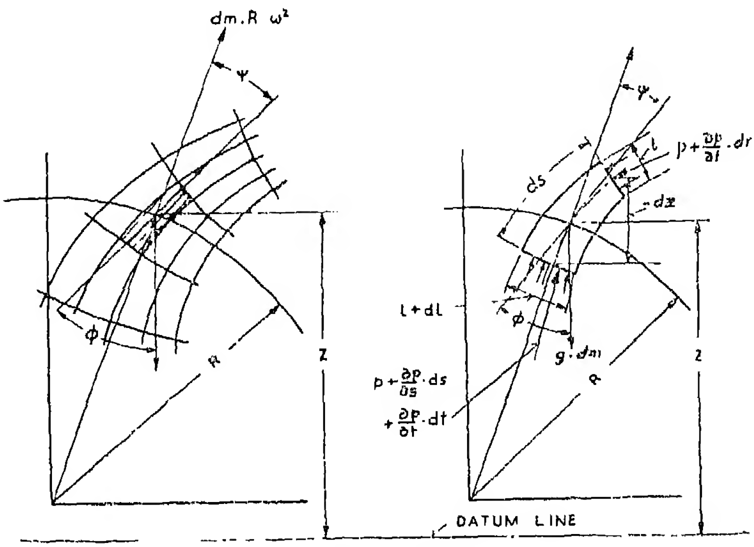


Fig V.2 Relative Motion of a Fluid Parcticle etc.

Acceleration in the direction of relative velocity w,

$$\frac{dw}{dt}=\frac{\partial w}{\partial t}+w \, . \, \frac{\partial w}{\partial s_w}$$

$$=w \, . \, \frac{\partial w}{\partial s_w}=w \, . \, \frac{dw}{ds_w}$$

Mass of fluid accelerated $=\frac{w_d}{g} \, . \, (b \, . \, l \, . \, ds_w)$

Forces acting on the element are :

a) weight $=w_d \, . \, b \, . \, l \, . \, ds_w$

b) centrifugal force $=\frac{w_d}{g} \, . \, b \, . \, l \, . \, ds_w \, . \, R \, . \, \omega^2$

c) pressure difference $=b \, . \, l \, . \, p-(p+dp) \, . \, b(l+dl)$

$$=-b \, . \, l \, . \, dp \ldots\ldots \left(\text{neglecting } \frac{dl}{l}\right)$$

Now, resultant force in the direction of stream line
$= \text{mass} \times \text{acceleration}$

$$\therefore \quad w_d \cdot b \cdot l \cdot ds_w \cdot \cos\phi - b \cdot l \cdot dp - \frac{w_d \cdot b \cdot l \cdot ds_w \cdot R \cdot \omega^2}{g} \cdot \cos\psi$$

$$= \frac{w_d}{g} \cdot b \cdot l \cdot ds_w \cdot w \cdot \frac{dw}{ds_w}$$

Simplifying,

$$w \cdot \frac{dw}{g} = ds_w \cdot \cos\phi - \frac{dp}{w_d} - \frac{R \cdot \omega^2}{g} \cdot ds_w \cdot \cos\psi$$

Substitute,

$$ds_w \cdot \cos\phi = -dz\,; \quad \text{and } ds_w \cdot \cos\psi = -dR$$

Then $\dfrac{w \cdot dw}{g} = -dz - \dfrac{dp}{w_d} + \dfrac{R \cdot \omega^2}{g} \cdot dR$

or $\quad w \cdot \dfrac{dw}{g} + dz + \dfrac{dp}{w_d} - \dfrac{R \cdot \omega^2 \cdot dR}{g} = 0$

Integrating,

$$\frac{w^2}{2g} + z + \frac{p}{w_d} - \frac{R^2 \cdot \omega^2}{2g} = c$$

Since $R \cdot \omega =$ circumferential velocity u, final form of Bernoulli's Equation for relative motion would be

$$\frac{w^2}{2g} + z + \frac{p}{w_d} - \frac{u^2}{2g} = c \qquad \ldots(\text{V.II})$$

for ideal fluid and ideal flow.

In practice, considering frictional and other losses,

$$\frac{w_2^2}{2g} - \frac{u_2^2}{2g} + \frac{p_2}{w_d} + z_2 = \frac{w_1^2}{2g} - \frac{u_1^2}{2g} + \frac{p_1}{w_d} + z_1 - H_{L(1-2)} \qquad \ldots(\text{V.II}a)$$

Bernoulli's theorem thus proved applies strictly to a single stream line. In problems of practical importance, as flow in pipes, however, the stream is made up of an infinite collection of stream lines or stream tubes. In general, the velocity varies from point to point in a cross-section of such a stream and the expression usually yields the average or the mean velocity.

Even under such circumstances Bernoulli's theorem can often be applied without introducing any grave error because the kinetic head, in practice, is small compared to other heads.

If kinetic head is considerable, it is expressed as $\alpha \times \dfrac{(\overline{v})^2}{2g}$, where $\overline{v}$ is the average velocity $= \dfrac{Q}{A}$, and α is a correction factor varying from 1 to 2 (See Fig V.3).

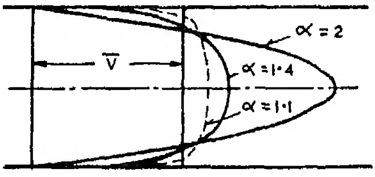


Fig V.3 Velocity Distribution Over the Section of Pipe.

$\alpha = 1$, when velocity is constant over the cross-section.

$\alpha = 2$, when velocity varies parabolically from zero at the boundary to a maximum at the centre, as in pipes.

APPENDIX VI

Combined Formulae for Turbines and Pumps

Turbine (Reaction Type)	Pump (Centrifugal Type)
1) Generally inward flow (centripetal type)	Outward flow (centrifugal type)
2) Work done/sec $=\frac{w \cdot Q}{g}(v_{u_1} \cdot u_1 - v_{u_2} \cdot u_2)$	Work consumed/sec $=\frac{w \cdot Q}{g}(v_{u_2} \cdot u_2 - v_{u_1} \cdot u_1)$
3) $\alpha_2=90°$ (*i.e.* radial discharge)	$\alpha_1=90°$ (*i.e.* radial inlet)
4) $\therefore$ W.D./sec$=\frac{w \cdot Q}{g} v_{u_1} \cdot u_1$	$\therefore$ W.C./sec$=\frac{w \cdot Q}{g} v_{u_2} \cdot u_2$
5) W.D./sec/lb (or kg) of water $=\frac{v_{u_1} \cdot u_1}{g}$ (Fundamental equation of reaction turbine)	W.C./sec/lb (or kg) of water $=\frac{v_{u_2} \cdot u_2}{g}$ (Fundamental equation of centrifugal pump)
6) Head utilised by turbine$=H$	Head supplied by pump$=H=H_{mano}$ *i.e.* Head shown by gauges.
7) $H=H_{static}-\Sigma H_L$ (Losses outside the turbine)	$H_{mano}=H+\Sigma H_L$ (Losses outside the pump)
8) $H-\triangle H=\frac{v_{u_1} \cdot u_1}{g}$ ($\triangle H$=Losses inside the turbine)	$H_{mano}+\triangle H=\frac{v_{u_2} \cdot u_2}{g}$ ($\triangle H$=Losses inside the pump)
9) $\eta_H=\frac{v_{u_1} \cdot u_1}{gH}$	$\eta_{mano}=\eta_H=\frac{g \cdot H_{mano}}{v_{u_2} \cdot u_2}$
10) $Q=\pi \cdot D_1 \cdot B_o \cdot v_{m_1}$ (neglecting thickness of vanes) $=(\pi \cdot D_1 - z_2 \cdot t) \cdot B_o \cdot v_{m_1}$ (taking the thickness of vanes into account)	$Q=\pi \cdot D_1 \cdot B_1 \cdot v_{m_1}$ $=\pi \cdot D_2 \cdot B_2 \cdot v_{m_2}$ (neglecting thickness of vanes) $=(\pi \cdot D_1 - Z \cdot S_1) \cdot B_1 \cdot v_{m_1}$ $=(\pi \cdot D_2 - Z \cdot S_2) \cdot B_2 \cdot v_{m_2}$ (taking the thickness of vanes into account)

If Positive (+) sign is used for Pumps
Negative (−) sign is used for Turbines.

a) Head $H=H_{stat}\pm\Sigma H_L$ $\quad H_{th}=H\pm\triangle H$

b) Power $P_t=\frac{w}{550}(Q\pm\triangle Q)\cdot H_{th}\pm\triangle P_{mech}$

c) Efficiencies $\eta_{th}=\left(\frac{H}{H_{th}}\right)^{\pm 1}$

$$\eta_{mech}=\left(\frac{P_h}{P_t}\right)^{\pm 1}=\left(\frac{P_t+\triangle P_{mech}}{P_t}\right)^{\pm 1}$$

$$\eta_t=\left(\frac{w \cdot Q \cdot H}{550 \cdot P_t}\right)^{\pm 1}$$

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